TACTOMETRIA.

SEU,

TETAGMENOMETRIA.

OR,

The GEOMETRY of Regulars

practically proposed; After a new, most artificial, exact and expeditious manner (together with the natural or vulgar, by way of mensurall Comparison) And in the *Solids*, not only in respect of Magnitude or Dimension, but also of Gravity or ponderosity, according to any Metal assigned. Together with severall usefull Observations and Experiments falling in by the way, concerning Measure and Weight.

And withall, the like artificial practical *Geometry* of regular-like Solids (as I term them) in both the foresaid respects: And moreover, of a Cylindricall Body, for liquid or Vessell-Measure (commonly called by the name of Gauging) as is for solid measure; it being therefore a most exact and expedite way of Gauging: With sundry new and exact Experiments, Observations, and Rules concerning the same.

And lastly, an A-TACTOMETRIE, or an APPENDIX, for the most ready and exact discovering of the dimensionall quantity of any irregular kind of Body, whether solid or concave, which in it selfe will not admit of an ordinary or orderly way of measuring. And this from certain new and exact Experiments also, made for that purpose.

A Work very usefull and delightfull for all such as are either ingenuously studious of, or necessarily exercised and employed in the practice of the Art METRICALL.

By J. W[illiam]

LONDON,
Printed by ROBERT LEYBOURN, for *Nathaniel Brooks*, at the Angel in *Cornhill*, *MDCL.*

Art
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And withall, the like artificial practical *Geometry* of regular-like Solids (as I term them) in both the foreſaid reſpects: And moreover, of a Cylindricall Body, for liquid or Veſſell-Meaſure (commonly called by the name of Gauging) as is for ſolid meaſure; it being therefore a moſt exact and expedite way of Gauging: With ſundry new and exact Experiments, Obſervations, and Rules concerning the ſame.

And laſtly, an A-TACTOMETRIE, or an APPENDIX, for the moſt ready and exact diſcovering of the dimenſionall quantity of any irregular kind of Body, whether ſolid or concave, which in it ſelfe will not admit of an ordinary or orderly way of meaſuring. And this from certain new and exact Experiments alſo, made for that purpoſe.

A Work very uſefull and delightfull for all ſuch as are either ingenuouſly ſtudious of, or neceſſarily exerciſed and employed in the practice of the Art METRICALL.

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Ὁ Θεὸς ἀεὶ γεωμετρεῖ: Vel, ὁ Θεὸς ἀεὶ πάντων μάλιστα γεωμετρεῖ. PLATO. De quo PLUTARCHUS *lib.* 8 Συμποσιακῶν, *prob.* 2.

Omnia quæcunq; à primæva rerum natura conſtructa ſunt, numerorum videntur ratione formata: Hoc enim fuit principale in animo Conditoris exemplar. BOETIUS Arithm. lib. 1. cap. 2.

Veniet tempus quo iſta quæ nùnc latent, in lucem dies extrahat, & longioris ævi diligentia. SENECA, nat. quæſt. lib. 7. cap. 25.

Eſt natura hominum novitatis avida. PLINIUS.

TO

All that are wel-willers to the Mathematiques, in generall: But more eſpecially to the candid, impartiall, intelligent, and practicall Reader.

THere is now two Luſtres, or one Decade of years elapſed, and ſome more time, (gentle and courteous Reader,) ſince I hapned firſt to hit upon this artificiall kind of Menſuration (or more artificiall practicall Geometry for regular and regular-like Magnitudes,) which I here deliver: and which was then but in two or three Particulars thereof, here firſt of all laid down in the three principall Propoſitions contained in the firſt Part, and the ſame demonſtrated practically. And ſo having ſince by degrees, very

much enlarged my Conceits & inventions herein (and indeed as far I think as poſſibly may be, & that only by way of Mathematical exercitation & recreation from other Studies and employments) I thought good at length, thus to put them together, (with other things by the way, pertinent thereto,) and ſo to exhibite and expoſe them to a publicke View, examination and triall in generall, and in ſpeciall to thy candid and courteous cenſure: which I was the more emboldened and encouraged to do, conſidering that when at firſt I propounded thoſe things which I had then conceived in this way, to ſeverall able Artiſts, not only in ſeverall parts of this Kingdome where I happened then to come, and with whom I had the opportunity to converſe, but alſo in ſome parts beyond the Sea (where not long after I fortuned to ſpend ſome time, for the proſecuting of other Studies, which I then cheifly aymed at and intended, and have ſince for the part followed and embraced) moſt of which were Profeſſours and Teachers of the Mathematicks, and ſo ſuch as are uſually ſooneſt acquainted with all the new mathematicall inventions that are any way made known) I found that it was to them a meer novelty, (according as I conceived it would be) and that moreover at firſt they ſomewhat doubted

doubted of the ſame, ſaying, that it were indeed an excellent way, if it would generally certain and true; and ſo to ſome others of leſſe judgement, it ſeemed to be a thing ſo very improbable (they having made no triall thereof) as that they would ſuddenly and unadviſedly conclude it to be impoſſible, only that it might hit right now and then by chance, but not conſtantly.

And then beſides this, I having for my further ſatisfaction herein, taken the pains (both at firſt, and alſo again of late) to make a ſtrict ſearch and enquiry into all the chiefeſt Authours which have hitherto treated of practicall Geometry either in Latin or Engliſh, and more eſpecially for inſtrumentall practice, in the way of manuall or mechanicall Menſuration; could not find the way here propoſed, ſo much as barely hinted by any of them in the leaſt kind. And ſo have I now here at length (by the favour and permiſſion of *God*) according to my earneſt deſire, brought that to a Generall, which at firſt I had conceived or apprehended only in a few Particulars, and ſo have compleated the Invention, as that I may think there is hardly any room left for additi on thereunto.

Here then firſt (friendly and ingenious Reader) ſhalt thou find the moſt artificiall and exquiſite quadrature of a Circle, in a practicall, or organi-

organicall way, as to the immediate obtaining of it's *Area* in any meaſure appointed. For as *Joſeph Scaliger* ſaith of the ſquaring of a *Circle* in a generall way, (or of the generall quadrature of a *Circle*) *Elem. Cyclomet. 1.* or *Elem. Cycloperimet. Defin. 5.* Τὸν Κύκλον τετραγωνίζειν, ἐστὶ πρὸς τῷ κύκλῳ χωρίῳ ἴσῃ ἐμβαδῷ, ἴσον δ' εὐγράμμον εὑρεῖν. *Circulum quadrare, eſt Circuli aream æquale Rectilineum invenire.* that is, To ſquare a Circle, is to find a right-lined Plane equall to the Area of the Circle. Here ſhalt thou find that rectiline Plane to be the very Square of the Diameter, (or of the Circumference) according to Quantity diſcrete; in which it is artificially diminiſht by a Line of meaſure, ſo as to be made equall with the Circle it ſelf, whereas naturally or geometrically it is greater then the Circle. And ſo I may ſay, that more properly, preciſely and nearly to ſquare a *Circle*, is to find an exact Square equall to the Circle given: And the like underſtand for the ſquaring of any other regular Figure; and ſo thou ſhalt here find the like artificiall quadrature of all rectiline regular Planes as of a Circle, by their lateral, diametrall, and diagonall lines, for the immediate producing of their ſuperficiall contents, which I have here performed (by way of practicall demonſtration) in two of the firſt of them.

Then

Then next ſhalt thou here find the like moſt artificiall and excellent cubing of a *Globe* or *Sphear*, for the immediate producing of its ſolid content in any meaſure aſſigned. And what was ſaid before of the ſquaring of a *Circle*, both general and particular or ſpecial, the like I may ſay of the cubing of a *Sphear*; That the ſame generally underſtood; is to find a right-lined or plain *Solid*, equall to the ſolid Area of the *Sphear*: and ſo more particularly and properly, to cube a *Sphear*, is to find an exact Cube equall to the Spheare given; which thou ſhalt here find to be made artificially (in quantity arithmetical) the very Cube of the Diameter (or of the correſpondent Circumference) which naturally, or in quantity geometricall, exceeds the Sphear it ſelf; and this not only in reſpect of ſolid meaſure, but alſo of gravity or ponderoſity, according to any Metall aſſigned: And the like underſtand for any other regular *Solid*: And ſo ſhalt thou here find the like moſt artificiall and admirable cubature of all the five famous plain ordinate Bodies in Geometry, or rectiline regular Solids, as of a Sphear, in both the foreſaid reſpects; and that not only as conſidered ſimply and abſolutely in themſelves alone, but alſo in relation to a Sphear, as being deſcribed either within or about the ſame. And what is here performed in all theſe regular So-

lids for their ſolid dimenſions, by way of cubature; the like is done in them for their ſuperficiall dimenſions, by way of quadrature; (and which therefore I conceive, may not altogether unaptly and improperly be termed the Quadrature of theſe Bodies, as to their ſuperficiall or externall part.)

And then ſhalt thou here find after the like artificiall manner, the dimenſion both ſolid and ſuperficiall of all ſuch Solids, as are not exactly ordinate or regular, but ſomewhat like the ſame, and ſo which I call regular-like Solids, as namely right Cylinders and Cones; and all Priſms & Pyramids conſtituted upon regular Baſes; and both which Dimenſions aforeſaid, may ſometimes happen in theſe kind of Bodies, to be artificially of the ſame nature with thoſe in exact regular Bodies: *viz.* cubatory and quadratary; as I ſhall ſhew in their Dimenſions; though indeed in their ſolid Dimenſion, there is alwaies a Quadrature, in reſpect of the Baſe. And here likewiſe what is underſtood for ſolid meaſure, muſt be underſtood for gravity or weight, according to any Metall propoſed.

And all theſe ſeverall dimenſions afore-named, are here performed by Lines of equall parts only, according to a decumane, decimane, or decimall diviſion, in which therefore conſiſteth the

the excellencie of the performance: and the ſame demonſtrated, not only in reſpect of the practicall uſe thereof, but alſo the theoricall grounds and reaſons, according to certain Propoſitions laid down for that purpoſe. And which Lines are here generally ſet forth by Number, denoting or expreſſing their magnitudes from any Meaſure given or appointed.

And now, although that learned Mathematician, Mr. *Edm. Gunter* (ſometime Profeſſour of Aſtronomy in *Greſham-Colledge* in *London*, and long ſince deceaſed) a man excellent for Inſtrumentall inventions, hath (among other Lines) upon his *Sector*, certain Lines of quadrature, (as he termeth them) as for to make a *Square* equall to a *Circle* given, by the ſemidiameter thereof, *& contrà*: and ſo for the like quadrature of certain rectiline regular Planes, by their ſides; yet thoſe are of a much different kind and nature from ours, (as any one may plainly ſee) being not ſeverall Lines of meaſure divided into parts any way, whereby to give immediatly of themſelves, the ſide of the equal *Square* arithmetically (as I may ſo ſpeak) or in quantity diſcrete, as from any certain, ſet, denominate Meaſure, according to a common way of meaſuring; as our Lines do, (& conſequently, the ſuperficiall content of the *Square*, for the Area of the Figure) but

only one Line, drawn twice over upon the *Sector*; or (if you will) two like, correſpondent, or congruall Lines, drawn upon each leg or ſhank of the Sector, from the Center thereof, and undevided; containing in them only certain points, in which the *Sector* muſt be opened, according to the ſemidiameter of the Circle given, or the ſides of the other Figures to which they belong, being there expreſſed by the numbers of their Sides; and ſo give the ſide of the equall Quadrat, only geometrically (as I may ſo ſpeake) or by Line, croſſing the Sector between the two points of quadrature at the ends thereof, and ſo parallel to the line for the ſemidiameter of the Circle, or ſide of other Figure, to which the Sector is opened: and which line being taken off from thence with Compaſſes, and ſo applied to any certain Line of meaſure divided, will then indeed give the ſide of the equall Square in the parts of meaſure, as our Lines of quadrature do: but yet Mr. *Gunter* doth not apply his Lines of quadrature to ſuch a uſe, or any way mention the ſame, but only to the laying down of the exact ſide of the Square equall to a Circle given, &c. and ſo to reduce the *Circle* into a *Square*, geometrically, after a mechanicall manner: and which can hold but only in a ſmall Circle, whoſe ſemidiameter (or other ordinate Plane, whoſe ſide)

ſide) may not exceed a convenient extent or opening of the legs of the *Sector*, (or of a pair of Compaſſes) ſo as to take off the ſide of the equal Quadrat, in its due place upon the *Sector:* for indeed, the largeſt *Sector* or *Compaſſes* that are uſually made, (or can be made convenient for uſe) will open or extend (at the utmoſt) but to a very ſhort line, in compariſon of thoſe which frequently fall out to be meaſured; and therefore his Lines (or way) of Quadrature, cannot extend to any large Dimenſions, and ſo ſerve for a generall meaſuring as ours do, if they ſhould be applied to the very ſame uſe that they are, namely, the ſuperficiall dimenſion of Circles, and other ordinate Planes, (in a quadratary way) according to any meaſure aſſigned, and thereupon can ſerve but to very ſmall purpoſe. Nor indeed had I acquainted my ſelf with thoſe his Lines of Quadrature, or any other upon his Sector, when I firſt apprehended and conceived in my mind this artificiall way of meaſuring which I here propoſe, (though I had often ſeen that Inſtrument, and had much ſtudied his Book in other things) nor till I had in a manner perfected the ſame throughout all the Dimenſions to be performed thereby, and ſo was come to the Cloſe of this Book. Neither did any of thoſe Artiſts, to whom I ever yet propounded the ſame (as a-

foreſaid) mention to me thereupon in the leaſt kind, Mr. *Gunter's* Lines of quadrature (or any other upon his Sector) in reference to any meaſuring; although ſome of them I am ſure (if not all) were well acquainted with Mr. *Gunter's Sector*, & all his other Inſtruments, & taught the uſe of them to others. But this only by the bye. Nor do his Lines of *Superficies* in generall, upon the *Sector*, being alſo two like, congruall Lines drawn on the two legs thereof from the Center, and divided unequally into 100 parts, ſerve for the meaſuring of Superficiall Figures, as to the immediate producing of their contents ſimply by themſelves, in any kind of meaſure given; but only in a way of proportion, to find out the ſuperficiall content of one Figure, by the ſuperficiall content of another like (or unlike) Figure given or known; together with ſeverall other uſes noted by him. And ſo his Lines of *Solids* in generall, being two like Lines, drawn upon the *Sector* in like manner, as the Lines of Superficies, and divided alſo into 100, (or rather 1000) parts unequally, ſerve only for the like uſes in Solids, that the Lines of Superficies do in Superficies.

And then for his more particular Lines of Solids (as I may term them) upon the *Sector*, called the Lines of inſcribed, and of equated

quated bodies; having reference in particular to the five foreſaid plain regular Solids, and a Spheare: the firſt ſort of them, in reſpect of the inſcription of the ſaid five bodies in a Spheare; and the other ſort, in reſpect of their equation to a Spheare, and alſo of one to another; are of a different nature and kind from our artificiall Lines pertaining to theſe bodies, and a Spheare, for the immediate producing of their ſolid contents in any Meaſure appointed, according to an exact, abſolute cubature, both ſimply in themſelves, & alſo in reſpect of inſcription & circumſcription to a Sphear given, as aforeſaid: but are the firſt of them, for the finding of the Sides of theſe five bodies, as being to be inſcribed in a Sphear, by the ſemidiameter thereof given; and the other, for the finding of their Sides, as being equall in magnitude to a Sphear, and this by the Diameter of the Sphear given; *& contrà*: or as to be made equall one to another by their ſides: and all this in a meer geometricall ſence; and thereupon theſe latter Lines of his, will give off from the *Sector*, the ſide of the *Cube* equall to a *Sphear*, by the Diameter thereof; & to any of the other regular bodies, by the ſides thereof given, in the ſame manner, that his Lines of quadrature give the ſide of the *Square* equall to a *Circle*, by the ſemidiameter thereof, and to the other re-

regular Planes or Superficies, by their ſides given; theſe Lines of equated bodies (as alſo thoſe of inſcribed bodies) being of the like kind and nature with thoſe of quadrature, (which by the ſame reaſon, may aſwell be called Lines of equated Superficies, for that they do not only equall a *Square* to a *Circle*, or other ordinate Plane given; but alſo equall them all one to another) and are drawn upon the *Sector* accordingly; & ſo are of no further or better uſe then they are.

Then laſtly, his Lines of *Metals* (ſo called) upon the Sector, (being inſerted with the lines of equated bodies, becauſe there was ſpare room, and much of the like kind with them, and ſo are contrived together on the ſame two lines, on each leg of the Sector, being drawn from the Center) are not like our lines of Metals (as I may ſo term them) which are for the immediate diſcovering of the weight of a Sphear, or other regular (or regular-like) body made of any Metall, in the very ſame (cubicall) manner, that their ſolid contents are obtained in any meaſure, by their reſpective artificiall Lines as aforeſaid; but thoſe (together with his *L*ines of Solids) do ſerve only to find the proportion, as it were, between ſeverall Metals (as he ſaith) in their magnitudes and weights, and that according to the experiments of *Marinus Ghetaldus*, in his book intituled

intituled *Archimedes promotus*; that is (as he ſaith) In like Bodies of ſeverall Metals, and equall magnitude, by having the weight of the one, to find the weight of the reſt; *& contrà*: together with two other uſes noted by him.

Theſe things (courteous and judicious Reader) I thought good here to inſert by the way from Mr. *Gunter*, to ſhew the difference between his Lines and ours; eſpecially thoſe of his, which for their uſe, may ſeem to come moſt nearly to ours, namely his Lines of Quadrature, and of equated Bodies: & which as they cannot be ſo generally uſefull as (or not conſiderable for their uſe, in reſpect of) ours, according to what I ſhewed before; ſo neither can they be altogether ſo exact in the performance of thoſe ſmall Dimenſions which they can reach unto.

And what I have here performed Geometrically, or by Line (or Scale) in an artificiall way of meaſuring; I have alſo ſet forth Arithmetically, or by Number, in a way of Proportion after the moſt exquiſite manner that may be, as from the naturall Meaſure; according to the ſame diviſion or partition of the Unity, as is of the Lines of meaſure, both naturall and artificiall; and by which therefore the artificiall meaſure may be readily deduced from the naturall, or the naturall Meaſure be reduced to the

artificiall: together with a multitude of other mericall concluſions beſides, in moſt of the geomericall Figures which I have here particularly handled, by way of practicall demonſtration; none of them having been done before by any man, that I do know of; except thoſe in the *Circle*, being ſet forth by Mr. *Gunter*, and from him by Mr. *Wingate*, and perhaps ſome others; but yet not all of thoſe Proportions in ſo ample termes, as I have here done them, which therefore I extracted again a-new.

And then moreover ſhalt thou here find, not only the moſt artificiall and expeditionall way of meaſuring thus all regular Bodies, and ſuch as do come very near a regular forme (which therefore I call regular-like) but alſo of ſuch as are of an irregular form; and firſt, of concave Bodies, or Veſſels for Wine and Beer, (which commonly do ſomewhat imitate the form of a Cylinder, and may be called Cylindroidall, and ſo admit of a Cylinder-like dimenſion, being firſt reduced by art to a Cylinder: to which end I beſtowed ſome conſiderable pains (and a little coſt too) in the making of ſundry experiments, for the diſcovering of the true contents of the Standard-meaſures for Wine and Ale or Beer, pertaining to the Citie of *London*, (which are kept at the *Guild-hall*) as being commonly

commonly taken for the moſt generally receiued Meaſures for this purpoſe, throughout the Kingdome; and by which I haue ſeen the Meaſures which haue been made for ſome eminent Towns farr remote from *London*, to be ſized and ſealed, (though here I wil not cõtend about them) and haue here accordingly fitted a gauging-Line to each of them, according to our artificiall way of meaſuring, being alſo therefore Lines or Scales of equall parts in a decimall diuiſion, by which the liquid content of any Veſſell will be obtained immediatly in Gallon-meaſure, after the ſame manner (and with the ſame expedition in a manner, the irregularity of the Veſſell being conſidered) that the ſolid content of any exact Cylinder is had artificially, according to any Meaſure appointed.

And then laſt of all, do I here ſhew (by way of Appendix) the moſt eaſie and exact way for the diſcouering of the ſolid capacities of all other irregular kinds of Bodies whatſoeuer, both ſolid and concaue, which of themſelues are altogether unmeaſurable in the uſuall way of meaſuring (or whoſe Dimenſions can in no wiſe be taken by a Line of meaſure) which therefore is a work of a contrary nature to all the former: and haue here exemplarily illuſtrated the ſame from experiment, in a certaine regular ſolid body,

eaſily and exactly meaſurable, for a confirmation thereof; (and which way alſo, hath not been ſet forth by any man before, that I do know of) Together with ſeverall other new mathematicall experiments and obſervations, very uſefull, and worth the noting. All which (friendly Reader) I cõmend to thy courteous conſideration & acceptance; hoping thou wilt receive no leſſe delight (and benefit alſo) in the peruſing and practiſing hereof, then I have taken delight and contentment in the ſtudy and exerciſe of the ſame, though ſurely with no ſmall pains and induſtry, beſide the expence of ſome time now and then, from my occaſions of ſerious concernment, according as the ſame would reaſonably permit: and which therefore I could do to no other end, then only to enlarge and advance (ſo farr as here I might) the practice and exerciſe of this ſo noble and admirable an Art, being drawn thereunto by that *Genius*, which hath heretofore much diſpoſed and inclined me to mathematicall contemplation and exercitation in generall: ſo that, thy friendly acceptation hereof, is all I expect for my labour. And if any after me ſhall happen to raiſe any further Concluſions from what I have here laid down, in any particular thereof; then ſurely will theſe my paines be yet thereupon ſo much the more to purpoſe. But

now

now as I muſt expect this work of mine will meet with ſome *Momaicall* or *Zoilan* Spirit, ſo I ſhal not regard the ſame, or be terrified thereat, ſeeing that the beſt conceits and inventions of men that were ever yet publiſhed to the world, have been obnoxious to the obloquies and obtrectations of ſuch malevolent and malignant ſpirits: and which hath been the complainr of the moſt learned men in all ages. And thus, courteous, ingenious, and ingenuous Reader, I friendlily bid thee fare-well, reſting,

London; the firſt of *May*, 1650.

Thine hearty wel-wiſher,

J. WYBARD, *DM.*

THE

Generall Contents of this Work, conſiſting (almoſt all) in meaſure alone.

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5. Of the gravity of a certain ſerreall Sphear or Bullet, both in *Aveirdupois* and *Troy-weight*, from a new and exact experiment. p.217

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8. Of the weight of water, in relation to it's ſolid meaſure; or the ponderall and dimenſionall quantity of water compared together, from ſeverall new experiments, for diſcovering the ſolid meaſure of any irregular body thereby, whether ſolid or concave. p. 269, 270 & 287.

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9 How to find out the ſolid content of any irregular body by the weight of water, &c. p.308.

10. How to find the exact quantity of water (or other-like liquid body) which is equall in magnitude or dimenſion to a ſolid body given. *ibid.*

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Whoſo-

WHoſoever ſhall think good to make uſe of the Lines for Gauging, or for any other Dimenſions deſcribed in this Book: or of any other Inſtruments for mathematicall practice in generall; they may have the ſame very accurately made by Mr. Chriſtopher Flower, *dwelling in the Bulwark, neer the Tower of* London: *beſides divers others about this City.*

Courteous Reader,

SEeing that hardly ever any Book paſſeth the Preſſe, free from Typographicall errours, notwithſtanding all the care and diligence that can be uſed (and in which I was not here wanting) eſpecially Books of this nature: and then that they being commonly ſet at the end of the Book, are ſeldome taken notice of, till the Book be read throughout; and ſo many times the miſtakes of the Printer are by ignorant or malevolent Readers, put upō the Author as errours: Therefore the chief of thoſe few faults which have here eſcaped, (the moſt of them being but meerly literall) I thought good to put at the beginning; that ſo thou mighteſt in the firſt place know them, and conſequently amend them: aſſuring thee withall, that of that multitude of Numbers which are contained in this Book (the moſt of them denoting Lines, and Proportions of meaſure; and ſo upon which dependeth the ſpeedy working and reſolving of many excellent practicall Propoſitions: and are therefore ſpecially to be regarded; and in which a fault cannot be eſpied by bare inſpection,) there is not one of them defective in any one figure thereof; ſuch was my (more then ordinary) care therein.

ERRATA.

PAge 4. line 7. put (before commonly. l. 8. *dele* of. l. 20. r. *partior*. p. 7. l. 28. for hath, r. both. p. 8. l. 22. r. in reſpect of the Meaſure from which it is taken, as alſo of the Figure &c. l. 30. *dele* the. p. 32. l. 11. r. *Sect*. 4. p. 59. l. 14. for matter of, r. manuall or. p. 75. l. 2. r. whoſe. p. 91. l. 24. r. ſhall. p. 173. the beginning of the laſt line ſave one, r. And ſo. p. 214. l. 2. *dele* the comma at Meaſure, and put one after Diameter. p. 316. l. 14. *dele* the firſt as. Other faults thou maiſt meet with, which being only literall, and ſo not worth the noting here; thou maiſt eaſily amend in the reading.

Tactometria,
Or the moſt Exquiſite practicall
Dimenſion of all Regular, and Regular-like Figures in generall.

And firſt, of the *Circle, Spheare, Cylinder,* and *Cone,* in ſpeciall. PART. I.

SECT. I.

Of the nature and diviſion of the Lines of Meaſure in generall, for the performing of all the aforeſaid Dimenſions.

Foraſmuch as to the due meaſuring of any Magnitude or Quantity continuall (in practicall Geometry) there is required ſome certain Meaſure firſt to be given or appointed: Therfore firſt and more generally, by Lines, we do her underſtand any right L aſſigned for a certain M

ſure; ſuch like as *Euclid, Elem* 10. *Defin.* 5. calleth

which is as much as certain, definite, determinate, ſpeakable or expreſſible by voyce, or otherwiſe expreſſible by Number: And ſo the moſt Noble, illuſtrious, and learned, *Fr. anc. Fluſ. Candalla*, a moſt diligent and induſtrious reſtorer of *Euclid's* Elements interprets it *Certa*, a line certain, as firſt put or propoſed and made manifeſt, and divided into parts certain and known: And it is alſo called *Fameſa*, a meaſure famous, that is, (as *P. Ramus*, and *Adr. Metius* do note) firſt ſpoken or expreſſed, &c. But moſt of the Latine Geometers do calll ſuch a line *Rationalis*, a Rationall line, for that (as *Ramus* ſaith in the places here cited) it is rationall to it ſelfe, as are all magnitudes equall among themſelves: and *Clavius* ſaith it is called *Rationalis*, becauſe it is alwayes put certain and known, whereas all other lines which are compared to this (as lines infinite in multitude, may be according to *Euclid* in the place forenamed) are not certain & known, though taken apart by themſelves, they are, ſeeing that every one may be divided into any number of equall parts, and being compared to this for their Meaſure, they are all either ſymmetrall or aſymmetrall, and ſo are ſaid to be either ῥηταὶ, or * ἄλογοι, in *Def.* 6, 7. explicable or inexplicable, rationall or irrationall; (but with this point we meddle not here,) and ſo we here underſtand this Line to be moſt properly called Rationall, as comprehending or containing in it ſelf the dimenſionall reaſon of all other lines meaſurable thereby. And therefore here we (for brevity) will with moſt Latine Tranſlatours and Commentators, and alſo our * Engliſh Tranſlatour of, and Annota-

Ram. Geom. lib. 1, El. 8. Et Schol. Mathemat. lib. 21 Adr. Met. Geom. pract. Par. poſter. ſeu Gæodeſ.

Clavius in Def. 5, El. 10.

*So put by *Theon*, whereas it ſhould rather be ἀῤῥήτους, as it is oppoſed privatively to ῥηταὶ, See Fluſſate upon the place, and alſo in Proem. 10. Elem.

* H. Billingſley, a Citizen of London, and Lord Mayor, An. 1596.

tour

tour on *Euclid's* Elements, underſtand any right line ſo firſt ſet, put or propoſed, by the name of the Rationall Line (and this may be applyed to any Meaſure whatſoever, and it is one of *Euclid's Data in Lib. Dator. Defin.* 1.) or (in reſpect of the enſuing work) the prime, ſimple, true, or naturall Rationall line. And this we mean when any where we ſay ſimply the Rationall Line.

Secondly, and more eſpecially by Lines we here underſtand any ſuch line augmented or diminiſhed by a certain convenient ſegment or portion of the ſame, for the more artificiall and ſpeedy menſuration of the aforenamed Figures: And this Line we may (not unaptly) call the ſecond ſuppoſed or artificiall Rationall Line, as being derived from the former, and ſo ſubſtituted in place thereof: like as in Numbers the Logarithmes are uſually called Artificiall Numbers, as being ſubſtituted inſtead of the naturall numbers, from which they are deduced, and whoſe place they ſupply in a moſt excellent and admirable manner, by performing all arithmeticall operations with that facility, expedition, and compendiouſneſſe (and exactneſſe alſo in ſome caſes, as I ſhall afterwards upon occaſion ſhew) which the naturall numbers themſelves cannot, for that by theſe, the two moſt tedious and troubleſome parts or ſpecies of Arithmetick (to wit, Multiplication and Diviſion) are wholly avoyded and aboliſhed, and that moſt difficult branch (or operation) thereof called the Extraction of Roots, is mightily abbreviated and facilitated: And ſo the Arithmetick performed thereby is uſually called artificiall Arithmetick.

Now ſeeing that every continuall or continued Quantity falling under Meaſure (in practicall Geometry) is referred and reduced to the diſcrete, that thereby its dimenſions may be made more manifeſt to us; ſo that Geometry hath

perpetually need of Arithmetick for the explicating and expreſſing of its magnitudes in their dimenſions: Surely no kinde of Numeration can be ſo accommodate to this thing, as that which conſidereth the Integer of Meaſure (as the Unit) in a decumane, decimane, or decimall ſolution, for that by this the practicall, inſtrumentall, or mechanicall part of meaſuring, or of the art *Metricall*, commonly and improperly called by *Ramus*, *Metius*, and of ſome others *Geodæſia* which properly ſignifies the diviſion or partition of right lined Superficies, as *Pediaſimus*, *de menſuratione & partitione Terræ*, well obſerves, ſaying *Terræ menſuratio duas in partes dividitur, Geometriam, ſcil. & Geodæſiam: Areæ namq; ſecundum artem menſuratio, & terræ menſuratio eſt et meritò Geometria vocatur; Unius verò & ejuſdem areæ, ſeu loci diviſio inter diverſas perſonas, partitio quædam eſt terræ, & jure optimo Geodæſia appellatur*, (and which from him *Clavius* noteth *Geomet. pract. lib. 6.*) for that the greeke words, Δαίω, δαίομαι, δαίζω, δαίζομαι, do (poetically) ſignifie the ſame that the Latine words *divido* and *portior*, and ſo of which, and the word Γῆ, *Terra*, comes Γεωδαισία, *i.e. Terræ diviſio ſeu partitio*) is made much more facile accurate and expeditious.

For indeed this Mathematicall ſolution of unity or continuity, is of all others the moſt abſolute and certain, and the moſt perſpicuous and rationall, and by how much the more numerous it is in the parts thereof, by ſo much the more exact it is, and conſequently the work effected by it. And what utility it hath brought to the *Mathematiques* in generall, may be ſufficiently witneſſed by that moſt noble and uſefull part of Geometry, called *Trigonometrie*, and that in the Radius of a Circle (which is the very Baſis and Root of all Trigonometricall operations) where (to wit) firſt, that greatly renowned Mathematitian *Johannes*

nes Regiomontanus, having for a long time used the Sexagenary solution (as *Ptolemy* and others before him) did at last bethink himselfe of the Decimall, as being much better (seeing that the Unit would perform a far greater Compendium then the Senarie) and indeed the best of all (and so put the Radius, to 10 millions of parts, and next after him, *Rheticus* in his great Trigonometrical Canon, to 10000 millions, and afterwards (to make that his Canon most absolute and perfect) he proceeded to 1000 millions (as I term them) or millions of millions, whereby the art of *Trigonometry* & cõsequently other *Mathematical* arts, as *Astronomy*, *Geography*, *&c.* depending thereupon) did become, in the practice, far more facile and expeditious then before: for where the first proportionall Term of the Trigonometricall proposition, is the Radius or totall Sine (which very frequently happens, or may be so made for the most part, as the learned *Pitiscus* excellently sheweth amongst other his compends in working) there the propofition or question is solved only by a * conjunct or manifold composition (or mutuall implication or induction) of the second and third terms, seeing that the Unit altereth nothing in a conjunct or manifold resolution, but the Number of composition immediately becomes the number of resolution, only distinguishing between the absolute or integrall, and the fractionall part thereof. And moreover the benefit of this solution of unity is excellently seen in that most excellent Arithmeticall operation, vulgarly called *The Extraction of Roots*, wherein (to wit) seeing the Roots of numbers not explicable or rationall (which the Algebrists or Cossists commonly call Surd Numbers, and so their Roots surd Roots) cannot be exactly had, then those numbers are reduced into some kinde of decimall parts (or parts of a great denomination, as *Ramus* termeth them) as *Centesms*

Pitiscus, Trigon. lib. 5.

* This is to be understood in working by the Naturall Numb.

teſms, Milleſmes, &c. and that figurate, as Quadrate, Cubique, &c. that thereby their Roots may be had more certain and neerer to the truth, then they can by the naturall or vulgar extraction, as *Ramus* ſheweth in the aforeſaid Books, where he (the firſt) ſhewed this kinde of Extraction which to ſome ſeems to have been the very foundation of Decimall Arithmetick, although *Ramus* hath no where elſe in his Mathematicall Works made any other uſe of this kinde of numbring, or made any mention of the ſame: But indeed, that of *Regiomontanus* in the Radius of a Circle ſeems to me to have given the firſt light thereof to the World; ſo that the Trigonometricall Numbers which now we uſe, may be termed Decimall, as they are derived from that Radius: For all the Sines to a Quadrant, and the Tangents to an Octant or ſemi-quadrant, are decimall parts or fractions of the Radius, but indeed the leaſt Secant is greater then the Radius: And ſo wee will here make uſe of this kinde of Numeration, as being the fitteſt for our purpoſe, as we ſaid before: And indeed for that this preſent work of ours cannot conveniently be performed by any other.

Ramus lib. Geom. 22 de Quadrato & 24 de Cubo.

See *Wingate* Arithm. 1 Book, 41 Chap. 3 Sect.

Now the Geometricall Figures which we have here firſt named, and ſo for which we have firſt extracted theſe kinde of metricall lines (or linear numbers) are thoſe foure which *Archimedes* himſelfe more eſpecially treated of, and which are as it were, the beginning and ground of all the reſt, namely, the *Circle*, *Spheare*, *Cylinder* and *Cone*, and accordingly the like Lines may be extracted for all ordinate Planes and Solids whatſoever, as we afterwards ſhew; for that to theſe foure may be aptly referred all other regular and regular-like Figures; As to the Circle may be aptly referred all ordinate Planes, to the Sphear, all exactly ordinate Solids; to the Cylinder, all

all ordinate-baſed Priſmes; and to the Cone, all ordinate-baſed Pyramids: And though a Cylinder and Cone, and the like, cannot properly be reckoned among Regular Figures, according to the ſtrict acception of an ordinate or regular Figure in Geometry, yet in reſpect of the regularity of their Baſes, and alſo the regularity and uniformity of their other, and more ſpeciall ſuperficiall part beſides (whether the ſame conſiſt of one entire plane only, as in the Cylinder and Cone, or of ſeverall planes equall and like, as in all right or erect regular-baſed Pyramids and Priſmes) the ſame may in a ſort be termed regular (eſpecially the Cylinder and Cone) and which therefore, for diſtinction ſake, I call regular like ſolid Figures.

But now the quantities of the artificiall metricall Lines, firſt extracted for the foure Figures firſt before named, according to any prime Rationall Line, and that to a Decumillenary diviſion of the ſame, are numerally thus.

I. *A Line for the moſt excellent ſuperficiall menſuration (or Diametrall Quadration) of a Circle, is* 1.1284 ſeré.

II. *A Line for the moſt exquiſite Solid dimenſion (or Diametrall Cubation) of a Globe or Spheare, is* 1.2407.

III. *A Line for the Sqnare Solidation (as I may term it) or Rectangular Parallelepipedation of a Cylinder, by its Diameter and Axis or Altitude (as to an exact quadrate Baſe) is* 1.0838.

IV. *A Line for the like dimenſion of a Cone, is* 1.5632 ſeré.

And ſo divers other the like Lines, for the dimenſion both of theſe, and alſo of other Figures hath Superficiall and Solid, I ſhall afterwards ſhew in their ſeverall places.

Every ſuch ſecond ſuppoſed, or artificiall Rationall Line, muſt be divided decimally, as is the firſt, true, or naturall Rational

Rationall Line, from which it is taken, according as the length thereof can conveniently beare: But if the firſt Rationall Line cannot well admit of ſo many parts as are here ſet down, then they may be abbreviated or contracted; as to *Milleſmes*, *Centeſmes*, or to prime or ſimple *Decimals* (or Tenths) only; though indeed our leaſt common Meaſure in uſe, (*viz.* an Inch or Pollicar) may be diſtinctly divided into 100 (or 1000) parts, if it be rightly handled according to the more artificiall (or Diagonall) way, as hereafter we ſhall have occaſion to ſhew. And ſo we have here ſet forth the Rationall line in parts of a large denomination, that ſo it might ſerve for more exactneſſe in uſe, becauſe the more parts, or the greater diviſionall denomination the Integer of meaſure (or the unite) is of, the more exact will be the work performed by it, as I noted before, and as afterwards I ſhall make plainly to appear.

Furthermore, every ſuch artificiall or ſuppoſed rationall line, may be ſaid to be two fold, to wit, generall or univerſall, and ſpeciall or particular: Generall, in reſpect of the number by which it is indicated and explicated, for that the ſame linear number, doth ſerve alike to all prime Rationall lines: And particular or ſpeciall in reſpect of the Figure (Superficiall or Solid) to which it is appropriated and applyed, becauſe that every ſuch particular Figure as is mentioned in this Book, doth peculiarly and properly claim to it ſelfe (and that ſeverall wayes) ſuch a line for its more artificiall and expeditionall menſuration.

Now the uſe of the foure artificial Lines before-going, for the meaſuring of the foure firſt Figures aforenamed, we will deliver in the three practical or problematicall Propoſitions following in the next Section; and withall the magnitudes and uſes of all the other artificiall metricall Lines pertaining to the ſaid foure Figures.

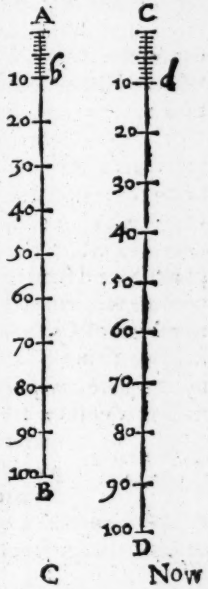
SECT.

SECT. II.

PROP. I.

If the Diameter of a Circle be taken by its proper Line of Meaſure (according to any Rationall Line) I ſay then, that the Quadrat of the Diameter ſhall be the Area of the Circle (according to the ſame Rationall line) which I prove thus demonſtratively.

LEt the right line A B be put as Rationall, and divided into an 100 equall parts (or firſt into 10 parts, and then one of thoſe only into ten parts will be ſufficient) And let the ſame line entirely taken, repreſent the Semidiameter of a Circle: So the Diameter will be entirely A B, 2. and thereupon the Circumference (according to the ancient, and ſtill vulgarly received and retained *Archimedean* proportion of the Diameter to the Circumference, *ſubtriple ſeſquiſeptimall*) will be $6\frac{2}{7}$, whoſe $\frac{1}{2}$ *viz.* $3\frac{1}{7}$ being infolded with the Semidiameter AB1 produceth the ſame for the *Area* of the Circle. Or hence according to the proportion of the Quadrat of the Diameter to the Circle it ſelf, *ſupertripartient-undecimall*: Or again, the ſame according to *Euclid's tetragoniſmall* reaſon, (as it is with *Hero*) by deduction of $\frac{3}{14}$ of the Diametrall Quadrat 4, *viz.* $\frac{6}{7}$, and there reſts $3\frac{1}{7}$ for the Circular Area as before.



C Now

Now that there may be a comparing of our new or artificiall menſuration with the common or naturall, and thereby a confirmation of the ſame: I draw the right line C D, for the ſecond, compound, ſuppoſed, artificiall (or quadratarie) Rationall Line, to the length of A B, the primary, ſimple, true, or naturall Rationall line, and moreover, 13 *ferè* of the ſame (for ſo much is the additament or additionall ſegment in centeſimall parts) and then I divide C D likewiſe into 100 equall parts (or firſt into 10 parts, and then only one of thoſe parts into 10 parts, which will be ſufficient, as in the Line A B) which done; I meaſure the line A B 2.00 (for the Diameter of the Circle) by the line C D (the Line or Scale of quadrature) and finde it to be thereof 1.77 (for the line A B, ſimply upon the line C D, falleth about the middle between .88 and .89, and ſo the double of A B in one entire line at length being meaſured by the Line C D will fall about the middle between 1.76 and 1.78, which is about 1.77) which ſquared, gives 3.1329 for the Area of the Circle, agreeing with the former area exactly in the integrall part: But now the fraction-part of that area, *viz.* $\frac{1}{7}$ being converted into *decu-milleſmes* or ſquare *centeſmes*, is .1428, which exceeds our meaſure by .0099, *viz.* 99 ſquare *centeſmes*, which difference is not conſiderable in common practiſe, as I ſhall afterwards plainly ſhew: but yet our meaſure wanteth of the true area, found by the more new & true terms of *Cyclometry*, or Circular Tetragoniſme (which here we uſe in all Cyclometricall operations) not ſo much by 12 ſquare *centeſmes*, *viz.* but 87 ſuch *centeſmes* as I ſhall ſtreightway ſhew: For thoſe of *Archimedes*, of the Diameter to the Circumference 7 to 22 *ſubtripla-ſeſquiſeptima*, and conſequently of the Square of the Diameter to the Circle 14 to 11 *ſupertriundecima*, or *ſupertripartiens-undecimas* (though ſufficient enough for any ordinary mechanicall uſe, as hee only

Archim. de dimē. Circ. prop. 2, 3.

only meant them, and that eſpecially in ſmaller Circles, yet) are in ſtrictneſſe of Art too large, and ſo give the area ſomwhat greater then indeed it is, and the more, the greater the Circle is, and hereupon hath ariſen that difficult and curious queſtion ſo much controverted among Artiſts about the Quadrature of a Circle; and in which many learned men have beſtowed great pains, as to the finding out of the neereſt proportion between the Diameter and Circumference, &c. among which that excellent Artiſt* *Ludolph van Ceulen (alias Cullen & Collen)* hath hitherto generally carryed away the greateſt commendations, having ſet forth the ſame in decimall terms to 36 places, which he willed to be engraven upon his Tombeſtone, as a Teſtimoniall and Memoriall of thoſe his painfully ſuſtained and finiſhed Labours (as *W. Snellius* noteth, who afterwards produced the very ſame number) and which I have ſeen upon the ſame in the great Church in *Leyden*, called S^t. *Peter's* Church, there being drawn thereupon a large Circle, and upon the Diameter-line an unit with 35 cyphers for the number of the Diameter, and round about the Circumference, the number for the ſame to 36 places; of which ſo many as are needfull here to deliver, as being ſufficient for ordinary uſe, are 3.14159, anſwering to the Diameter 1.00000, or more briefly 3.1416, the Diameter being 1.0000; which according to the foregoing *Archimedean* terms of 7 and 22, will be 3.1428, &c. But thoſe of that famous Mathematician* *Metius* the elder, ſomtime Geometrician to the Eſtates of the confederated or United *Belgick* Provinces, of 113 and 355 do agree with the *Ceulenian* terms to the firſt ſeven places, *viz.* 3141592, anſwering to the Diameter 1000000, for that he demonſtrateth the proportion

* Lib. de Circ. & Ad ſcrip. Here alſo is ſeen the excellent uſe and benefit of decimall Numeration in the Quadrature of a Circle. *Snel.* lib. Cyclomr prop. 31. See *Lansberg.* Cyclomet. lib 1. Poriſm 3. to 29 places, agreeing with *Ceulen.*

portion of the Periphery to its Diameter, to be leſſe then $3\frac{27}{120}$ (that is $\frac{3}{10}$) but greater then $3\frac{15}{106}$ (that is $\frac{3}{7}$) whoſe *intermedian proportion is (ſaith he) $3\frac{16}{113}$ (or $\frac{355}{113}$) and which is a little larger then that of *Ludolph Van Cullen*, yet ſo, as that the difference is leſſe then $\frac{1}{10000}$. And this proportion doth give the Area of the foregoing Circle $3\frac{16}{113}$, which is by Decimall converſion of the fraction-part into ſquare Centeſms, 3.1416, agreeing exactly with that which is produced by the *Ceulenian* proportion, and this is the true Area of the Circle: but the Area $3\frac{1}{7}$ produced by the vulgar or *Archimedean* proportions, being decimally to ſquare Centeſmes 3.1428 exceedeth this Area by 12 Square Centeſmes (and ſo neere the old and new *Cyclometry* do here agree) & ſo the area of the circle found by our way 3.1329. wanteth of the true area but 87 ſquare centeſmes, as I ſaid before: and this defect (not conſiderable in common practiſe, as I noted before) happeneth, in regard that the double of the firſt line meaſured by the ſecond, falleth not preciſely on 1.77, but is ſomwhat more, though indeed upon the ſaid line it ſelfe, it is very hardly diſcernable, being but very little: Wherefore if both the Rational Lines were divided into more parts, as 1000, 10000, &c. then the work would prove ſtill more and more exact (but indeed here we could not draw a line actually capable of a greater decimall diviſion, or denomination of parts then 100, according to the plain, vulgar, ſimple, or naturall diviſion of a Line, which here chiefely for plainneſſe we have uſed:) For firſt, if the prime rationall Line (A B) be divided into 1000 parts; then the ſecond (C D) will be of the ſame. 1.128, and the

* *Metius* in lib. adverſ. Quadratur. Circ. *Simonis* à Quercu.

Et *Adr. Met.* in Lib. Geomet. pract. part. prior. Cap. 10. prop. 3. & part. poſt. Cap. 4. prop. 1.

* The mean Arithmeticall is in the leaſt Terms $3\frac{1801}{12720}$ which is Decimally 3.14158805, &c. and the mean Geometricall (or mean proportionall) is decimally 3.14158802493, &c. But his mean $3\frac{16}{113}$ is decimally 3.14159292, &c. which exceedeth both the other, & that of *Cullen* is 3.14159265, &c.

the firſt compared with the ſecond (being alſo divided into the like number of parts) wil be .886 or neer thereabouts, which falleth about the middle between .88 and .89 of the ſaid ſecond line being 100, for theſe two converted into Milleſmes are but .880 & 890, and ſo the double of the firſt Line meaſured by the ſecond will be 1.772, or neere thereabouts, which is ſomwhat more then 1.77 upon the ſecond Line centeſimally divided, for 1.77 converted into Milleſmes, is but 1.770, which ſhews that 1.77 was ſomwhat too little for to produce the true content of the Circle: Now 1.772 being ſquared, gives 3.139984 for the area of the Circle (which is in ſquare Centeſmes, 3.1400 *ferè*) and the area produced by the trueſt terms of Cyclometry, both *Ceulenian* and *Metian*, is to Milli-milleſms, or ſquare Milleſms, 3.141593 *ferè*, (whereas the vulgar area $3\frac{1}{7}$ is 3.142857) which our area is wanting of by .001609 *ferè*, *viz.* 1609 ſquare Milleſms, or but 16 ſquare centeſms, and agrees with the ſame in ſquare tenths, *viz.* 3.14) which comes much neerer the matter then the centeſimall operation, and indeed as neare as need be deſired: but yet this inconſiderable difference (by way of defect in ours) happens alſo for the reaſon aforeſaid, in that the double of the firſt line meaſured by the ſecond is not juſt 1.772, but ſomwhat more (as the firſt line ſimply applyed to the ſecond, is not preciſely 0.886 but ſomwhat more) and therefore if the firſt be divided into 10000 parts; the ſecond will be thereof 1.1282 *ferè*, according to the ancient or *Archimedean* Cyclometry, but 1.1284 *ferè* according to the later or *Ceulenian* and *Metian Cyclometry*: and ſo the firſt line compared with the ſecond (being alſo 10000) will be 0.8862 (or ſomwhat more) and thereupon the double of the firſt, being meaſured by the ſecond, will be 1.7724, which ſhews that 1.772 was ſomwhat too little to produce the true area, for this in decumilleſms is but 1.7720, but our number is 1.7724 (and in-

deed ſomwhat more) which ſquared yields 3.14140176 for the Circular area, agreeing yet much neerer with the true area, which is now 3.14159265 (to which comes very neere that of $3\frac{16}{113}$ being by decimall converſion of the fractionall part into ſquare decu-milleſms 3.14159292, beginning but now to exceed the true area produced by the *Ceulenian* Cyclometricall terms) our meaſure wanting thereof now but 19089 ſquare decumilleſms, which by contraction of the parts is hardly 2 ſquare centeſms.

And if we proceed one operation further, namely, to a centu-millenary ſolution of the rational Lines (where A B being 100000, C D will be thereof 1.12838 *ferè*) our meaſure will be found to agree with the true naturall meaſure, exactly to ſquare centeſms, and to want thereof hardly 14 ſquare milleſms: for A B 2.00000, being meaſured by C D, made 100000, wil be found thereby 1.77245, (which ſhews that the artificiall or quadratary number in the precedent operation, was not exactly 1.7724, but would fall about the middle between that and 1.7725 upon the artificiall line of meaſure, and ſo gave the area of the Circle ſhort of what it ſhould be) which being ſquared affords 3.1415790025, for the area of the Circle, which wanteth ſtill of the true area (being now correſpondently 3.1415926536 *ferè*) 136511 ſquare centumilleſms, which by contraction of the parts is but 1365 ſquare decu-milleſms, and about 14 ſquare milleſms, and not one ſquare centeſm.

And if we go on yet further to a milli-millenary ſolution of the Rationall Lines (where A B being 1000000, C D will be of the ſame 1.128379) the naturall and artificiall Meaſure will be found to agree exactly to ſquare milleſms: For A B 2.000000 entirely taken, being meaſured by C D (put 1000000) wil be found 1.772454 *ferè*, whoſe Quadrat is 3.141593182116 *ferè* for the ſuperficies of the Circle, which now exceedeth the true meaſure (being here correſpondently

ly 3.14159265359o *ferè*) by 528526 ſquare milli-milleſms, which by abbreviation or contraction, is 5285 ſquare centu-milleſms, or but about 53 ſquare decu-milleſms, and not one ſquare milleſm.

By all which it is already ſufficiently evident, that the more parts the Unit or the Integer of meaſure given, is divided into, the more exact will be the work performed by it, as I noted before, and as may be alſo further ſeen in the following dimenſions.

But indeed if the Rationall Lines be divided but into 100 parts they will be ſufficicient for any ordinary uſe: For the greateſt difference ariſing here between the true naturall meaſure and ours, being that in the firſt operation (by centeſms) where the ſuperficiall content of the Circle found by our meaſure 3.1329. falleth ſhort of the true content (found by the trueſt terms of Cyclometry) 3.1416, by 87 ſquare centeſms, is in vulgar terms but $\frac{1}{115}$ of the prime rationall Line A B (as the Integer of the meaſure given) ſquared, which is but as one part of a ſquare unit divided into 115 parts; and ſurely this difference is of no moment in common practiſe. And whereas in the other operations, the differences happening between the true content and ours, in parts of a greater denomination, may ſeem to ſuch as do not well apprehend this matter, to be very great; yet being reduced to vulgar Arithmeticall terms (which are better underſtood by them) will appear to be ſtill leſſer and leſſer, and as nothing: As in the fifth or laſt operation of this firſt Example or *Demonſtration*, being under a ſolution of the Rationall Lines to a million of parts, where the area of the Circle found by its proper Line of diametrall quadration, exceeds the true area by 528526 ſquare milli-milleſms, that is, 528526 parts of the prime Rationall Line being 1000000 parts ſquared, and ſo reſolved into 1000000,000000 parts: which difference, though it may ſeem great by reaſon

ſon of the multitude of the decimall fraction-parts, yet conſidering the greatneſſe of their denomination, and being reduced into common arithmeticall terms (whoſe numbring part is 1) they will appear to be as nothing, being hardly $\frac{1}{1892054}$ of the intire prime Rationall Line (as the Integer or Unity of Meaſure) ſquared, which is but as one part of a ſquare Unit containing 1892054 parts.

And if the Line of Meaſure firſt given, be ſo ſhort as that it cannot be diſtinctly divided into 100 parts, according to its own ſelfe ſimply, then may 10 parts reaſonably ſuffice: As ſuppoſe here a Rationall Line to be of the former line A B, .1, and divided equally into 10 parts, *viz.* A *b*. (which is A B 0. 10) then the ſecond Line, or Line of quadration, *viz.* C *d*. will be thereof 1.1 (which is as A B 0.11) anſwering analogically to A B 1.10, and which wanteth of the true additionall parts .03 *ferè*, becauſe the totall additionall centeſimall ſegment of A B is .13 *ferè*, as was noted before) And let the Diameter of a Circle, be A *b*, 1.5 (which is as A B 0.15) then the Peripherie wil be (according to the ſame diviſion) A *b* 4.7 (which is as A B 0.47) whoſe moiety A *b* 2.35 (or A B 0.235) together with the ſemidiameter A *b* 0.75. produceth the Circle it ſelfe in ſquare Integers and parts of A b, 1.76. Now the diameter A *b* 1.5 being meaſured by C *d* (being alſo of a denarie or ſimple decimall diviſion) becomes but 1.3, whoſe ſquare is 1.69 for the ſuperficies of the Circle; which wanteth of the true content 7 ſquare primes or tenths only, *viz.* 7 parts, of the prime Rationall Line, or Line of Meaſure firſt given, A *b* 10 parts (as the Integer or Unit of meaſure) ſquared, and ſo reſolved into 100 parts, which in vulgar Arithmeticall eſteem are hardly $\frac{1}{14}$ of the ſame line ſquared, the ſaid line it ſelfe being then neere 4 parts. Or the diameter being taken in centeſimal parts only of the Line A B, *viz.* 0.15 the content of the Circle wil be only in

ſquare

ſquare centeſms, .00176, exceeding the Square of the quadratarie parts CD 0.13, *viz.* 0.0169 by 7 ſquare ſeconds, or centeſms only, which in common accompt make but $\frac{1}{1429}$ *ferè* of A B ſquared (the line it ſelfe being then neer 38 parts) which is much leſſe then $\frac{1}{14}$ for the difference between the two Meaſures: ſo that you may here ſee, how much 100 parts in the Line of Meaſure are better then 10; and therefore they that underſtand the more artificiall way of dividing lines (commonly called the diagonall way) may better divide the leaſt Line of Meaſure, into 100 parts: For this latter aſſumed Line A *b* being but .1 of the firſt Line A B, and but about $\frac{1}{4}$ of an Inch or *Pollicar* may be diſtinctly divided into 100 parts, being taken according to the power thereof, ſo as that any number of centeſms may be taken exactly therefrom.

And now for a further illuſtration of this our artificiall menſuration (or diametrall quadration) of a Circle, I will adde to the former one demonſtrative example more, and that in a larger Circle, whereby the verity hereof may be further manifeſted. Therefore admit the diameter of a Circle to be of the Prime Rationall line in generall, under a centeſimall ſolution, 14 (which may be here again for example ſake, A B, 14.00) ſo the Circumference will be (according to the common Cycloperimetricall Terms) exactly 44, and there-

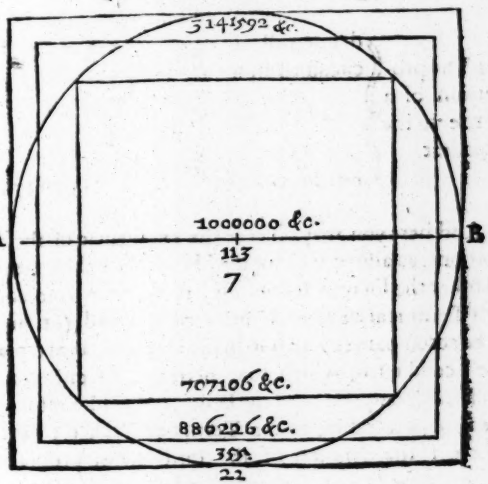
by the Area, exactly 154. But according to the later and better terms in a decimall expreſſion before declared; the circumference will be but (AB) 43.98 (according to the Cycloperimetricall terms in a more vulgar expreſſion, agreeing with the decimall to the firſt 7 places as before noted, it is $4\frac{111}{113}$ which is decimally alſo 43.98 as before) and ſo the Area (to ſquare centeſims of the Rationall Line) but 153.9380 (or $153\frac{106}{113}$, which agrees exactly with the other to ſquare centeſims, being by decimal converſion 153.9380) Now the ſaid diameter (A B) 14.00, being referred to our Line of Quadrature (or ſecond Rationall Line *C D*) for its meaſure, will be found but 12.41 *ferè*; whoſe Quadrat is 154.0081 *ferè* for the Area of the Circle, agreeing almoſt exactly with the Area found by the vulgar or *Archimedean* Cyclometry, which indeed differs here but little from the true Area, *viz.* 620 ſquare Centeſims by way of exces, or 620 of 10000, which make in vulgar accompt about $\frac{1}{16}$ of a ſquare unit. And our Meaſure exceeds the true meaſure by 701 ſquare centeſims only, or 701 of 10000, which in vulgar terms make about $\frac{1}{14}$ of a ſquare unit. But if the ſaid *Diameter* be taken by its *Line* of Quadrature in a milleſimall partition, then it will be found, 12.407, and ſo being ſquared will give 153.933649 for the Circular ſuperfice, comming now much neerer the true one, *viz.* 153.938040; it differing therefrom by way of defect, only ſo much as $\frac{1}{225}$ *ferè* of a ſquare unit; though indeed it came before as neer the ſame as need be deſired for any ordinary uſe.

And here wee now gather theſe profitable and delectable proportionall concluſions in the Circle, and that to a millimillenary ſolution of the unite.

1 The

1 The proportion of the Diameter to the

Circumference			3.141593 *ferè*
Side of the Square equall to the Circle (which is the most neare, precise, and proper squaring of a Circle.)	as 1.000.000	to	.886227 *ferè*
Side of ℈ inscribed square			.707107 *ferè*, $\sqrt{q\frac{1}{2}}$



Contrarily.

2 The proportion of the Circumference to the

Diameter,			.318310 *ferè*
Side of the Square equall to the Circle, (which is the second most proper quadrature of a Circle.)	as 1 to		.282095
side of ℈ inscribed Square			.225079

3 The proportiõ of the Quadrat. { Diametrall or circumscribing / Circumferential } to the Circle it self, as 1 to { ·785398 / ·079577 }

Contrariwise.

4 The proportion of a Circle to the Quadrat { diametrall or circumscribing / circumferentiall } as 1 to { 1.273240 ferè. / 12.566371 ferè. }

And here you may observe the excellencie of these proportions, as also of those of the like kinde in the Sphear, and all other the Figures following; in that the antecedent or first Term is always an Unit, accompanyed (or supposed to be accompanyed) with so many Cyphers, as are places in the second term, which may here signifie either a millimillenary composition of the Unity, or the like resolution of the same, according as the Terms are taken; by means whereof the solution of the question or proposition is mightily facilitated and expedited, according to what I said in the beginning: and therefore in the extraction both of these in the Circle, and also of those in all the other Figures following, I have used all possible industry and exactnesse.

Prop.

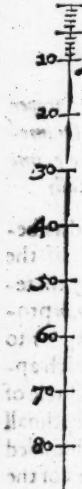
PROP. 2.

Of the ſolid dimenſion of a Spheare.

If the Axis or Diameter of a Spheare, be taken by its proper Line of ſolid Meaſure; I ſay, that the Cube of the Diameter ſhall be the ſolidity of the Sphear, according to the prime Rationall Line: which I thus demonſtrate practically.

ADmit the former Rationall Line A B (divided as before) and let the *Diameter* of a Spheare be of the ſame Line entirely taken 2.00, ſo the ſolidity thereof (according to the commonly received *Archimedean* proportion of the Cube of the *Diameter* to the Spheare, 21 to 11, ſuper-decupartient-undecimall) will be $4\frac{4}{11}$: which appears alſo by the Circumference of the greateſt Circle of the Spheare, $6\frac{2}{7}$ (in reſpect of the ſubtriple ſeſquiſeptimall proportion of the *Diameter* to it) for this being infolded with the diameter, gives the totall convex ſuperficies of the Sphear, $12\frac{4}{7}$, whole $\frac{1}{3}$, *viz.* $4\frac{4}{21}$ being infolded in like manner with the Semidiameter (A B 1.00) or the $\frac{1}{6}$ *viz.* $2\frac{2}{11}$ (anſwering to the ſquare Plane or Baſe of the inſcribed Cube) infolded with the whole diameter, produceth the totall ſphericall ſolidity $4\frac{4}{11}$ as before.

Now for the more artificiall and expeditious dimenſion of this Spheare (and ſo of all others by the ſame reaſon) and for the comparing of this our new way with the vulgar or naturall: I augment the line A B to .24 of the ſame, (for juſt ſo much is the additionall ſegment in Centeſimall parts) for the ſecond, ſuppoſed, compound, artificiall, (or Cubatory) Rationall Line, which is here the Line E F, ſo that E F is A B 1.24, and then dividing the ſame into 100 pares alſo: I meaſure the diameter of the Spheare A B 2.00 (being in one line at length) by the Line E F, (the Line or



Scale of Cubature) and finde it to be of the same 1.61, which Cubed, gives 4.173281, for the solid content of the Spheare, agreeing exactly with the former solidity in the integrall part: But the fractionall part of that solidity, $\frac{4}{21}$, being converted into milli-millesms, or Cubicall centesms, gives .190476, which our measure comes short of by 17195 cube-centesms, and which in common practise is not considerable, being in vulgar terms but $\frac{1}{58}$ of a Cube unit. But yet our solidity wanteth of the truest solidity not so much by 1686 Cube-Centesms, which in vulgar terms are but about $\frac{1}{593}$ (for so much the former common solidity doth here exceed the true solidity:) For indeed the true solidity of this Spheare will be found (according to the truest proportion of the diameter to the greatest Circles circumference, noted before in the dimension of a Circle, and so according to the truest proportion of the Cube of the *Diameter* to the Spheare it selfe noted afterwards) to bee (in the more common expression) $4\frac{64}{339}$. For the *Diameter* being 2, the greatest or true circumference will be $6\frac{32}{113}$ and so by the mutuall implication of these two, the totall Spherical superficies, $12\frac{64}{113}$, whose $\frac{1}{3}$ *viz.* $4\frac{64}{339}$, together with the semidiameter, or the $\frac{1}{6}$ *viz.* $2\frac{32}{339}$ with the *Diameter*, will produce the solidity of the Spheare, $4\frac{64}{339}$ as before; which the vulgar solidity $4\frac{4}{21}$ exceeds by $\frac{12}{7119}$, and which is very neer equivalent to $\frac{1}{593}$: Now this solidity, $4\frac{64}{339}$ being expressed decimally to cubick centesms, is 4.188790, and so the true solidity will be found to be by the decimall operation: For the *Diameter* being 2, the greatest Periphery will be found 6.283185, and thereby the totall superficies

12.566370

12.566370, whoſe $\frac{1}{3}$ being 4.188790 will here be the ſolidity of the Spheare, or the $\frac{1}{6}$ *viz.* 2.094395 being augmented by the *Diameter*, will produce 4.188790 for the ſolidity, as before; which the ſolidity found by our way, 4.173281, wanteth of but 15509 cube-centeſms, and which being reduced to more vulgar terms, make not ſo much as $\frac{1}{64}$ of a cubicall Integer or unit. And yet this inconſiderable defect happeneth for the like reaſon that was before declared in the meaſuring of the Circle: For that here the meaſure taken by the *Line* E F for the *Diameter* of the Spheare (being the double of the *Line* A B) is not preciſely 1.61, but rather more, though upon the *Line* it ſelfe, it cannot be diſtinguiſhed therefrom, being ſo very little, but only by number. (as the firſt *Line* A B ſimply, compared with the ſecond E F, is neither .80 nor .81 exactly, but will fall about the middle between them both, as you may perceive if you divide the *Line* A B into 1000 parts (which may eaſily be done) for then the *Line* E F will be thereof 1.241 *ferè*, and then the firſt compared with the ſecond (being alſo divided into 1000 parts) will be 0.806, which pointeth out about the middle between .80 and .81, upon the *Line* E F being 1000: for theſe two are in milleſms .800 and .810, but the true meaſure upon the *Line* E F is .806.) And therefore the *Line* E F being made 1000 parts, A B 2.000 (for the diameter of the Spheare) will be thereby 1.612, whoſe Cube is 4.188829-2928 for the ſolidity of the Spheare, which now inſenſibly differeth from the true ſolidity, being here correſpondently 4.188790205 *ferè*, it differing therefrom (by way of exceſſe) only 62723 cube-milleſms (or by contraction, about 63 cube-centeſms) that is, 62723 parts of the prime Rationall *Line* A B (being 1000) cubed, and ſo reſolved into 1000000000 parts, which in vulgar terms make hardly $\frac{1}{15943}$ of a Cube-integer or unit.

Again,

Again, ſuppoſe the *Diameter* of a Sphear ro be of the leſſer, or ſimple decimall Rationall *Line* A *b*, 1.5 (which is in parts only of the greater or centeſimall *Line* A B, 0.15) ſo the greateſt periphery wil be of the ſame, 4.7 (in parts only of A B 0.47) as formerly in the ſecond practical demonſtration npon the Circle) which two infolded together, do produce the ſuperficies of the Spheare in ſquare or ſuperficiall Integers and prime decimall parts of A *b*, 7.05 (in ſquare centeſimall parts only of AB, 0.0705) whoſe $\frac{1}{3}$ *viz.* 2.35 (or 0.0235) being infolded with halfe the *Diameter* A *b* 0.75 (or A B 0.075) or the $\frac{1}{6}$, *viz.* 1.175 (or, 0.01175) with the whole *Diameter*, A *b* 1.5 (or A B 0.15) do produce the totall Spheare in ſolid Integers and prime parts of A *b*, 1.762 (in ſolid centeſimall parts only of A B 0.001762) Now A *b* 1.5 for the *Diameter* of the Sphear, upon E *f* the proper correſpondent *Line* of Cubation (being A *b*, 1.2 (*viz.* A B 0.12) which in Centeſms make but 1.20, wanting indeed of the true additionall parts .04, for that the totall ſegment of A B in centeſms, to be added, is .24, as before hath been ſhewed) divided alſo into 10 parts, is 1.2, whoſe Cube is 1.728, for the ſolid content of the Sphear, wanting of the true content, only 34 cube-primes, or ſimple decimall parts, *viz.* 34 of 1000, which in a common arithmetical accompt, are only about $\frac{1}{35}$ of the *Line* A *b* (as the Integer of meaſure) cubed, and which difference will be in the centeſimall operrtion (by parts only of the *Line* A B) but 34 cube-centeſms, *viz.* 34 of 1000000, or more vulgarly 1 of 29412 *ferè*: For that the Diameter of the Sphear being put A B 0.15, the ſame will be upon the correſpondent *Line* of Cubation E F 0.12, whoſe Cube, 0.001728, is for the ſolidity of the Spheare, which wanteth of the true ſolidity, 0.001762, only as aforeſaid.

And here we may obſerve by the way, how this Sphear and

and the ſecond Circle before ſuppoſed, do agree, to wit, in that they both having the ſame diametral number, have alſo the very ſame dimenſionall or areall number, the one ſuperficiall, the other ſolid; *viz.* in Integers and parts of the Line *A b*, 1.76, &c. in parts only of the Line *A B* .0176, in the Circle, and .00176 &c. in the Spheare: which will further appear, if their common Diameter be taken according to a more ample diviſion (or partiall denomination) of the Rationall Line, and ſo conſequently, the ſuperficiall content of the one, and the ſolid content of the other, how far ſoever they be extended decimally, (and then alſo indeed they will be found ſomwhat greater.) And this may alſo plainly appear by the more uſuall or common arithmeticall expreſſion of their Diameter (and ſo of their other parts of dimenſion) *viz.* *A b* $1\frac{1}{2}$, (or *A B* $\frac{3}{200}$, in the leaſt proportionall terms to *A B* 0.15) and ſo their periphery, in the reaſon of the common (or *Archimedean*) Cycloperimetricall terms (7 and 22) *A b* $4\frac{5}{7}$; or rather of the *Metian* terms, 113 and 355 (before noted) $4\frac{161}{2260}$ (in proper parts anſwering to *A B* $\frac{3}{200}$ is *A B* $\frac{1065}{21300}$) whoſe $\frac{1}{2}$ *A b* $2\frac{161}{452}$ (or *A B* $\frac{1065}{42600}$) together with the ſemidiameter *A b* $\frac{3}{4}$ (or *A B* $\frac{3}{400}$) do produce the area of the Circle, $1\frac{1387}{1808}$; which converted into ſquare primes, or ſimple decimals, (according to the diviſion of the Line *A b*) is 1.76: or the area in parts only, $\frac{3195}{1808000}$ converted into ſquare ſeconds or centeſims (according to the partition of the Line *A B*) is 0.0176, as before. Then the Diameter, and ſo the Circumference of the Spheare being the ſame with thoſe of the Circle, theſe two conjunctly, produce the Sphericall ſuperficies according to *A b*, $7\frac{31}{452}$, (in parts only according to *A B* $\frac{3195}{904000}$) whoſe $\frac{1}{6}$ *viz.* $1\frac{483}{2712}$ (or $\frac{3195}{27120}$) together with the diameter $1\frac{1}{2}$ (or $\frac{3}{200}$) or the $\frac{1}{3}$ *viz.* $2\frac{483}{1356}$ (or $\frac{3195}{135600}$) with the Semidiameter $\frac{3}{4}$ (or $\frac{3}{400}$) do produce the Sphericall ſolidity, $1\frac{4761}{1424}$, which in the decimall expreſſion to milleſms, or cubicall prime de-

cimall parts (according to the diviſion of the Line *A b*) is 1.767 (and ſo much it is really; exceeding the ſolidity formerly caſt up, by 5 cube-primes:) And the ſolidity in parts only, $\frac{9585}{5424,000}$ is in milli-milleſms, or cube-centeſms (according to the diviſion of the Line *A B*) 0.001767. So as that the ſuperficiality of the Circle, and the ſolidity of the Sphear being put in vulgar fractional terms (either proper in reſpect of parts only, or improper in reſpect of Integers and parts) *viz.* the Circle $\frac{3195}{1808,00}$, and the Sphear $\frac{9585}{5424,000}$, they will be found to be proportionall among themſelves, *viz.* as the Denominatours are each to other, ſo alſo are the Numerators; for as 1808..00 to 5424..000, ſo 3195 to 9585, and ſo convertibly.

And here alſo (as before in the Circle) I have firſt uſed ſmaller numbers in demonſtrating the uſe of our artificiall Line, for the diametrall cubation of a Sphear: And therefore for a further & fuller clearing of the ſame, I ſhall adde one example in a larger Spheare, whoſe diameter or Axis let be according to any Rationall Line whatſoever, 21, exactly (which for the preſent purpoſe may here alſo be *A B* 21.00) ſo its cube 9261, and thereupon the totall Spheare it ſelfe (according to the fore-mentioned vulgar proportionall terms of the

the diametrall Cube to its Spheare, ſuper-decupartient-undecimall) will be exactly 4851. For the Diameter being 21, the greateſt circumference of the Spheare will be (according to the moſt vulgar Cycloperimetricall terms aforeſaid) exactly 66, and thereby the totall *Sphèricall*, exactly 1386, a ſextant whereof, *viz.* 231 (anſwering to the Baſe of the inſcribed Cube which ſubtends the ſame) being infolded with the Diameter (or a trient, *viz.* 462, with the ſemidiameter $10\frac{1}{2}$) produceth the ſolidity of the Sphear (as before) 4851. which may be obtained diverſe other wayes, (as *Clavius* ſheweth) but theſe are the readieſt. But now according to the moſt approved (decimal) Cycloperimetricall terms before declared, the diameter being 21.00, the greateſt circumference will be but 65.97 (or according to the *Metian* terms before noted, agreeing with the decimall to the firſt 7 places, it is $65\frac{110}{113}$) and ſo the Sphericall, or the ſuperficiall Area (to ſquare centeſims of the Rationall Line) but 1385.4424 *ferè* (or $1385\frac{50}{113}$) whoſe $\frac{1}{6}$ *viz.* 230.9071 *ferè*, or 230.907060 (or $230\frac{85}{678}$) together with the diameter, or the $\frac{1}{3}$ *viz.* 461.8141 (or $461\frac{92}{113}$) with the ſemidiameter 10.5 (or $10\frac{1}{2}$) produceth the totall Sphear or ſolid Area (to cube-centeſims of the Rationall Line) but 4849.048261 (or $4849\frac{93}{678}$, agreeing neerely with the decimall operation, this latter being in decimall terms, 4849.048672) which is leſſe then the ſphericall ſolidity found out firſt, by very neare two integers or units. Now the ſaid diameter (*A B*) 21.00, being taken by its Line of Cubature (*E F*) will be found 16.93 *ferè*, whoſe Cube is 4852.559557 *ferè*, for the ſolidity of this Spheare, exceeding the ſolidity firſt found, by 1.5, and the ſecond by 3.5, the reaſon of which differences I have declared partly in the former operations upon the Spheare and the Circle, and more fully hereafter; though in ordinary meaſuring, this difference

Clav. Geom. pract lib. 5. cap. 5. prop. 7

is of but little account, eſpecially in a Spheare of ſo great a magnitude. But if the diameter be taken by a Line of 1000 parts, then it will be found 16.926, which cubed, yields 4849.119178176, for the ſolidity, differing now from the true ſolidity, 4849.048260, &c (by way of exceſſe) only as much as $\frac{1}{74}$ of a Cube-unit. By which the verity of our diametrall-cubick dimenſion, or diametrall cubation of a Sphear, ſufficiently appeareth.

Another ſuperficiall (or quadrate) dimenſion of a Circle, and ſolid (or Cubick) dimenſion of a Sphear: *viz.* By their Circumferences: and the Artificiall Lines of Meaſure for the ſame.

And here note, that what we have now done in the Diameter of the Circle and Sphear, to make the Square of the one, equal to its Circle, and the Cube of the other equall to its Spheare: The like may be done alſo for their Circumferences: ſo the Line for ſquaring the Circumference of a Circle, will be of the prime Rationall Line (under a decu-milleſimal partition) 3.5449. and for Cubing the greateſt or true Circumference of a *S*phear, will be 3.8978: and which perhaps may be of more generall uſe then thoſe for the diameters, becauſe that commonly the Circumference of a Circle and *S*pheare (in materiate things, where the Center is not apparent) muſt firſt be had, before their diameter can exactly, (eſpecially in a *S*pheare) for then that muſt be had by proportionall argumentation; though indeed the diameter of a *S*pheare may be taken at firſt as well (if not better) by a paire of Callaper Compaſſes, where the ſame may be had ready upon occaſion, and then the Line for the diameter may better be uſed.

The ſuperficiall dimenſion of a Sphear, only by ſquaring its Diam. & Circumference. And the artificiall Lines of Meaſure for performing the ſame.

And other *L*ines alſo of the like nature may be fitted to the *D*iameter and Circumference of a *S*phear, as to *S*uperficiall Meaſure: As namely to make their particular

Qua-

Quadrats, equall to the convex ſpherical ſuperficies: So the Line of meaſure for this purpoſe, will be for the diameter, of the Rationall Line (deficiently) * 0.564 &c. (*viz.* as *AB* 10000, &c.—.4358 &c. frō the ſegment of diminution) and for the (greateſt or true) Circumference (redundantly) * 1.772 &c. And hence we may here raiſe this practicall propoſition. *viz.*

* *vid.* Diam. ad periph. 1 *vid.* 1—68-169. &c.

* *vid* Periph. ad Diam. 1. as in page 19. and ſee page 30. & 31. Numb. 2.

If the Diameter or Circumference of a Sphear, be taken by their proper and peculiar artificiall Lines for ſuperficiall meaſure; That then their ſeverall Quadrats, ſhall be equall to the Superficiall Area of the Sphear (or to the Spherical) according to the prime or naturall Rationall Line.

AND ſo ſuppoſing the former Sphear, whoſe diameter being firſt put (of the prime Rationall Line) exactly 21, the ſpherical convex ſuperfice was found (according to the true or greateſt Periphery 65.97 &c.) 1385.4424. Now the ſaid Diameter being taken by its proper Line of Quadrature (in a centeſimall partition) will be found 37.22, whoſe Square is, 1385.3284, for the ſpherical ſuperfice; agreeing almoſt exactly (in the very parts of meaſure) with the other.

And ſo again likewiſe the greateſt Periphery of a Sphear, being firſt put (of the Rationall Line) exactly 66, the true Diameter will be found (of the ſame) 21.008 &c. and thereby, the ſpherical ſuperficies, 1386.5579 *ferè*. Now the ſaid Periphery being meaſured by its proper Line of Quadrature (under a centeſimall ſolution) will be found, 37.24 *ferè*, which ſquared, gives 1386.8176 *ferè*, for the convex ſuperfice, or the Surface of the Sphear; exceeding the other,

only about $\frac{1}{4}$ of a ſquare integer (or unit) as of the Meaſure firſt appointed.

And here we now gather theſe uſefull and excellent proportionall Corollaries in the *Sphear*, *viz.*

Periphery of the greateſt Circle, *& contra*: is the ſame with that of the Diam. of a Circle to its Circumference, *& contra*.

1 The proportion of the Axis or Diameter to the

Side of the Cube equall to the *Sphear* (which is the neereſt, preciſeſt, & propereſt cubing of a Globe or Sphear.)

Side of the Quadrat equal to the convex ſpherical ſuperfice, (which may not altogether unaptly, be termed the ſquaring of a *Sphear*; but moſt truly the ſquaring of the *Spherical*: and that the neereſt and propereſt alſo.)

as 1. to

.806040

(*ferè*.

1.772454

2 The

2. The proportion of the greateſt or true Circumference to the

Side of the Cube equall to the *Spheare* (which is the next moſt proper cubing of a *Spheare.*)	as 1. to	.256556
Side of the Quadrat, equal to the convex ſuperficies; (which may be termed the next moſt proper ſquaring of the *Sphæricall.*)		.564189

3. The proportion of the Cube of the

Axis or Diameter	to the *Sphear* it ſelf as 1. to	.523599
Greateſt Periphery		.016887 *f.*

Contrariwiſe,

4 The proportion of the total Sphear to the Cube of the

Axis or Diameter.	as 1. to	1.909859
Greateſt Periphery.		59.217626

5 The proportion of the Quadrat of the

Diameter.	To the totall ſuperficies as 1. to	3.14159, &c. being the ſame with that of the Diam. to the Circumf.
Periphery		0.318310 *ferè*, being the ſame with that of the Circumference to the Diam.

6 The

Converſly.

6 The proportion of the totall ſuperficies of a Sphear to the quadrat of the

- Diameter — Is the ſame with that of the Peripheriall Quadrat to the totall ſuperficies, (or the Circumference to the Diameter.)
- Periphery — Is the ſame with that of the Diametrall Quadrat to the totall ſuperficies (or the Diam. to the Circumf.)

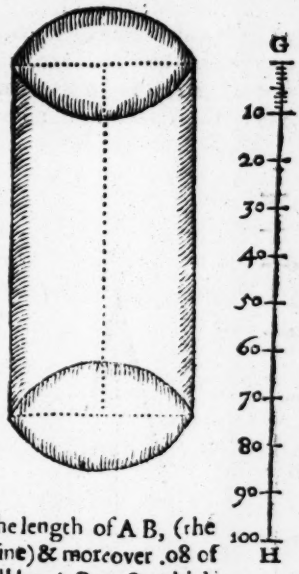
As for the proportion of the Diameter and greateſt Periphery to the ſide of the inſcribed Cube; I ſhall deliver the ſame afterwards, among the proportions in the five plaine Regular bodies inſcribed in a *Spheare*, *Part* 2. *Sect.* 3.

PROP. 3.

Of the ſolid dimenſion of a *Cylinder* and *Cone*.

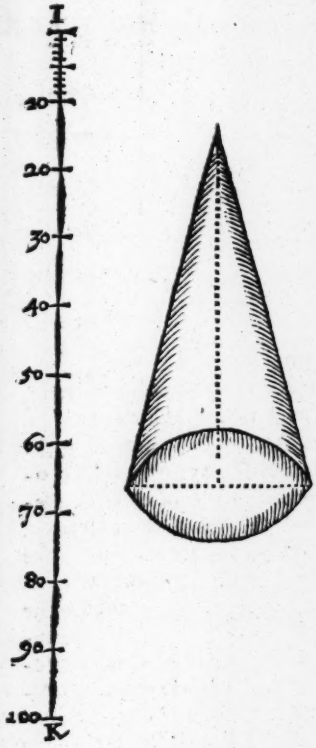
If the (baſiall) Diameter, and the Axis of a right Cylinder, be taken by their proper Line of Meaſure, and the Quadrat of the Diameter be augmented by the Axis; I ſay, that the reſulting rectangle, regular-baſed Priſma *or* Parallelepipedon, *ſhall be ſolidly equall to the Cylinder. And the like in a Cone. Both which I do here practically confirm.*

AS ſuppoſe here the former Rationall Line A B, and let the Axis of a right Cylinder, be only A B 0.75, and the Diameter A B 0.25, So the Circumference being of the



the ſame 0.78, the Baſe will be (in ſuperficiall centeſims only of A B) 0.0487, which augmented by the Axis wil produce the ſolidity of the Cylinder (in ſolid centeſimall parts only of A B) 0.036525. Now for the more artificiall and expeditious dimenſion of the Cylinder, & alſo a comparing of it with the former, vulgar or naturall way, and thereby a comprobation of the ſame: I draw the Line G H (for the ſecond, ſuppoſed, or artificial Rational *Line*) to the length of A B, (the primary, true, or naturall *Line*) & moreover .08 of the ſame, and ſo G H will be A B 1.08, which I alſo divide here as the *Line* A B. Then comparing the Axis A B 0.75 and the diameter A B 0.25, with their proper and peculiar *Line* of meaſure (or *Line* of rectangle-parallelepipedation) G H; I finde the Axis to be of the ſame, 0.69, and the *Diameter*, 0.23, whoſe Square 0.0529, for the Baſe, being drawn through the Axis, doth produce the Rectangle regular-baſed Priſm, or Parallelepip. 0.036501, for the Cylindricall ſolidity, which falleth ſhort of the former meaſure but 24 Cube-centeſims only, which in common accompt, make but $\frac{1}{41667}$ *ferè* of A B cubed: which diffetence would be 24 Cube-primes (or $\frac{1}{42}$ *ferè*) if the Cylinder were meaſured by the Lines of 10 parts (according as one

of the Circles and Sphears before propoſed) as *Ab* the naturall Line, and ſo the correſpondent artificiall Line, whoſe quantity would here be G H o. 1. (being *Ab* 1.1 *ferè*, viz. *AB* 0. 11 *ferè*) for then it would be in Integers and parts, 36.525 by the cómon or natural way, & 36. 501, by our way.

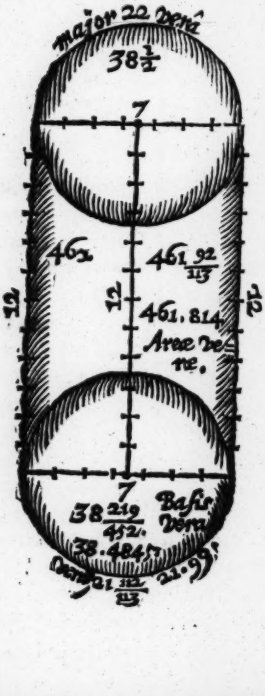


And by the ſame artificiall *L*ine of meaſure which is uſed in a Cylinder, may the ſolid content of a Cone be alſo obtained; the Diameter of its Baſe, and alſo its Axis being taken thereby, and ſo a Cylinder raiſed thereupon, (according to the foregoing propoſition) whoſe ſub-triple will be the content of the Cone; ſeeing that a Cone is the ſub-triple of a Cylinder of equall baſe and altitude (by *Eucl.* 12. *prop.* 10.) As let the baſiall Diameter, and the Axis of a right Cone be the ſame with thoſe of the foregoing Cylinder, *viz.* naturally, A B, 0.25, (the ſaid Diameter) and A B 0.75 (the Axis) then will the ſolid content of the Cone

Cone be naturally, 0.012175, as being a trient of the Cylinder 0.036525. And ſo the Diameter of the baſe being artificially G H, 0.23, and the Axis G H, 0.69, the ſolid content of the Cone will be artificially, 0.012167, as being the ſub-triple of the Cylinder. 0.036501, which wanteth of the true or naturall meaſure, only 8 ſolid Centeſims, and which in a more vulgar expreſſion, are but $\frac{1}{125000}$ of the prime Rationall *Line* A B (as the Integer of meaſure) cubed, the ſaid *Line* it ſelf ſimply (as the Root or Side) being then juſt 50 parts.

But indeed (which is the moſt abſolute, compleat, and compendious way) the very ſame artificial dimenſion which is uſed in a Cylinder, wil hold good in a Cone, and ſo may be as properly and fitly applyed thereunto, (according to the laſt Propoſition) whereby the ſolid content thereof ſhall be immediately produced, as that of a Cylinder (even as if it were a Cylinder) notwithſtanding the continuall diminurion of its body between the Baſe and the top or point. And the artificial *Line* of meaſure for this purpoſe; I noted at the beginning, to be of the Rationall *Line* in generall, 1.5632 *ferè*, and which for the preſent purpoſe will here be (according to the foregoing practicall demonſtrations) of the centeſimall *Line*, A B 1.56, which is here the *Line* I K, (divided as the former *Lines*) By which the baſiall Diameter of the fore-ſuppoſed Cone, (naturally A B 0.25) being taken, the ſame I finde to be thereof, 0.16, whoſe Quadrat, 0.0256, is for the Baſe of the Cone: And then the Axis (naturally *A B* 0.75) being alſo meaſured by the ſame *Line*, becomes thereof, 0.48 *ferè*, which wholly infolded with the whole artificiall Baſe, produceth the ſolidity of this acute-angled Cone, 0.012288 *ferè*, which exceedeth the true ſolidity before-noted, *viz.* 0.012175, by 113 ſolid centeſms only, and which in vulgar terms, make but $\frac{1}{8850}$ *ferè*, *viz.* as one part of a ſolid Integer or unit, divided into 8850 parts.

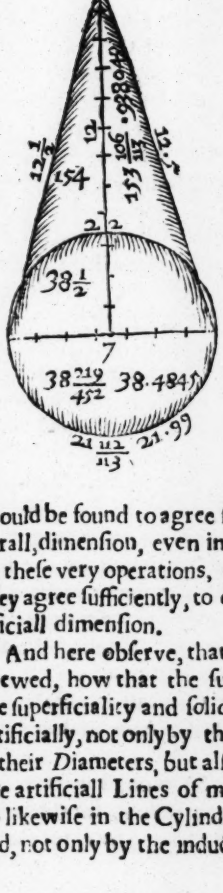
And thus having first wrought by smaller numbers of dimension (or parts of measure) in the Cylinder and Cone, as before in the Circle and Spheare, for the demonstrating of their artificiall solid dimensions: I will here adde one example in larger numbers of dimension, which shall serve for both these Solids together, whereby the sufficiencie of this our artificial dimension in the same may further appear. Let therefore the Diameter of a Cylinder be of the prime Rationall Line (whatsoever it be) 7. (or which may be here again for demonstration-sake, *A B* 7.00) so the Base thereof wil be immediately (according to the common Tetragonismal terms, super-tripartient-undecimall, before mentioned) $38\frac{1}{2}$. Or the Diameter being 7, the Circumference wil be (by the common Cycloperimetry) 22, and thereby the base $38\frac{1}{2}$: or by *Euclids* tetragonismall reason (before declared) by diminishing the diametral Quadrat 49 by $\frac{1}{12}$ of the same, *viz.* $10\frac{1}{2}$, and so there remains $38\frac{1}{2}$ for the Basial Area, as before. But by the later and better Cycloperimetricall Terms, the *Diameter* being (*A B*) 7.00 the Periphery wil be but (A B) 21.99 (or by the more vulgar expression of



of the ſame terms, it will be $21\frac{113}{115}$, which agrees decimally with the former) and thereupon the Baſiall Area (to ſquare centeſims of the Rationall Line) 38.4845 (or $38\frac{219}{453}$, agreeing with the decimall Area.) Then let the Axis of the Cylinder be (A B) 12.00, into which being drawn the ſaid Baſiall Area, there will ariſe the ſolid Area of the Cylinder, 461.814120 (or $461\frac{92}{113}$, which by the firſt baſe $38\frac{1}{2}$, is 462 exactly) Now the ſaid Cylindricall *Diameter* (A B) 7.00 being tried by its proper Line of Quadrature (G H) will be found 6.46 *ferè*, for the artificiall Diameter, which ſquared, gives 41.7316 *ferè*, for the Artificiall baſe: and the Altitude (A B) 12.00, being tryed by the ſame Line, will be found 11.07, for the artificiall Axis of the Cylinder, by which the ſaid Baſe being increaſed, there reſulteth the Cylinder (according to the naturall or prime Rationall Line) 461.968812 *ferè*, agreeing with the former exactly in ſolid Integers (of the ſame Line) and not conſiderably differing therefrom in parts, it amounting but to $\frac{1}{4}$ of a Cube-Integer or Unit, at moſt, by way of exceſſe.

And ſo a Cone being of equall Altitude and Diameter (in the Baſe, and ſo of equall baſe) with the foreſaid Cylinder, the ſub-triple thereof ſhall be the totall Oxygoniall Cone, *viz.* 153.938040 (or $153\frac{10}{115}$) which by the firſt baſe $38\frac{1}{2}$, is 154 exactly; and which agreeth exactly with the Circle laſt handled whoſe *Diameter* was put 14) as appeareth by the moſt uſuall dimenſion of a Cone, by infolding the Baſe with a trient of the Altitude, which here being (A B) 4.00, produceth the Conicall ſolidity as before, (or a trient of the baſe with the Axis prodnceth the ſame.)

Now the Baſiall *Diameter* (A B) 7.00, by its proper Line of Quadrature (I K) will be 4.48 *feré*, for the artificiall *Diameter*, whoſe Quadrat is 20.0704 *feré*, for the ar-



tificiall Baſe of the Cone: And the Axis or altitude (A B) 12.00, will be found by the ſame Line, 7.68 *ferè*, (for the totall artificiall Axis of the Cone) which infolded with the whole Baſe, produceth the totall Cone in the naturall meaſure (according to the prime Rationall Line) 154.140672 *ferè*, which agrees neerly with the moſt true meaſure, *viz.* 153.938040, it exceeding the ſame only about $\frac{1}{7}$ of a Cubick or ſolid Integer or Unit. And if theſe two dimenſions were performed by Lines of a more ample or numerous diviſion, as 1000, &c. then they would be found to agree ſtill more and more with the naturall dimenſion, even in the very parts of meaſure: though in theſe very operations, which we have here performed, they agree ſufficiently, to demonſtrate the verity of our artificiall dimenſion.

And here obſerve, that as in the Circle and Sphear, we ſhewed, how that the ſuperficiality of the one, and both the ſuperficiality and ſolidity of the other, were obtained artificially, not only by the bare Quadration and Cubation of their *D*iameters, but alſo of their Circumferences, by the like artificiall Lines of meaſure accommodated to them: So likewiſe in the Cylinder and Cone, may the ſolidity be had, not only by the induction, implication, or involution of

of the Quadrat of their Baſiall *D*iameter into their Axis (as we have already ſhewed in both of them) but alſo of the Quadrat of their Baſiall circumference into their Axis, theſe being (both of them,) taken by one and the ſame *L*ine of meaſure: So the *L*ine for ſquaring the circumference (of the Baſe) of a Cylinder, (for its Baſe) will be of the Rationall *L*ine, 2.32489: and for ſquaring the Baſiall circumference of a Cone, 3.3531; which two Lines being divided as the former, and the Baſial circumferences (as alſo the Axes) of theſe two Bodies taken thereby, may ſomtimes prove to be of more ſatisfaction then thoſe for the baſiall *D*iameters, to wit, where the baſiall centers of theſe Bodies are not apparent, (eſpecially if the Baſes of the Cylinder or Cone to be meaſured be very large) according as I noted before in the Circle and Sphear. But indeed, where the *D*iameters may be firſt exactly taken, either in theſe two Figures, or in the other two beforegoing, it will be much eaſier and readier in practiſe, then by the Circumferences, in regard both, that the artificiall *L*ines of meaſure ſerving thereunto, are much ſhorter, and alſo that the *D*iametrall numbers being much leſſer or ſmaller then the circumferentiall, the Arithmetical operations following thereupon, in caſting up the ſuperficiall and ſolid contents, wil be ſooner expedited, unleſſe the ſame be performed Geometrically (as I may term it) or Inſtrumentally, *viz.* by Scale and Compaſſes, or the like (as I ſhall in the very cloſe of this Book by way of concluſion declare) for then the latter may be performed thereby, even as ſoon as the former.

The ſolid dimenſion of a Cylinder and Cone by their Baſiall Peripheries, and their Axes together. And the artificial Lines of meaſure for the ſame.

And now you may obſerve here by the way, how that although the baſe of a Cylinder and Cone be a Circle, and its Area be here had alſo, by the only quadration of

of the *D*iameter or Circumference, as before was done in the Circle, whereby the ſame artificiall *L*ines of meaſure that are there uſed, might alſo perhaps ſeem to ſerve here, (and which indeed in ſome ſort may, as I ſhall ſtraightway ſhew) yet for the obtaining of the ſolidities of theſe Bodies, by the implication or induction of the Quadrat of their Baſial *D*iameter or Circumference into their Axis (either of theſe two baſiall *L*ines, with the Axis, in each of theſe two Bodies, being taken by one and the ſame proper, diſtinct, artificiall *L*ine of meaſure, as before hath been ſufficiently ſhewed) the *L*ines there appropriated to a Circle cannot here hold for their Baſes; for the *D*iameters or Circumferences hereof being taken by thoſe *L*ines, wil be either greater or leſſer (in the number or parts of meaſure; according to quantity diſcrete) then by their own proper *L*ines; (and ſo their Quadrats for the Baſes, and conſequently the ſolid contents wil be greater or leſſer then they ought to be; what artificiall *L*ine of meaſure ſoever, the Axis ſhould be meaſured by.) For ſo in a *C*ylinder and *C*one, the *D*iameter of the Baſe being taken by the artificiall *L*ine belonging to the *D*iameter of a *C*ircle, for its qnadrate dimenſion thereby (being ſimply and abſolutely conſidered in it ſelfe alone as a *C*ircle) will be found in the *C*ylinder to be ſomwhat leſſe, and in the *C*one much greater, (in the number or parts of meaſure, as in a Quantity diſcrete) then being taken by its proper artificiall *L*ine for the quadrate dimenſion of the Baſe, as in reference to the ſolid dimenſion of the *C*ylinder and *C*one, performed wholly by the ſaid *L*ine of Meaſure, from the Baſiall *D*iameter and the Axis together.

And then, both in a *C*ylinder and *C*one, the circumference of the Baſe being taken by the artificiall *L*ine, belonging to the circumference of a *C*ircle ſimply, for its quadrate dimenſion thereby; will be found leſſe (as falling under

der the notion or nature of a Quantity diſcrete, as aforeſaid) then by its proper, reſpective, artificiall *Line*, for the ſquaring of the baſe thereby, as in relation to the ſolid dimenſion of the *Cylinder* and *Cone*, wholly performed by the ſaid *Line*, from the Baſiall *Circumference*, and the Axis together.

As in the Cylinder and Cone laſt handled; where the Diameter of the Baſe being put naturally (*A B*) 7.00, the ſame was there found to be artificially (*G H*) 6.46 *ferè*, and (*I K*) 4.48 *ferè*, and which will be found by the Line (of Quadrature) pertaining to the Diameter of a Circle, ſimply (*C D*) to be 6.20, *viz.* ſomwhat leſſe then that of *G H*, and much greater then that of *I K*.

And ſo there again, the Circumference of the Baſe being naturally (*A B*) 22 *ferè*; the ſame will be found to be artificially, by the Line for ſquaring the (Baſiall) Circumference of a Cylinder (as in reference to its ſolid dimenſion by the Circumference and Axis together) 9.46; and by the Line for ſqnaring the baſiall Circumference of a Cone (as in relation to its ſolid dimenſion in the like manner) 6.56, and which by the Line for the Circumferentiall quadration of a Circle, ſimply, will be found 6.20 (as the Diameter before, by its proper Line of quadrature (C D) which is leſſe then either of the other.

Therefore, if the baſiall Diameter or Circumference of a Cylinder and Cone, be taken by the Lines of (Diametrall and Circumferentiall) quadration, properly, peculiarly, and ſimply pertaining to a Circle, and ſo its Quadrat be made the Baſe of the Cylinder or Cone; then muſt the Axis be taken by the Prime Rationall Line: (And ſo the dimenſion will be mixt.) For that here the Area of the Baſe, will fall immediately, in the true, naturall, meaſure, (as under the dimenſionall reaſon of the prime, true, or naturall Rational

Line) according as hath been demonſtrated before in the dimenſion of a Circle.

As here the Baſiall diameter of the Cylinder and Cone, naturally (*A B*) 7.00 taken by the Line *C D* is 6.20 (or 6.2, as I ſhewed even now) whoſe Quadrat. 38.4400 (or 38.44 only) for the Baſe, (which was found before, to be moſt truly and naturally, 38.4845) being drawn into the Axis, put before, naturally, (*A B*) 12.00, will give the Cylindricall ſolidity (in the dimenſionall reaſon of *A B*) 461.280000, (or 461.28 only) and ſo the Conicall ſolidity, 153.76; which differ (by way of defect) from the true, naturall ſolidities (produced wholly by the Line *A B*) *viz.* of the Cylinder, 461.814120, about $\frac{1}{2}$ of a cubique Integer or Unit, as of *A B* cubed: and ſo of the Cone, *viz.* 153.938040, about $\frac{1}{6}$ of a cube-integer only, as of the ſame Line Cubed; And the ſolidity of the Cylinder, produced artificially by its proper Line of meaſure (*G H*) *viz.* 461.968812 *ferè*, differeth therefrom (by way of exceſſe) only about $\frac{1}{6}$ of a cube-integer; and the ſolidity of the Cone produced in like manner, by its proper artificiall Line of meaſure (*I K*) *viz.* 154.140672 *ferè*, only about $\frac{1}{7}$ of a cubique or ſolid Integer of the naturall meaſure (as of *A B* conſidered cubically) as before hath been ſhewed,

All which ſeverall ſolid dimenſions (both naturall and artificiall, and mixt of both) doe ſo neerly agree one with another, as that their differences are altogether inconſiderable. And the ſame will happen in the ſolid dimenſions of theſe two Bodies, by the other Line of Quadrature pertaining ſimply to a Circle, as to the quadration of their Baſiall Peripheries, for their baſiall Area's, and withall by the prime or naturall Rationall Line, as to the dimenſion of their Axes or Altitudes.

And

And now, as for the Axis (or Altitude) of a right or erect Cone, you may here obſerve, that though the ſame cannot immediately be taken Inſtrumentally; or by a Line of Meaſure (as it is within the body of the Cone) as that of a Cylinder (being parallel to, and ſo agreeable with the ſide,) by reaſon of the inequality of its Body betweene the Baſe and the Cuſpe, or verticall point; but being obtained purely Geometrically, muſt be had by mediation of the Side (as being the ſide of a rectiline rectangle Triangle, ſubtending the right angle, and therefore potentially equall, or equally potent to the two containing, comprehending, or including ſides. by *E.*1.*p.*47.) and of the baſiall Ray, (being one of the ſides about the right angle (and moſt commonly the leſſer,) and ſo the ſide of the Cone, and the radial line of its baſe, being firſt taken Inſtrumentally or Mechanically, the Axis will be had Trigonometrically, according to the reaſon of the fore-cited *prop.* of *Euclid.*) Yet may the ſame be obtained moſt readily (out of the Cone) by a Line of meaſure, if from a Plane conſtituted in the top, or verticall point of the Cone, parallel to the Baſe, you let down a Perpendicular-line to the Plane on which the Baſe is, (and which may fall preciſely upon the Baſiall periphery) for that (being meaſured) ſhall be equall to the true Axis of the Cone. And thus alſo may the altitude of any oblique, ſcalene, or inclined Cone be taken; and alſo of an oblique, ſcalene, or inclined Cylinder; if from the top of the Cone to the Plane in which the Baſe is ſet, or from the ſuperiour baſe of the Cylinder, to the Plane of the inferiour baſe, be let fall a perpendicular-line; for that (being meaſured) ſhall be the altitude of the oblique, or inclined Cylinder or Cone; and ſo being infolded with the whole baſe, if a Cylinder (whether meaſured by the naturall or the artificiall Line of meaſure) ſhall make the ſolidity of the ſame: or

being infolded with a trient of the baſe, if a *Cone*, (or the baſe with a trient of that) and meaſured by the naturall Line; or elſe the whole perpendicular of altitude with the whole baſe, if meaſured by the artificiall *Line* proper to a *Cone*, ſhall produce the ſolidity of the oblique *Cone*; ſeeing that ſuch a *Cylinder* and *Cone*, is equall to a right *Cylinder* and *Cone*, having the ſame baſe and altitude with it, (according to the reaſon of E. 12. p. 11, & p. 14.)

See alſo E. 11. p. 31, & E. 1. p. 35, 36, 37, 38.

And after the ſame manner will the true altitude of any Pyramid, whether right or oblique, be had, as of a right or oblique *Cone*, and the altitude of an oblique or inclined Priſme, as of an oblique or inclined *Cylinder*; the dimenſion of which bodies, eſpecially the Pyramidal (both ſolidly and ſuperficially) according to our new artificiall way (together with the naturall or vulgar, by way of dimenſionall compariſon) I ſhall ſhew next after the dimenſion of right-lined ordinate Planes or Superficies, ſeeing that upon any of them may be erected or conſtituted a Pyramid or Priſme; as a Cone or Cylinder upon a Circle.

SECT.

Sect. III.

Of the Superficiall Dimenſion of a Cylinder and Cone.

AND as we have here ſhewed the moſt artificiall and expeditious dimenſion of the *Cylinder* and *Cone*, in reſpect of their *Stereometry*, or ſolid meaſure; ſo we ſhall likewiſe demonſtrate their dimenſions in reſpect of *Planometry*, or ſuperficiall Meaſure. And the artificiall *Lines* for the performing hereof, I finde, for the *Cylinder*, (in reſpect of its Diameter & Side together) to be of the prime Rationall *Line*, the ſame with that which was formerly found to be for the obtaining of the Superficies of a Sphear, by the only quadration of the Diameter, *viz.* 0.564, &c. And for the Cone (in relation to its baſiall diameter, & its ſide conjunctly) 0.79788 (which according to the parts of diminution, is as A B 1.0000c___.2021a) and in relation to its baſiall periphery and ſide together, it will be 1.4142, &c, (*viz.* $\sqrt{q}$ 2) which ſeverall Lines being exactly ſet off from the prime Rationall Line, and then divided as the ſame, and ſo the baſiall diameter, and the ſide of a Cylinder, and of a Cone; and alſo the baſiall Circumference, and the ſide of a Cone together, be taken by their peculiar, reſpective, diſtinct artificiall Lines of Meaſure, and ſo multiplyed together, their ſeverall products ſhall be the ſuperficiall contents of the Cylinder and Cone, according to the prime Rationall Line: where, by the ſuperficies of a Cylinder

and Cone, muſt be underſtood without their Baſes; for ſo the Superficies of theſe two Solids are generally taken by Artiſts, and are called by the Latin Geometers, eſpecialy *Ramus*, (in one word) *Cylindraceum*, and *Conicum*, that is as much as to ſay in Engliſh, the Cylindraceall or Cylindricall, and the Conicall; and ſo the Superficies of a Spheare is called by them *Sphericum*, *viz.* the Sphericall. But now for a briefe dilucidation of theſe ſuperficiall dimenſions in the Cylinder and Cone, I ſhall lay down an example in each of theſe three artificiall Metricall Lines (though here I do not draw them, but only expreſſe them by number) whereby the verity of theſe our dimenſions alſo, may plainly appeare, to thoſe that ſhall pleaſe to make tryall thereof. And firſt we may hereupon raiſe this practicall Propoſition, *viz.*

If the (baſiall) Diameter, and the Side of a right Cylinder; And the baſiall Diameter or Periphery, and the Side of a right Cone, be taken by their proper and peculiar, diſtinct artificiall Lines of Meaſure, and the ſame be ſeverally infolded together: That the reſulting rectangle Parallelogram, *ſhall be equall to the Cylindricall and Conicall Superficies, (according to the reaſon of the prime Rationall Line.)*

THerefore, ſuppoſe here firſt the Cylinder laſt handled, whoſe Diameter was put (of the Prime Rational Line) exactly 7, and ſo the Circumference (according to the moſt common Cycloperimetricall terms) was exactly 22, and the altitude (which is the ſame with the ſide) was put exactly 12, and theſe two infolded together, do produce the Superficies of the Cylinder (without the two Baſes) exactly, 264: whereby this Cylinder becomes abſolute in all its

its dimenſions; But by the other Cycloperimetricall terms, the Circumference was but 21.99 &c. (or $21\frac{113}{115}$) whereby, the Superficies becomes now (to ſquare centeſims of the Rational Line) but 263.8938, (or more vulgarly $263\frac{101}{115}$) Now if the Diameter of the Cylinder naturally 7, be meaſured by its proper artificial Line of ſuperficiall meaſure (being as A B 0.56, made 100) the ſame will be found, 12.41 *ferè*, and the Side of the Cylinder naturally 12, being meaſured by the ſame Line, will be found, 21.27 *ferè*, which two multiplyed together, produce 263.9607 *ferè*, for the ſuperficies of the Cylinder, agreeing with the true ſuperficies, exactly in Integers (or Units) of meaſure, and differing therefrom in the fraction-parts (by way of exceſſe) but as $\frac{1}{14}$ of a ſquare integer or unit. And if the firſt or naturall Line of meaſure be made 1000, and ſo the ſecond or artificiall *Line* (being thereof 0.564) be alſo made 1000, then the ſaid Diameter taken thereby, wil be 12.407, and the Side wil be 21.269, which two multiplyed together, will produce the Cylindrical Superfice, 263.884483, differing from the true one, *viz.* 263.893783 (now by way of defect) only as $\frac{1}{108}$ *ferè*, of a ſquare integer or unit of the appointed meaſure. Which excellent compend in the ſuperficiarie dimenſion of a Cylinder by its Diameter and Side only, will further and fullier appear, if it be compared with the moſt naturall or vulgar way, in reſpect of the Side and Diameter only given: where, by the Diameter, the Circumference being proportionally obtained, muſt then be multiplyed into the ſide, to make the Superficies (for that the Superficies of a right Cylinder is moſt naturally a Plane made of the Circumference and the ſide thereof) and the obtaining of this by the Diameter and Side; Mr. *Oughtred* in his Book entituled *The Circles of Proportion, Part* 1. *Chap.* 7. *Sect.* 10. delivers in this proportionall manner.

As

As 7 to 22, Or 1 to 3.1416.

So the Diameter and ſide multiplyed together,

To the Superficies (*viz.* without the two baſes.)

And ſo in this our Cylinder, the Diameter being put 7, and the ſide 12, I ſay,

As 7 to 22, or rather, 1 to 3.14159 &c. (or 113 to 355)

So 7 into 12, *viz.* 84,

To 264, the Superficies,

Or rather 263.89378 &c (or $263\frac{10}{113}$) as firſt by the Circumference and ſide together: which two ſeverall operations you ſee do conſiſt of two ſeverall Multiplications, and one Diviſion, whereas ours conſiſteth of one Multiplication only, to wit, of the Diameter and Side together.

And from this Analogicall operation uſed by Maſter *Oughtred*, for the diſcovering of the ſuperficies of a right Cylinder, by its ſide and (baſiall) Diameter, you may here note, how that it is demonſtrated by *Archimedes*, *Lib.* 1. *de Sph. & Cylind. prop.* 13. that the Superficies of a right Cylinder (without the Baſes) *is* equall to a Circle, whoſe Semidiameter is a mean proportional *Line* between the ſide of the Cylinder, and the Diameter of its Baſe.

And ſo in this our Cylinder, the ſide being 12. and the Diameter 7, the mean Proportionall between them, wil be $9\frac{3}{19}$, or rather by an immediate decimal extraction of the parts, 9.16515 &c (whereas the other is decimally by converſion of the parts, but 9.15789 &c.) which being put as the Semidiameter of a Circle, and ſo the Square thereof 84, the Area of the Circle wil be found 263.89378 &c. agreeing exactly with the Superficies of the Cylinder. Or the Area is by the *Metian* Cyclometry, $263\frac{101}{113}$, for the Superficies of the Cylinder, as before.) For ſeeing here, that the Semidiameter of the Circle equall to the Superficies of a right Cylinder is the mean proportionall betweene the Side and (ba-

(baſiall) Diameter of the Cylinder; and that the Analogie holds the ſame from the Quadrat of a Circles Semidiameter to the Circle it ſelfe, as from the Semidiameter to the Semicircumference, or the * Diameter to the Circumference, (only the one Analogie is Lineall, and the other Superficiall.) Therefore the Analogie of the ſide and (baſial) Diameter of a right Cylinder infolded together, to the Superficies thereof (without the two Baſes) wil hold the ſame, as of the Diameter of a Circle to its Circumference.

* *Quæ eſt ratio totius ad totum, eadem eſt ratio dimidii ad dimidium; Et ſic alicuius partis ad aliquam partem conſimilem ſeu respondentem.*

Again, ſuppoſing the baſiall Diameter and the ſide of a right, erect, or Iſoskelan Cone, to be the ſame with thoſe of the foregoing Cylinder, *viz.* 7 and 12; the ſuperficies will be halfe the ſuperficies of the Cylinder: for that the ſuperficies of a right or Iſoskelan Cone is a rectangle Plane made of the baſiall ſemi-periphery and the ſide (or the baſiall periphery and the ſemi-ſide) and ſo the true Conicall ſuperfice or ſurface, will be 131.9469, (or $131\frac{107}{113}$, agreeing decimally with the former, or by the moſt common accompt, 132 exactly) And the finding of the true, genuine ſuperficies of a right Cone by the ſide and baſiall Diameter, the foreſaid Mr. *Oughtred* alſo ſheweth in the 8 Sect. of tha ſame Chapter before named, by this Analogie, *viz.*

As 7 to 22, Or 1 to 3.1416:
So the Semidiameter of the Baſe multiplyed with the ſide,
To the Superficies (*viz.* without the Baſe)

And ſo the ſide of the Cone being here put 12, and the baſiall Diameter 7, I ſay,

As 7 to 22 (or 113 to 355) or rather 1 to 3,14159 &c.
So $3\frac{1}{2}$. (or 3.5) with 12, *viz.* 42. to 132 exactly:

Or more truly, $131\frac{107}{113}$, or 131.9469, juſt as before, for the

Conical. So that in both theſe wayes alſo, are two ſeverall Multiplications and one Diviſion, beſides the bipartion or mediation of the baſial diameter: or the ſame is performed by halfe the ſide with the whole *D*iameter, (for both come to one paſſe.)

And from hence you may obſerve, that *Archimedes, lib. 1. de Sph. & Cylind. prop.* 14. (from whence the foreſaid Analogie is deduced) demonſtraterh the ſuperficies of a right or Iſoskelan Cone (without the Baſe) to be equall to that Circle, whoſe ſemidiameter is the mean proportional line between the ſide of the Cone, and the *S*emidiameter of its Baſe. And ſo in this our Cone, the *S*ide being 12, and the baſiall *S*emidiameter $3\frac{1}{2}$ (or 3.5) the mean proporticnall between them is $6\frac{6}{113}$, or rather by immediate decimall production of the parts, 6.48074 &c. (whereas the other is by decimal reduction, but 6.461538 &c.) which being put for the ſemidiameter of a Circle, and ſo the Quadrat thereof 42, the Area wil be found 131.9469 &c. agreeing exactly with the foregoing *S*uperficies of the Cone: (or the Circular Area, is from the *Metiau* Cyclometry, $131\frac{107}{113}$, which is decimally, 131.9469 &c. agreeing exactly with the true Conicall *S*uperficies, found out decimally at firſt.

Archim. lib. 1. de Sph. & Cyl. pro. 15

And here you may by the way take notice of the next Propoſition of *Archimedes* concerning the ſuperficiary dimenſion of a Cone, laid down Analogically thus, *viz.* That the ſuperficies of every Iſoskelan Cone, hath that rationality to the Baſe, as the ſide of the Cone hath to the *S*emidiameter of the Baſe. For ſo here,

As 12 (the ſide)
To $3\frac{1}{2}$, or 3.5 (the Radius of the Baſe)
So $131\frac{107}{113}$, or 131.9469 (the true ſuperficies)
To $38\frac{210}{413}$, or 38.4845 the Baſe;

(And

(And ſo was the ſame found to be before, in the ſolid dimenſion of a Cone, and of a Cylinder,) And contrariwiſe will the reaſon of the Baſe be to the ſuperficies of the Cone, as of the Semidiameter of the Baſe to the Side of the Cone: For ſo,

As $3\frac{1}{2}$ or 3.5, to 12;

So $38\frac{219}{451}$ or 38.4845, to $131\frac{107}{113}$, or 131.9469.

Now the Baſe being added to the foregoing Superficies, there will ariſe the totall Superficies of the Cone, $170\frac{195}{451}$, or 170.4314 &c. which by the moſt common accompt, will be $170\frac{1}{2}$, the Conical being juſt 132, and the Baſe $38\frac{1}{2}$.

And the very ſame Analogie with the former, holdeth in the right, erect, or rectangle Cylinder, from the Superficies to the double of the Baſe, (or both the Baſes joyntly) and contrarily; becauſe that the Superficies of the *Cylinder* is double to the Superficies of the *Cone*, having the ſame Side, and *D*iameter in the Baſe, as I noted before: And ſo here,

As 12 (the Side)

To $3\frac{1}{2}$ or 3.5 (the baſiall ray)

So $263\frac{101}{113}$ or 263.89378 &c. (the Superficies)

To $76\frac{219}{226}$, or 76.9690 &c. (the double of the Baſe, or the aggregate of the two Baſes.)

And ſo contrarily, will the Analogie hold from the double of the Baſe, or the two baſes conjunctly ($76\frac{219}{226}$, or 76.9690 &c.) to the *Cylindrical* Superficies ($263\frac{101}{113}$ or 263.89378 &c.) as from the Radius of the Baſe ($3\frac{1}{2}$ or 3.5) to the ſide of the *Cylinder* (12.) And laſtly, if to the foreſaid *Cylindricall* Superficies $263\frac{101}{113}$, or 263.8938 *ferè*, you adde the aggregate of the two Baſes, $76\frac{219}{226}$, or 76.9690, you will have the total externall ſuperficies of the *Cylinder*, $340\frac{195}{226}$, or 340.8628 *ferè*, which by the moſt vulgar ac-

compt, would be 341 exactly; the *Cylindraceal* (or *Cylindrical* Superfice) being juſt 264, and the two Baſes together, 77

But now for a more ſpeedy dimenſion of the Superficies of the foregoing *Cone*, and firſt by the Side and baſial diameter only, together: I ſay, that if the ſide of the *Cone* (naturally 12) be meaſured by its proper Line of Meaſure for this purpoſe (which is in centeſimal parts only of the prime Rational Line, 0.80 *ferè*) under a centeſimal ſolution; the ſame will be found thereby, 15.04; and the baſial *Diameter* (naturally 7) meaſured by the ſame Line, will be found 8.77, which two being multiplyed together, do produce 131.9008, for the *Conical* Superfice, which wants of the true content, only $\frac{1}{33}$ *ferè* of a ſquare or ſuperficial Integer or Unit of the Meaſure firſt aſſigned. And ſo again, if the baſial Periphery of this *Cone*, naturally, 22 *ferè*: be taken by its proper, reſpective, artificial Line of Meaſure for this purpoſe, (being of the prime Rational Line, in a centeſimal ſolution, 1.41) centeſimally divided; the ſame will be found thereby, 15.56 *ferè*: and the Side of the Cone being alſo taken by the ſame Line, will be found 8.48; which two multiplyed together, do yeild 131.9488 *ferè*, for the Superficies of the *Cone*, which agreeth ſomwhat more neerly with the true *Conical*, *viz.* 131.9469, then that which was produced in this kinde by the baſiall *Diameter* and the Side together, *viz.* 131.9008. for this latter (by the baſial periphery and the Side) differeth from the true one (by way of exceſſe or redundancie) but 19 ſquare or ſuperficial centeſms *ferè*, *viz.* 19 of 10000, which in a vulgar Arithmetical expreſſion, are hardly $\frac{1}{526}$ of the prime Rational *Line* (as the Integer or Unit of meaſure) ſquared: ſo that both theſe wayes for the finding of the ſuperficies of a *Cone*, conſiſt alſo but of one Multiplication only, as that for the *Cylinder*.

But

But for finding the ſuperficies of a *Cylinder* in this manner, by an artificiall Line of meaſure, peculiarly fitted to the *Circumference* and Side together, the ſame is not to be done, ſeeing it is performed immediately by the naturall Line of meaſure, or the Prime Rationall Line it ſelfe, with the ſame expedition as that in the *Cone* by an Artificiall Line; for that the *Plane* made of the *Circumference* and Side of the *Cylinder*, taken by the prime or natural Line of meaſure firſt appointed (according as I ſaid at firſt) doth conſtitute the true *Cylindraceall*, or the Cylindricall ſuperfice. And ſo from the premiſſes it appears, that as a Cylinder in regard of ſolid dimenſion, is the triple of a Cone having the ſame Baſe and Axis, or Altitude: So in reſpect of ſuperficiary dimenſion, it is the double of a Cone having the ſame Baſe and Side or *Longitude*.

And as for the Baſes of a Cylinder and Cone, if they be required in their ſuperficiary dimenſions, the ſame may moſt readily be obtained by either of the *Lines* of Quadrature pertaining to a Circle, according as the Diameter or Circumference thereof ſhall happen to be taken: For ſo the baſiall Diameter or Circumference of the foregoing Cylinder and Cone (naturally 7.00, and 22 *ferè*, *viz.* 21.99) being meaſured by the ſaid Lines of diametrall and circumferentiall quadration (under a centeſimall ſolution) will be found each of them ſeverally; to be 6.20 (as I noted before in their ſolid dimenſions) which ſquared, yeilds 38.4400 for the Baſe; and the true baſe was formerly found, 38.4845; from which ours differs, (by way of defect) not ſo much as will make in vulgar terms, $\frac{1}{22}$ of a ſquare Integer or Unit.

And thus having ſufficiently declared and demonſtrated the dimenſion both Solid and Superficiall of a Cylinder and Cone (both theorically, and) practically, according to the Inſtrumentall part of Geometry; and that as well naturally

(by way of metricall compariſon) as artificially, for the confirmation and verification of our artificial way of meaſuring: I ſhall next lay down the ſame Dimenſions numerally, in Terms analogicall (by way of compariſon) from the naturall Meaſure to the artificiall; and that under an ample or numerous ſolution of the unity (according as I did in the Circle and Spheare) by means whereof, the artificiall Meaſure may readily be deduced from the naturall; or the naturall meaſure be reduced to the artificiall: And firſt for the Cylinder,

The true or natural Meaſure is to the artificial, in reſpect of the

- Axis, or Altitude, conjunctly with the (baſial) Diameter
- Axis, or Altitude, conjunctly with the (baſial) Circumfer.

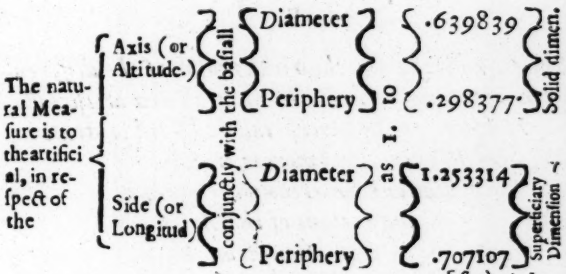
as 1. to

- ·92264 *ferè* — Solid meaſure.
- ·430127. — Solid meaſure.

Side (which in a right, erect, or rectangle Cylinder, is equall to the Axis or Altitude) with the (baſial) *D*iameter, as 1. to 1·772454 *ferè*, (being the ſame with that which was noted before in the Spheare, for the proportion of the *D*iameter to the ſide of the Quadrat equall to the Spherical.) — Superficiall Meaſure.

Then

Then for the Cone;



The laſt of which propoſitions, is the ſame with that of the Diameter of a Circle to the ſide of its inſcribed Quadrat, noted before in the dimenſion of a Circle.

SECT.

SECT. IV.

Shewing briefly, the theoricall reason of the differences happening between the naturall and artificiall Measure, in the superficiall and solid contents of Figures. And moreover, some observations concerning the grounds and reasons of the Artificiall Mensuration in generall.

AND now again, as for the differences happening between the superficiall and solid Contents of Figures, found by the naturall or vulgar way of measuring, and our artificial way; we have formerly shewed, how small and inconsiderable they generally are; and also the practical, instrumental (or Geometricall) reason thereof; *viz.* that the severall Lines of dimension in the severall Figures (either naturally belonging to them, or artificially and commonly ascribed to them) as namely, the Diameters and Circumferences of the Circle and Sphear; and so the Diameters, Circumferences and Axes, or Altitudes, and Sides, of the Cylinder and Cone, beforegoing, (and so of all the other Figures following respectively) taken by their proper, respective artificiall *Lines* of Measure, are seldome or never exact and precise indeed in the parts of measure, but either deficient or redundant in the same, and so give the areall

areall contents of those figures either a little lesser or greater, then indeed they are (though for the most part lesse, especially in the two first decimal partitions of the Lines of measure, *viz.* into Centesms, or only into prime or simple Decimal parts, or Tenths) as appeared formerly (and will also afterwards) by a continuall decimall augmentation (or subdecuplation) of the parts of those Lines, whereby the Contents were had still neerer to the truth; Which reason ariseth from (and so dependeth upon) the more true, natural, theoricall (or the Arithmetical) reason of these differences, lying in the extraction of the Square and Cube-Roots: For that the Roots of numbers not exactly Square and Cubical, cannot be exactly had, but are alwayes defective, so as that they being inverted, or drawn again into themselves, do not render the numbers precisely, out of which they were extracted; but the further that the extraction is extended or continued decimally, by the adjection or apposition of more Figures, (or of Cyphers where there is need) the neerer still to the truth will the Root be had; as I noted at the * beginning, where I took occasion to speak of the excellencie of Decimall Arithmetick, as in reference to the work in hand: So that what errour (though inconsiderable) may at any time arise in our work, doth proceed

* Where I might also have particularly expressed one thing more (to the young practitioner) amongst other particulars concerning the decimal solution of Unity, (and which I may here, not altogether un-oppertunely do, though it be there included and understood in the Generall,) *viz.* That thereby, all the tedious and troublesome operations of Arithmetick in Fractions, by the vulgar way, are wholly avoyded: for that here all fractional numbers, whether comming alone by themselves, or together with integrall numbers, are wholly and universally wrought as integrals, without any manner of preparatory operation, (as Reduction or transmutation of terms, one way or other,) which in the working of vulgar Fractions is necessarily required.

ceed primarily and principally from the extraction of the Squate and Cube-Roots; our artificiall Lines of meaſure giving immediately the Square and Cube-Roots of the Figures to which they are appropriated and applyed, (or the ſides of their equall Squares and Cubes, as nearly, almoſt as may be) according as their dimenſion is ſuperficial or ſolid, as being naturally (as it were) procreated or derived from them; except it be in the Cylinder and Cone, and the other regular-like Solids following; as regular-baſed Pyramids and Priſms: but elſe in all truly ordinate Superficies and Solids; as the Circle and Spheare beforegoing, and ſo the equiterminall and equiangular Superficies and Solids following, it holdeth ſo: And ſo likewiſe in the Cylinder and Cone, and other regular-like Solids, both for ſolid and ſuperficiarie dimenſion, where there is a congruity between their Lines of dimenſion, by which their ſolid and ſuperficial Conetnts are obtained, as the Diameters, Peripheries, Sides, and other dimenſionall Lines of their Baſes with their Axes or Altitudes for ſolid dimenſion, and with the ſides of their Bodies for ſuperficiall dimenſion. And therefore if there were no defection in the aforeſaid Radicall extractions, there would be none in our work; for that all our artificiall Lines of meaſure (or Lines of artificiall meaſure) being thus Radically produced, are themſelves in the nature of Roots quadrate and cubique. And ſo alſo, all the Numbers or Terms of Quadrature and Cubature, &c, by which the ſides of the equall Squares and Cubes of Figures, (in the naturall Menſuration) and alſo ſome other particular lines, or numbers of dimenſion, in the artificiall Meaſure, deduced from the naturall) are proportionally obtained ſeverall wayes (according to unity) as from their ſeverall lines of dimenſion before named (and which alſo are immediately given by

our

our artificiall Lines of meaſure) are produced by Radical extraction, from the reſpective figures, to whoſe dimenſion (ſuperficiall or ſolid) they ſerve, as being their Roots, or the ſides of their equall Squares and Cubes, according to their foreſaid ſeverall dimenſionall Lines, put unitly. But indeed, the greateſt errour that can commonly ariſe here, (in reſpect of the difference of our meaſure from the true meaſure, whether ſuperficiall or ſolid,) will be of no moment, as I have ſhewed before (and ſhall alſo ſhew after) in ſeverall examples. And if our meaſure agree with the true meaſure, but exactly to integers or units (as it almoſt always doth, and much neerer alſo; as even to ſmall parts of an Integer or unit, (ſuperficiall or ſolid) of the appointed Meaſure) it will be ſufficient in any matter of mechanicall Menſurations; for which, this our artificiall way of Meaſuring, was chiefely deviſed and intended.

Concerning the grounds and reaſons of the Artificiall meaſure in generall.

But now further, as to the ground and reaſon, briefly, of this kinde of dimenſion (or of theſe artificial metricall Lines,) the ſame may be underſtood to be two-fold, to wit, generall or univerſall; and ſpeciall or particular; that conſiſting in Unity alone; this in the ſolution of Unity: For the generall reaſon is by it ſelfe abſolute, ſimple, and certain, without relation or limitation to any the proper, compounding, denominate (or other) parts of the Rationall Line, (as being the Integer or Unit of meaſure) but conſidereth the ſame generally (and abſolutely in it ſelfe) as ſome one entire or whole thing, (of which I ſhall (God willing) ſpeake more fully afterwards, in the cloſe of the ſecond part of this Book, as being a place convenient.) But the particular reaſon (whith I ſhall chiefely inſiſt and proceed upon) is limited and confined to the certain, ſet, or commonly known parts of every particular rationall

onall Line (they being conſidered diſcretely or Arithmetically (taken as a common, known Meaſure, and that, according to ſeveral places & cuſtoms, (or ſuch like other parts, as the Geometer or Artificer ſhall in his minde think fit privately to impoſe on the ſame for his uſe) and ſo commeth by them to that of the Generall. (Every particular, figurate Quantity or Magnitude, meaſurable in this artificiall manner, being here conſidered in thoſe parts, according to their powers Quadrate or Cubick (or the Parts conſidered in their ſaid powers Arithmetically, in every particular Figurate Magnitude, according as the dimenſion is ſuperficiall or ſolid.) For every common or cuſtomary Line of meaſure is uſually divided into certain denominate parts, of which it doth primarily and properly conſiſt: As our Foot is vulgarly ſaid to be divided into 12 parts immediately, called Inches, which do compoſe or conſtitute the ſame: Or (as in divers Countries beyond the Sea) a foot may be underſtood with us to be compoſed firſt (and that moſt nearly) of 4 Palms, or Hand-breadths; a Palm (being compoſed) of 4 Digits, or Finger-bredths; and a Digit of 4 Grains or corns of Barley; according to the Latin Diſtich,

Quatuor ex granis, Digitus componitur unus:
Eſt quater in Palmo digitus: quater in Pede palmus.

Vitruv. Lib. 3. Architect. cap. 1.

And this according to the deſcription of the ancient Roman Foot by *Vitruvius* and others: So that thus the Foot contains 16 Digits, anſwering to 12 inches with us, for that a * Palme (namely *Palmus minor*) is ſaid to be 3 of our Inches, and ſo 4 Palms 12 Inches: But we in *England* uſually taking no notice of the Palme and Digit in meaſuring, but only of the Foot and Inch (beſides the Yard and Ell &c.) divide the

* See Circles of Proportion, Part 1 Chap. 9. Sect. 4.

Foot

Foot immediately into 12 parts, called by the Latines *unciæ* & *Pollices* or *Pollicaria*, and ſo alſo was the Roman Foot anciently divided (and ſtill is) and ſo is the Foot in many other places. So that every ſuch greater Meaſure is commonly compoſed of the next leſſer, being ſome certain times reiterated or repeated: As our greateſt common Geometricall Line of meaſure, *viz.* a Perch or Pole (for Land-meaſure) is commonly compoſed of * 16½ Feet, a Foot being compoſed of 12 Inches (as I ſaid before) and an Inch of three barley-corns in length, picked out of the middle of the Eare: but a Barley-corn being the leaſt of all Meaſures (or rather no meaſure at all, being but the very beginning of meaſure, as an Unit is uſually counted no number it ſelfe, but only the beginning of Number) cannot be compoſed of any other.

* Statute meaſure *An.* 33. Edw. 1. other (cuſtomary) meaſures there be, as the Pole or Perch of 18 feet, uſuall for Wood-land meaſure, &c.

And although Arithmetick in generall, naturally taketh no notice of theſe leſſer Meaſures as the proper compoſing parts of the greater Meaſure given; but immediately conſiders every particular Meaſure (as an Unit) according to a ſimple or naturall Arithmeticall diviſion into parts (or a diviſion into parts ſimply or arithmetically denominate) as halves, quarters, and the like; or more eſpecially (and more exquiſitely) as Decimall Arithmetick; into Tenths, Centeſms, &c. and ſo taketh the Contents of Superficies and Solids, to ſuch parts thereof, Square and Cubick (or otherwiſe Superficiall and Solid) in generall; as Square and Cubick parts of a Foot, and of an Inch; ſquare parts of a Perch or Pole; and ſo alſo, ſquare parts of an Acre, &c. Yet this being underſtood only by Artiſts, and ſo not ſufficient to ſatisfie the vulgar: theſe ſimple or naturall Arithmeticall parts (or parts meerly diviſionall) muſt at laſt be reduced

to the proper compounding, denominate, or commonly known parts of the Meaſure given or appointed; (or the Geometricall or menſurall parts of the ſaid Meaſure (as I may term them) they being by themſelves alone, put as meaſures certain, and intire, and ſo compounded of, (or diviuall into) other the like kinde of parts, or inferiour Meaſures, (according as I noted even now) and ſo are to be conſidered as Quantities continuate:) As the parts or fractions of a Foot into Inches; of a Perch or Pole into Feet, (if they be required) and ſo likewiſe of an Acre into Perches, &c.

And now again, the ſpeciall, particular, or partiall reaſon aforeſaid, of this artificiall dimenſion (or of the Artificiall Lines of Meaſure) or the Reaſon of the Parts, as I may term it, from what I have declared before:) may be conſidered in a twofold reſpect: *viz.* either more generally; as relating to the denominate, compounding or Geometricall parts of every Meaſure firſt given or appointed (as the prime or naturall Rationall Line) only for the diſcovering or producing of the artificiall Lines of meaſure, as conſidered in the generall Reaſon, (or the Reaſon of the Whole;, as I may call it, from what I have ſaid before concerning the ſame) ſuch as are all the artificiall Lines beforegoing, and alſo the other following, being expreſſed by Quantity diſcrete, or number, which ſhew their Magnitudes (or quantities in meaſure) from the intire, prime, or naturall Line of Meaſure in generall, from which they are to be taken; And which therefore I may call (not unaptly, I conceive) their Indicant or Exponent Numbers. Or more particularly, preciſely and properly; as relating meerly to the foreſaid parts of every particular Meaſure given (conſidered diſcretely or Arithmetically) for the conſtituting of the ſaid artificiall Lines of Meaſure ſo, as to give the

ſuper-

ſuperficiall and ſolid contents of Figures, quadratelyand cubickly, &c. (or the ſides of their equall Squares and Cubes, &c.) from their ſeverall lines of dimenſion belonging to them, and by which ſeverally they may be thus artificially meaſured; as the forementioned artificiall Lines do) only, (or for the moſt part) according to thoſe parts, (being conſidered meerly in the nature of ſuch parts of the prime or naturall Line of meaſure appointed; which otherwiſe taken apart by themſelves alone, may be put as meaſures entire (or Integers of meaſure) as I noted even now; and then are conſiderable in our generall Reaſon of Meaſure, or Reaſon of the Whole, before named.) And which Lines therefore being moſt properly conſidered and laid down from the foreſaid parts of the naturalMeaſures appointed, (as before, the Lines are from the whole intire Meaſures themſelves) will alter continually in every particular Figure, (in reſpect of quantity diſcrete, according to their Arithmeticall Indices or Exponents, which expreſſe their magnitudes in the forenamed parts of Meaſure) not only in regard of the different dimenſionall lines thereof, by which it may be propounded to be artificially meaſured, as aforeſaid, and ſo to which they are reſpectively fitted: (as the artificiall Lines beforementioned do, according to their like Indices or Exponents before declared; eſpecially for the moſt part; though it doth happen otherwiſe ſometimes, as that one and the ſame artificiall Line, is found either wholly throughout, or ſufficiently in part, (that is, in reſpect of the fractionall or decimall part of the naturall Line of meaſure, from which it is taken, ſhewed by the Arithmeticall Index or Exponent, how

Here note another the like artificiall dimenſion of Figures (as the former) not mentioned before; which is only according to the compounding, denominate (or Geometrical) parts of Meaſures given or appointed.

how far ſoever the ſame be continued or extended decimally) to ſerve unto ſeverall Dimenſions; as I ſhew afterwards, in *Part* 2. *Sect.* 3.) but alſo, in reſpect of each one and the ſame particular dimenſionall line thereof, by it ſelfe alone, according as the ſaid parts of the Meaſure propoſed, do (arithmetically) alter: whereas the arithmeticall Exponents of the firſt mentioned Lines, relating generally to any whole Meaſure appointed (as I noted at the beginning) do continue the ſame (without alteration) in every particular, diſtinct Dimenſion of one and the ſame Figure, according to its ſeverall lines of Dimenſion aforeſaid. And ſo theſe two ſeverally mentioned (or ſuppoſed) ſorts of artificiall Lines, will hereupon differ in every ſeverall dimenſion of one and the ſame Figure, in reſpect of quantity diſcrete or Arithmeticall, (according to their Indices or Exponents of meaſure) though they doe not, in reſpect of quantity continuate or Geometricall; or of Meaſure it ſelfe in generall, as I ſhall ſhew by and by: And then alſo they differ herein; That as the former artificiall Lines being immediately of the whole intire naturall *Lines* (either redundantly or deficiently, according as their Arithmeticall Exponents do ſhew) conſidered without reſpect of parts, compounding or Geometricall, (but only Arithmeticall, or meerly diviſionall, as is alwayes neceſſarily required, for the exactneſſe of meaſure) and ſo giving the Areall contents of Figures accordingly; are themſelves to be conſidered (in the artificiall Dimenſion) as whole intire *Lines* of meaſure in like manner: Theſe latter mentioned (or ſuppoſed) *Lines* muſt (for the contrary reſpects aforeſaid) be conſidered (in the like Dimenſion) as *Lines* of meaſure containing (or compounded, as it were, of) certain parts, anſwering (Arithmetically) to the primary or compounding parts of the naturall *Lines*, from which they are (moſt properly) derived, and ſo

which

which they do artificially repreſent, and conſequently, according to which (chiefly) they give the ſuperficiall and ſolid contents of Figures (as aforeſaid.) And which Lines (for the reaſons before alleadged, as alſo for diſtinctions ſake) we may well call, the particular or ſecond artificiall Lines (or the Lines of parts) as the other may be called the prime or integrall artificiall Lines (or Lines of the Whole) And ſo the Meaſure ariſing there from, may accordingly be called, the one, the prime or integrall Meaſure (or Meaſure of the Whole) as having its denomination ſimply and abſolutely from the whole intire naturall Line of Meaſure appointed: And the other the particular or partiall Meaſure (or Meaſure of the parts) as being denominated chiefly from the parts of the ſaid Line of Meaſure.) And therefore, as thoſe firſt Lines do artificially repreſent the reſpective naturall Lines from which they are taken, conſidered ſimply and intirely in themſelves, as the Integer (or Unity) of Meaſure ſeverally: So theſe ſecond Lines, do accordingly repreſent the ſaid naturall Lines, as they are compoſed of certain denominate, or menſurall parts; (*viz.* of ſome inferiour or leſſer Meaſure, conſidered as a part of ſome greater Meaſure, and ſo ſome certain times iterated or enumerated, for the making up of the ſame, according as I lately ſhewed,) And ſo theſe ſecond Lines are really none other then the firſt, divided into the like number of parts, as are the compoſing, conſtituting, (or Geometricall) parts of the reſpective naturall Lines from which they are deduced; and which parts of theſe (ſuppoſed) ſecond artificiall Lines, we may conveniently call (by way of diſtinction from other parts) their prime or Geometricall parts: and theſe being then divided ſeverally into ſome certain number of parts, as is requiſite for the exactneſſe of Meaſure, as aforeſaid (according as the correſpondent parts of the naturall

Lines are) eſpecially decimall; we may call the ſame, their ſecond or Arithmeticall parts, (or the particles of meaſure, as being indeed only the parts of the other Parts.) So that theſe ſecond Lines are never exactly decimall, as the prime Lines perpetually are, unleſſe it happen, that the prime or compounding parts of the naturall Meaſure appointed be in a decimall number, for then as the naturall Line, ſo likewiſe the correſpondent particular, or ſecondary artificiall Line will be exactly decimall; their ſaid prime parts being divided decimally; but yet however, they do generally perform the Dimenſions wrought by them, after a decimal manner (though ſecondarily, *viz.* after a reduction of the Meaſure taken by them, into their prime parts, &c. As firſt (for example) in the naturall Menſuration; the moſt common, mechanicall way of meaſuring by the foot with us, is, as the ſame is vulgarly divided into 12 Inches, which are, as its compoſing, denominate, or Geometricall parts (for ſo every meaſure is commonly ſaid to be divided into the parts, of which it is properly compoſed) and each Inch divided into ſome certain parts (as is neceſſary for exactneſſe in meaſuring) which may beſt be decimall: and ſo the lines meaſured hereby, do fall out moſt frequently, in Feet, Inches, and parts of Inches together: Now if the ſides or other dimenſionall lines of any Superficies or Solid propounded to be meaſured, be thus found (in this mixt meaſure (as they will for the moſt part) then muſt the ſame be firſt of all reduced into Inches, &c. before the content of the Figure, ſuperficiall or ſolid, can be conveniently caſt up, ſeeing that the Meaſure thus taken is mixt (as it were) of ſeverall parts or kindes of meaſure, and ſo is of different denominations; and therefore muſt be reduced into one kinde of meaſure (or one menſurall denomination:) And then if the parts of the Inch be decimall, the work will be afterwards performed in

in a decimal manner, in Inch-meaſure only: But if the ſaid lines thus meaſured, be found wholly in Feet (without Inches, &c.) then will the contents be had immediately in whole Foot-meaſure, wch otherwiſe muſt be had in the like meaſure by Reduction from Inch-meaſure. And ſo ſeeing that the ſecond artificiall Lines doe repreſent the naturall Lines, only as being compoſed of certain denominate, or menſurall parts, and ſo are to be conſidered themſelves accordingly, as aforeſaid: Therefore, if the ſide or other dimenſionall Line of any Figure meaſured thereby, for the obtaining of its content ſuperficiall or ſolid, by way of Quadrature, Cubature, &c. and ſo artificially repreſenting the Side of the equall Square or Cube, &c. be found mixtly, in Integers and prime parts, &c. of the ſame Lines; (as for the moſt part they will) then muſt the Meaſure ſo taken, be firſt reduced wholly into their ſaid prime parts, &c. for the caſting up of the content as aforeſaid, (according to what was ſhewed before in the naturall Menſuration, after the moſt plain or vulgar way, by a common naturall Line of parts) And then thoſe artificiall prime, or menſurall parts, being divided Decimally; the dimenſion of the Figure propoſed, will be performed in a Decimall manner, according to thoſe parts only: But if the ſide, or other line of the Figure to be meaſured, doe fall out in Integers only of the ſaid artificiall Lines, (*viz.* in whole Lines without any parts;) then will the Areall content be obtained immediately, and exactly, in Integers of the naturall Meaſure appointed (according to the reaſon of the prime artificiall Lines, or Lines of the whole Meaſure) which otherwiſe can be had in the integrall meaſure (or the meaſure denominated only from the whole naturall Line) only, by way of reduction from the primary, compounding, denominate, or

metricall parts of the ſame, in which it is firſt found, as before was ſhewed.

And now therefore it appears from hence, that the artificiall Dimenſion performed by theſe ſecond or particular artificiall Lines, or Lines of parts, ſo called; or the artificiall Lines, as they are conſidered meerly in the particular or ſpeciall reaſon of the artificiall Dimenſion (or Reaſon of the Parts) taken in the latter reſpect; is not conſiderable in compariſon of the artificiall Dimenſion performed by the prime or integrall (or more generall) artificiall Lines, or Lines of the Whole; or the artificiall Lines, as they are conſidered in the generall reaſon of the ſaid artificiall Dimenſion (or Reaſon of the Whole) they giving the ſides of the equall Squares, and Cubes, &c. of Figures (and ſo their ſuperficiall and ſolid contents accordingly) immediately in Integers (and decimall parts) of the prime Rationall Line, or the naturall Meaſure appointed; which the particular Lines do give (for the moſt part) mixtly in Integers and compounding parts and particles together (or in Integers and parts primary, and ſecondary) and ſo we muſt come at laſt to the Areall content in the former Meaſure, by way of reduction from thoſe Parts, as aforeſaid. But yet however; for the variety of operation and Art, in this kinde of Menſuration, I thought it would not be amiſſe to manifeſt thus much concerning this latter way; that ſo the ingenious Reader that ſhall pleaſe to exerciſe himſelfe (practically or experimentally) in this artificiall way of meaſuring, may (by comparing the effects or reſults of the generall and particular, or ſpeciall reaſon thereof together, both in the extraction or production of the artificiall Lines themſelves, & alſo in the ſaid two ſeverall wayes of working by them) receive a more full ſatiſfaction

faction, and the thing it ſelfe be accordingly confirmed.

And now ſeeing that the aforeſaid parts of Meaſures given, are various (as in quantity Geometricall, ſo uſually in quantity Arithmeticall) according as the Meaſures themſelves are in magnitude various: therefore I ſhall firſt and principally proſecute our ſaid ſpeciall or partiall Reaſon of the artificiall dimenſion, and that chiefly in relation to the firſt, or more generall acception or conſideration thereof, (namely, for the producing of the artificiall Lines, as they are conſidered in the generall Reaſon, by the parts of the Meaſure given (or the prime artificiall *Lines* by the ſecond, as we have differenced or diſtinguiſhed them firſt of all, in Quantity diſcrete, or in the Number of their meaſure from the naturall *Line*, according as the ſame is underſtood, either without or with the foreſaid kinde of parts.)

And this I ſhall accordingly lay down in three ſpeciall Theorematicall Propoſitions (contained in the ſecond part of this Work) anſwering to the three principall Problematicall or practicall Propoſitions beforegoing (laid down in the Figures particularly handled in this firſt Part, namely the CIRCLE, SPHEARE, and CYLINDER, together with the CONE) and which will generally ſerve for all other ordinate or regular, and regular-like Figures whatſoever, for the like occaſion (according to what I noted at the beginning) And the laſt of theſe only I ſhall demonſtrate or illuſtrate by Number, as being ſufficient for all, though indeed there needeth no manner of demonſtration or illuſtration of any of them, they being all ſo very plain and perſpicuous.

PART II.

Containing the moſt artificial and expeditious practicall Dimenſion, of all right-lined ordinate or regular, and regular-like Figures in generall.

SECT. I.

Propoſing the foreſaid Dimenſion in all right-lined regular Planes *or* Superficies *in generall: And demonſtrating the ſame particularly in two of the firſt of them.*

THEOREME I.

Exhibiting particularly, the fore mentioned Lines, for the Quadrature of a Circle; from our particular or ſpeciall ground and reaſon before declared. And conſequently, the Lines for the like dimenſion of all rectiline or angular ordinate Planes in generall.

IF the Diameter of a Circle equal to the Quadrat from the Parts aforeſaid, of the Rationall *Line*, be found; The ſame ſhall be the reſpective Line of Quardature, according to the Parts: And the reaſon between that, and the correſpondent or congruall Tetragoniſmall *Line*, according to

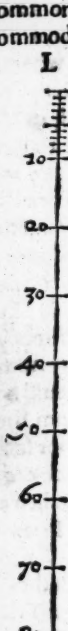
to the whole *Meaſure* will be ſuch as the reaſon between the Parts, and the Whole; which is as the reaſon of their Squares. And the like for the *Periphærie*.

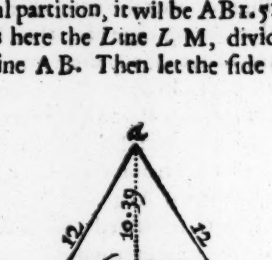
AND the ſame Reaſon holdeth (as aforeſaid) in all other ordinate, (or in al angular or rectiline regular) Planes in general; as the equilateral and equiangular *Trigon*, *Pentagon*, *Hexagon*, and all ordinate or regular *Polygons* whatſoever, in reſpect of the ſeveral dimenſional lines belonging to them, and by which they may be propounded to be thus artificially meaſured; as their Diametral, Diagonal and perimetral or *Lateral* lines, &c. Of which Figures, we will here practically demonſtrate our artificial Dimenſion (as alſo the natural, by way of metrical compariſion) in the two firſt of thoſe here particularly named, to wit, the *Trigon*, and *Pentagon*, (according as we did firſt of all in the only oblique-line or curviline ordinate Plane, namely the Circle;) and the rather alſo, for that theſe two do concur to the compoſition of four of the five famous ordinate plain Bodies, (or five angular or rectiline regular *Solids*) commonly called the *Platonical* Bodies, namely, the *Trigon*, to the *Tetra-edron*, *Octa-edron* and *Ficoſa-edron*; and the *Pentagon* to the *Dodeca-edron*. And then I ſhall next of all after, ſpeak briefly ſomewhat in the like kind, concerning the ſecond angular or rightlined regular Plane, (which goes to the compoſition of the other ordinate plain Body named *Hexa-edron*) called ſpecially and peculiarly (by way of excellency above and ſo diſtinction from all other quadrangular and quadrilateral Planes) by the *Greeks*, Τετράγωνον, and Τετράγωνον, and by the *Latines* more diſtinctly, *Quadratus*, and *Quadratum*. And ſeeing, that the ſides of the aforenamed Figures in general, are chiefly conſiderable, being the only proper natural lines of dimenſion pertaining to them, as bounding in & including their *Area's*, (they being conſidered ſimply & abſolutely

ly

ly in themſelves alone, as ſuch Figures, without any further circumſtances) and ſo are only apparent of themſelves naturally in any materiate thing coming under the form of any ſuch Figure, to be meaſured; which the other lines of dimenſion aforeſaid uſually are not, (according to what I noted formerly in the Diameters of the Circle, *Sphear*, *Cylinder* and *Cone*,) as being indeed no proper, natural, or eſſential parts of the ſame, but only adventitial, adſcriptitial, or accidental, (as it were) as happening to them after they are made; being uſually adſcribed to them by Geometricians, as helps chiefly, for the obtaining of their Area's (in any Meaſure aſſigned) after the natural or vulgar way of meaſuring; which being once had, theſe lines are again of no account; and ſo are uſually drawn in books, obſcurely, by ſmal points or pricks only. Therefore I will here demonſtrate the dimenſion of the two forenamed Figures only from their ſides. And indeed theſe Figures in general, ſhould be rather denominated from their ſides, as it were from the Cauſe, then from their Angles, as from the effects: for ſo a Triangular Figure is defined by *Euclid*, *El.* 1. *def.* 20 &c. from its three ſides; whereupon it might better be called a *Trilater*, or *Tripleuron*, (as from the Cauſe) then a Triangle or Trigon (from the effect) as *Ramus* ſpeaketh *lib.* 6 *Geomet. El.* 6.) eſpecially ſeeing that three ſides with three angles, are in no wiſe reciprocal, or convertible. For a Triangular Plane, may be quadrilateral, quinquilateral &c. as *Ramus* there ſheweth and alſo *Schol. mathemat. lib.* 6. but a trilateral Plane cannot be quadrangular, or quinquangular &c. But ſeeing Cuſtome and uſe hath taken up the name of Triangle for Trilater; and ſo of all other the like Figures from their angles rather then from their ſides; therefore we may moſt conveniently here retain their Appellations accordingly, but in the *Lateral* ſence. And ſo now firſt to

demonſtrate the artificial dimenſion of an ordinate Trigon, and that by its ſide only; Suppoſe here again, the former common Rational *Line*, A B; and to this I would ſo accommodate another Line, as that the ſide of an Iſopleural or Iſogonial Trigon, being meaſured thereby; its Quardrat ſhould be equivalent to the Area of the Trigon, meaſured by the *Line* A B. Now the length of this other *Line*, I find (according to the reaſon of the precedent Theoreme and alſo the general reaſon of Meaſure beforeſaid) to be A B 1.52; (or further in a decumilleſimal partition, it wil be A B 1.5197 *ferè*,) which is here the *Line* L M, divided as the prime *Line* A B. Then let the ſide of ſuch a





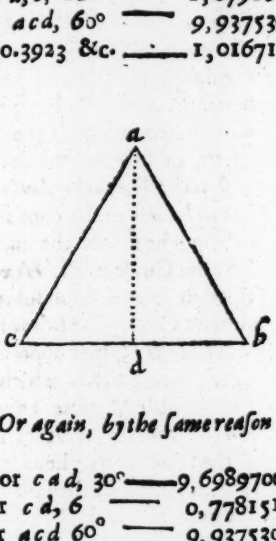
Trigon be A B, 12.00; So the Diametral or Perpendicular-line thereof, will be found by *E* 1. *p.* 47, and alſo by *E* 6. *p.* 31. to be A B, 10. 39. For ſeeing that in the *Iſopleuron* or *Iſogon*, *a*, *b*, *c*, the Perpendicular *a*, *d*, biſects the ſame into two *Orthogons*, or rectangle *Trigons*, *viz.* *a*, *d*, *b*, and *a*, *d*, *c*, the Sides *a*, *b*, and *a*, *c*, will be the *Hypotenuſal*

potenuſals (ſpecially ſo called) of the ſaid two Rectang'es; whole Power ſeverally, 144, being diminiſhed by the Power, of the leſſer containing ſide of the right angle, *viz.* *b d*, or *e, d*; (being ſuppoſed A B 6.00) *viz.* 36, there will remain the Power of the greater and common containing ſide, *a, d*, 108; whoſe Root irrational, (A B) 10.39 &c. is the greater and common containing ſide, *a, d*, ſought for, or the Perpendicular (of Altitude) of the *Iſopleuron*: with which will be found to accord the Trigonometrical calculation of the ſame. For ſeeing that in a plain or rectiline Triangle, all the 3 angles are equal to two right angles, by *E* 1. *p.* 32. the Angle of the *Iſopleuron*, or *Iſogon*, in general, muſt needs anſwer in meaſure to a Sextant of the Circular Periphery deſcribed from any angular point, according to either ſide of the Trigon about the angle, as the Radius; and thereupon, the ſide oppoſite to, or ſubtending the ſame angle, muſt by conſequence, anſwer to, or ſubtend the ſame Circular or Peripherial arch; and ſo is the *Hexachordon* (as I may briefly term it) or rather *Hecto-chordon* or the chord *Hexagonall* of the ſaid Circle, as being the ſide of the inſcribed ordinate *Hexagon* & thereupon the Circle it ſelf, *Hexachordal*. And ſo the ſide of the Trigon in general, muſt ſubtend the double arch of its exactly ambient Circle, (the ſides of ſ Trigō being indeed the *Subtenſes* or *Chords* of their oppoſite Angles, *viz.* of the Circular or Peripherial arches which do compaſſe thoſe Angles & are their double Meaſures) and conſequently, is the *Tri-chordon* (as it were) or rather *Trito-chordon*, or *Hypo-dia-triτe*, of the ſame Circle; being the *Hypotenuſe Inſcript*, or *Chord* Trigonal thereof: the ſaid ambient Circle being Trichordal. Whereupon, (the *Iſopleuron*, or *Iſogon*, being biſected into two Orthogonials, by its Perpendicular, as aforeſaid) it followeth, according to the reaſon of the ſe-

B. *Pitiſc.* Trigonom. *lib.* 3. *Axiom.* 2. *Conſect.* 1 and 2. And Trigonom. Britan. *lib.* 2. *par.* 1. *cap.* 2. *Prop.* 3. and *cap.* 4. *probl.* 4 and 5.

cond proportionall Axiome of plaine, or rectiline Triangles in *Pitiſcus*, and more particularly, *Conſect.* 1, and 2. thereof; and ſo by the third general Propoſition (or like Axiom) concerning the ſame, in *Trigonometria Britanica*; moſt plainly, and briefly, (eſpecially with help of the artificial or Logarithmical Numbers) thus;

As *a, d, b*, or *a, d, c*, 90° —— 10, 0000000 Rad.
to *ab*, or *a, c*, 12 —— 1, 0791812.
So *abd*, or *acd*, 60° —— 9, 9375306. *S, A.*
to *ad*, 10.3923 &c. —— 1, 0167118.



Or again, by the ſame reaſon;

As *b, a, d*, or *c a d*, 30° —— 9, 6989700. *S, A*
to *b, d*, or *c d*, 6 —— 0, 7781512
So *abd*, or *acd* 60° —— 9, 9375306. *S, A*
to *ad*, 10.3923 —— 1, 0167118. as before.

And

And other wales alſo may the ſame be found out; either by Sines alone, or by Sines and Tangents together, according to *Axiom.* 1 *Planor Pitiſc.* and *Conſect.* 1 and 2. And *Prop.* 1. and 2. Planor. in Trigon. Brit. and more particularly *cap.* 4. before cited, *probl.* 1. 2. 3. But this way of working is the moſt plain and vulgar.

Now the Diameter or Perpendicular of the Trigon, *a d*, 10.3923 being drawn into the ſemi-baſe (or ſemi-ſide) *b d*, or *c d*, 6; there reſulteth the Area of the Trigon (according to the reaſon of *E* 1. *p.* 41.) 62.3538. Or the ſame Trigonal Area will alſo be produced, by comparing each ſide of the Trigon ſeverally, with the ſemi-aggregate of all the ſides, and then infolding the ſaid ſemi-aggregate and the difference of each ſide thereto, continually together; for ſo the Root quadrate of the total Reſult, ſhall be the Area of the Trigon: As;

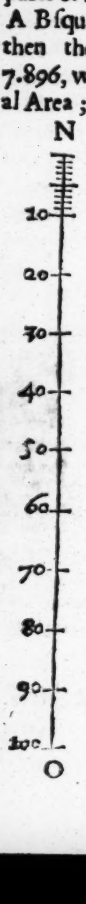
The Side	12	
The 3 ſides together	36	
The halfe	18	
The difference of each ſide thereto	6. *dif.* 1.	
1 Particular product	108.	
	6. *dif.* 2.	
2 Particular product	648	
	6. *dif.* 3.	
3 Or total Product	3888	
Root quadrate,	62.3538 &c.	

See upon this kind of Triangular dimenſion (or *Geodeſie*, as *Ramus* terms it) *Ramus lib.* 12 *Geomet. theor.* 9. but more eſpecially the reaſon thereof demonſtrated by him, *in fine lib. ult. Schol. mathemat.*

agreeing exactly with the Trigonall Area found before.

Now

Now the ſide of the Trigon, put A B, 12.00, being meaſured by its proper Line of quadrature L M, will be found thereof but 7.90 *ferè*, whoſe Quadrat is 62.4100 *ferè*, for the Area of the *Iſopleuron*, which exceedeth the true Area, in the fraction-part, only 562 ſquare-centeſms, *viz.* 562 parts of 10000, or in more vulgar terms $\frac{1}{178}$ *ferè*, of the Line A B ſquared. And if the Line L M be made 1000 parts; then the ſide of the Trigon will be found thereby, 7.896, which ſquared, will give 62.346814, for the Trigonal Area; which wanteth now of the true Area, found by both the former ways, 62.353819, only as much, as will make in vulgar terms, $\frac{1}{1781}$ *ferè* of the Line A B ſquared, as being the Meaſure aſſigned. And the very ſame Area will be artificially produced by the Perpendicular or Diameter of the Trigon; a Line of meaſure being accordingly fitted thereunto; which will be found by the reaſon of the fore-going Theoreme, &c. to be of the intire prime Rational Line in general, .1.3161 *ferè*, (which is $\sqrt{99}$3) ſuch as is here the Line N O, according to the Line A B.



The artificiall Dimenſion of Triangles in generall.

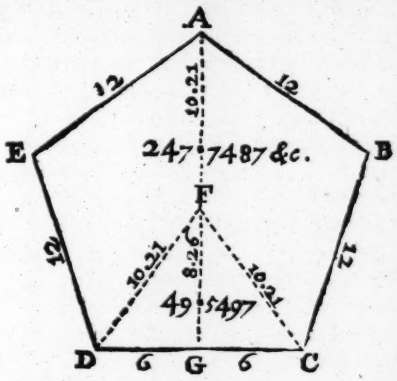
And here our artificial Menſuration, may be applied to any other kind of Triangle, either regular-like, or altogether irregular, (as I may term them) *viz.* Iſoskelan or Skalene, for the finding of their areal contents; but it will be in a way ſomwhat different from that of the regular or Iſopleural and Iſogonial Triangle; (being indeed according to the vulgar

or

or natural way of measuring,) *viz.* in that, as there the Area is had immediatly by the only squaring of some one dimensional line of the Triangle, as the Side or Perpendicular, &c. and so the same is the Quadrat thereof in Quantity arithmetical or measure numeral: there it will be the Rectangle made of the whole Base and Perpendicular together (as it is naturally and really the half of that Rectangle, by *E.* 1. *p.* 41. before cited) which two lines, if they happen at any time to be equal, then will the Area be produced under the form of a perfect Quadrat, from either of them, as that of an *Isopleuron*, or regular Trigon; seeing that the Rectangle made of them both, is no other then the exact Quadrat of either of them singly: And so therefore, after this latter way may an *Isopleuron* also be artificially measured, the way being general. And this resembles the artificial dimension both solid and superficial of regulo-like Solids, as the Cone, and all regular-based Pyramids, &c. and so differeth from that particular and peculiar dimension of the ordinate or regular Triangle (and of all other regular Planes) as the dimension of regular-like Solids doth from that of Solids exactly regular: And so, as the solid content of any oblique, Scalene, or inclined Cone or Pyramid, &c. is obtained both naturally and artificially, as that of a right, upright, or Isoskelan Cone or Pyramid &c. (as was shewed before in the dimension of a Cone, and shall be afterwards in the dimension of Pyramids) So is here the superficial content of any oblique Scalenal or irregular Triangle obtained (as well artificially as naturally) like that of an upright or Isoskelan Triangle by any side thereof, put as the base, and the perpendicular let fall from the opposite angle thereto, whether that side need to be continued out or produced, or not; seeing that every Skalene Triangle is equal to an Isoskelan, having the same

ſame baſe and perpendicular of altitude, (or they being conſtituted upon the ſame baſe, or equal baſes, and in the ſame Parallels, as *Euclid*, and his interpreters do ſpeak) by *E* 1. *p.* 37. and 38. as every oblique or Skalene Cone and Pyramid, &c. is equal to a right, or Iſoskelan Cone and Pyramid, upon the ſame (or equal) baſe, and of the ſame altitude, by the reaſon of *E.* 12, *p.* 11. and 14. beforecited in the Cone, &c. and *p.* 5. and 6. afterwards in the Pyramids, &c. And now the *L*ine of meaſure, for the performing of this latter or general artificial dimenſion of Triangles, wil be the ſame with that, which was noted formerly for the ſuperficial dimenſion of a Cone, by its ſide and baſial Periphery together, *viz.* of the Rational *L*ine in general, 1.4142, &c. $\sqrt{q}$ 2. And here the proportion of the natural Meaſure to the artificial, in the two forenamed lines of a Triangle together, will be the ſame with that in the foreſaid Conical dimenſion; *viz.* 1. to .7071 &c. $\sqrt{q}$ $\frac{1}{2}$. But indeed ſeeing that the Area of any Triangle may be obtained as readily in a manner by the natural *L*ine of meaſure, or common way of meaſuring, as by this latter kinde of artificiall Line, or general artificial way of Triangular dimenſion: (according to what I noted even now about the ſame.) Therefore I ſhal preſſe this point no further, then what is only for variety of Art and operation in this kind.

Suppoſe



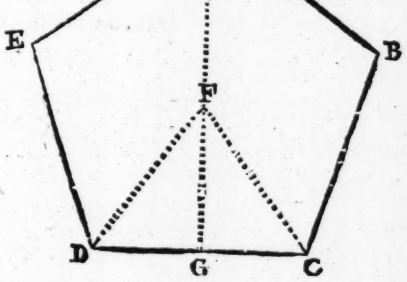
SUppoſe next the ſide of the ordinate *Pentagon*, A B C D E, to be the ſame with that of the ordinate *Trigon* beforegoing, (as from the Rational *Line* A B,) 12.00; then the true Area thereof, will be found, 247.7487 &c. and not 240, as *Ramus* makes it, ſhewing the Geodeſie (as he terms it) of ordinate *Polygons* in general, and particularly of an ordinate *Pentagon*, whoſe ſide is 12; where he makes the Ray of the inſcribed Circle, (which in this pentagonal Figure, is F G) to be exactly 8, (and conſequently the Ray of the circumſcribing Circle F C or F D, juſt 10) and thereby the Area, but 240; and which from him *P. Ryff* alſo hath in his Epitomy of *Ramus*: And ſo *Orontius Finæus* alſo meaſures this *Pentagon*; and thus they take it one from another (as *Ramus* from *Finæus*, and *Ryff* from *Ramus*) without any further examination; And hence they give the ſaid *Pentagon*, abſolute in all its dimenſional Numbers. But indeed

Ram. Geom. *lib.* 19. *el.* 1. *P. Ryff* in Epitome *Ramæ.* And *Oront. Fin. lib.* Geom. pract. *cap.* 24.

deed, the ſide of an ordinate Pentagon, being 12, the Ray of the inſcribed Circle, wil be truly, 8.26 *ferè* ; and the Ray of the circumſcribing Circle (which here we need not) wilbe 10.21 *ferè*, and ſo the Pentagonal area will be exactly (to ſquare centeſms) 247.7487. All which we ſhall here demonſtrate Trigonometrically. For ſeeing that an ordinate *Pentagon* is a Triangulate, conſiſting of, (or reſolvable into) five equal and like Iſoskelan or equicrural Trigons, meeting vertically in the center of the Pentagon, (or of its circumſcribing Circle) and ſo whoſe Baſes are the ſides of the *Pentagon*, and whoſe ſhanks are Raies of the ſaid Circle ; ſuch as is the Triangle C F D ; the vertical (or Centricall) angle, F, will anſwere to a Fifth of the ſaid circumſcribing Circle's circumference, as being meaſured thereby; which the ſide of the *Pentagon* C D (as the baſe of the Iſoskelan Trigon) ſubtending, the ſame is conſequently the *Pempta-chord* (as it were) or *Hypodia-pempte* of the foreſaid Circle, as the *Hypotenuſe*, *Inſcript*, or *Chord Pentagonall* thereof ; the ſaid circumſcribing Circle being Pentachordal. Which *Iſoskeles* being biparted or biſected into two Orthogonials, *viz.* C G F, and D G F, by its perpendicular F G (which is a Ray of the inſcribed Circle, as aforeſaid) whoſe Hypotenuſals (ſpecially ſo called) F C, and F D, as the two equal ſides of the *Iſoskeles*, are two Raies of the *Pentagon's* circumſcribing Circle, as aforeſaid ; the ſemi-baſe of the *Iſoskeles*, C G or D G, as the leſſer containing ſide of the right angle in the two foreſaid orthogonial Trigons ; will be 6 (as being the ſemi-ſide of the *Pentagon*) and he Angle C F G or D F G, will anſwer to a *Tenth* of the foreſaid ambient Circle's circumference, as being halfe the vertical angle of the *Iſoskeles* C F D ; and ſo conſequently, the baſial or greater acute angle F C G, or F D G, will be the complement of the other two angles to a Semi-circle (according to *E.*1.*p.* 32) or the Complement

plement of that other acute angle to a Quadrant: and hereby the perpendicular of the *Iſoskeles* F G, as the perpendicular let fall from the center of the *Pentagon* to its ſide, (or the Ray of the inſcribed Circle) being the thing next to be inquired, will be found. And therefore in the rectangle Triangle, C G F, or D G F, ſeeing all the angles are given, together with the leſſer containing ſide C G or D G; the greater and common containing ſide F G required, will be had theſe two ſeveral ways following: and firſt according to *Axiom.* 2 *Planor. Pitiſc.* and *Prop.* 3 *Trigon. Brit.* &c. before-cited, moſt eaſily and readily by Logarithmo-trigonometrical ſupputation, or artificial Trigonometry, thus;

As C F G, *or* D F G, 36°	——	9,7692187.	*S A*
to C G, *or* D G, 6.	——	0,7781513.	
So F C G, *or* F D G, 54°	——	9,9079576.	*S A*
to F G, 8.26 *ferè, viz.* 8.25829 &c.	——	0,9168902.	



Or ſecondly; ſeeing that in a recti-line rectangle Triangle, any ſide may be put for the Radius of a Circle, by *Axiom.* 1 *Planor. Pitiſc.* and *Prop.* 1 *Planor. Trigonom. Brit.* Therefore in the foreſaid Rectangle C G F, or D G F, the leſſer containing ſide C G or D G given, being put as Radius; the greater (and common) containing ſide, F G ſought for, will (by the foreſaid *Axiom*, and *Prop.* 2. *Planor. Trigon. Brit.*) be the Tangent of its oppoſite angle F C G, or F D G; Whereupon it followeth accordingly, in this Trigonometrical Reaſon;

As C G, *or* D G, —— 10, 0000000. *R.*
to F G —— 10, 1387390, *T A*, 54°.
So C G, *or* D G, 6 —— 0, 7781513
to F G, 8.258 *&c.* —— 0, 9168903 *as before.*

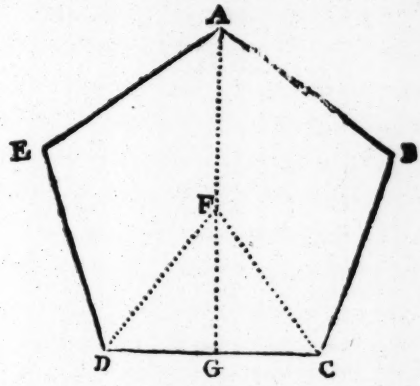
Which being infolded with the ſemi-perimeter of the *Pentagon*, A B C G, 30, there will reſult the Area of the *Pentagon*, 247.7487, as I ſaid at firſt. Or the ſaid F G, as the perpendicular of the foreſaid *Iſoskeles* C F D, being infolded with the ſemi-ſide of the *Pentagon*, C G or D G, as the ſemi-baſe of the *Iſoskeles*, produceth the Area of the *Iſoskeles*, C F D, 49.54974, &c. which augmented by the number of the compoſing Iſoskelan Trigons, produceth the total *Pentagon*, 247.7487 &c. as before; and which is moſt readily and accuratly obtained by Logarithmical numeration, only by a ſimple compoſition of Numbers, thus;

F G, 8.258 *&c.* —— 0, 9168903 } *A*
C G, *or* D G, 6 —— 0, 7781513 }

Iſoskeles C F D, 49.5497 *&c.* 1, 6950416 } *aggreg.*
5 —— 0, 6989700 } *A*

Pentagon, 247.748745 *&c.* —— 2, 3940116. *aggreg.*

Or



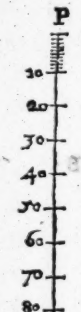
Or, according to the firſt operation, more briefly thus;

FG, 8.258 *&c.* —— 0,9168903 } *A*
ABCG, 30 —— 1, 4771213 }

Pentagon, 247.7487 *&c.* —— 2, 3940116, *as before.*

Now for a trial of this dimenſion artificially, by a Line for the quadrature of an ordinate *Pentagon* by its ſide, being here the Line P Q, whoſe quantity I find (by the reaſon of the foregoing Theoreme, &c) to be of the Rational Line A B, (defectively) 0.7624 *ferè*, the ſubductional ſegment being A B, 0.2376 *ferè*, and which Line being divided in like manner; the ſide of the *Pentagon*, ſuppoſed A B 12.00, meaſured thereby, will be found, 15.74, which ſquared, yields 247.7476, for the Area of the *Pentagon*, which differeth from the true Area (deficiently) only 11 ſquare-centeſms, or

11 of



11 of 10000, which in more vulgar terms, is hardly ſo much as $\frac{1}{909}$ of the Line A B. (as the Meaſure given) ſquared; And this from a centeſimal partition only of the artificial Line of meaſure, P Q. which is as near the truth, as need be deſired.

Now as to the Radius of the circumſcribing Circle, which *Ramus*, & the others beforenamed, do make exactly 10, (though here we make no uſe thereof, yet for the truth's ſake) we will demonſtrate the ſame to be 10.21 *ferè*, thus: And firſt in the rectangle Triangle C, G, F, or D G F aforeſaid; the leſſer acute angle C F G or D F G, and the leſſer ſide about the right angle, *viz.* C G or D G (as the ſemi-ſide of the *Pentagon*) ſubtending the ſaid acute angle, as the right Sine thereof, being firſt of all given: the *Hypotenuſe* F C, or F D, being the Ray of the ambient Circle, ſought, will be found thus,

As C F G, *or* D F G,	36°	9,7692187.	*S A.*
to C G, *or* D G,	6	0,7781513.	
So C G F, *or* D G F,	90°	10,0000000.	*R.*
to F C, *or* F D,	10.21 *ferè*, *viz.* 10.2078 *&c.*	1,0089326.	

Or again ſecondly in the ſame Triangles, the greater acute angle F C G, or F D G (being half the Pentagonal angle) and the greater and common ſide about the right angles, *viz.* F G, ſubtending the two ſaid equal acute angles, being only firſt given; the Hypotenuſal ſide, F C, or F D, inquired, will be had thus;

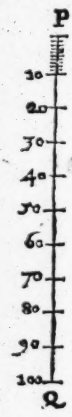
As

As F C G, *or* F *D* G, 54° —— 9,9079576 *S A*
to F G, 8.25829 *&c* —— 0,9168902
So F G C, *or* F G *D*, 90° —— 10, 0000000 *R.*
to F C, *or* F *D*, 10.2078 *&c.* 1, 0089326, *as before.*

Or again, thirdly, in either of the ſaid rectangle Triangles, where the foreſaid greater acute angle being only known, and the leſſer ſide about the right angle, *viz.* C G, or *D* G, and the ſame put as Radius; the ſide ſubtending the right angle, *viz.* F C, or F *D* inquired, wil be the Secant of the ſaid greater acute angle, F C G, or F *D* G, (as the greater and common ſide about the right angle, and the common Subtenſe of the ſaid two acute angles, *viz.* F G, was ſaid before in the like caſe, to be the common Tangent of the ſame angles) whereupon it followeth in this Trigonometrical Analogie;

As C G, *or* *D* G —— 10, 0000000, *R.*
to C F *or* *D* F —— 10, 2307813, *Sec.* 54°.
So C G *or* *D* G, 6 — 0, 7781513.
to C F *or* *D* F, 10.2078 *&c.* — 1, 0089326, *as before.*

Or fourthly, in either of the ſaid Rectangles, the leſſer acute angle C F G, or *D* F G, being only given, and the greater containing ſide of the right angle, *viz.* F G, and the ſame put for Radius; the Hypotenuſal F C, or F *D*, will be the Secant of the ſaid leſſer acute angle, (as the leſſer containing ſide C G or *D* G will conſequently be the Tangent of the ſame acute Angle.) Therefore it followeth;



11 of 10000, which in more vulgar terms, is hardly ſo much as $\frac{1}{909}$ of the Line A B. (as the Meaſure given) ſquared; And this from a centeſimal partition only of the artificial Line of meaſure, P Q. which is as near the truth, as need be deſired.

Now as to the Radius of the circumſcribing Circle, which *Ramus*, & the others beforenamed, do make exactly 10, (though here we make no uſe thereof, yet for the truth's ſake) we will demonſtrate the ſame to be 10.21 *ferè*, thus: And firſt in the rectangle Triangle C, G, F, or *D* G F aforeſaid; the leſſer acute angle C F G or *D* F G, and the leſſer ſide about the right angle, *viz.* C G or *D* G (as the ſemi-ſide of the *Pentagon*) ſubtending the ſaid acute angle, as the right Sine thereof, being firſt of all given: the *Hypotenuſe* F C, or F *D*, being the Ray of the ambient Circle, ſought, will be found thus,

As C F G, *or* *D* F G, 36° —— 9,7692187. *S A.*
to C G, *or* *D* G, 6 —— 0,7781513.
So C G F, *or* *D* G F, 90° —— 10,0000000. *R.*
to F C, *or* F D, 10.21 *ferè*, *viz.* 10.2078 *&c.* —— 1,0089326.

Or again ſecondly in the ſame Triangles, the greater acute angle F C G, or F *D* G (being half the Pentagonal angle) and the greater and common ſide about the right angles, *viz.* F G, ſubtending the two ſaid equal acute angles, being only firſt given; the Hypotenuſal ſide, F C, or F *D*, inquired, will be had thus;

As

As F C G, *or* F *D* G, 54° —— 9,9079576 *S A*
to F G, 8.25829 *&c* —— 0,9168902
So F G C, *or* F G *D*, 90° —— 10,0000000 *R.*
to F C, *or* F *D*, 10.2078 *&c.* 1,0089326, *as before.*

Or again, thirdly, in either of the ſaid rectangle Triangles, where the foreſaid greater acute angle being only known, and the leſſer ſide about the right angle, *viz.* C G, or *D* G, and the ſame put as Radius; the ſide ſubtending the right angle, *viz.* F C, or F *D* inquired, wil be the Secant of the ſaid greater acute angle, F C G, or F *D* G, (as the greater and common ſide about the right angle, and the common Subtenſe of the ſaid two acute angles, *viz.* F G, was ſaid before in the like caſe, to be the common Tangent of the ſame angles) whereupon it followeth in this Trigonometrical Analogie;

As C G, *or* *D* G —— 10,0000000, *R.*
to C F *or* *D* F —— 10,2307813, *Sec.* 54°.
So C G *or* *D* G, 6 — 0,7781513.
to C F *or* *D* F, 10.2078 *&c.* — 1,0089326, *as before.*

Or fourthly, in either of the ſaid Rectangles, the leſſer acute angle C F G, or *D* F G, being only given, and the greater containing ſide of the right angle, *viz.* F G, and the ſame put for Radius; the Hypotenuſal F C, or F *D*, will be the Secant of the ſaid leſſer acute angle, (as the leſſer containing ſide C G or *D* G will conſequently be the Tangent of the ſame acute Angle.) Therefore it followeth;

As F G ——— 10, 0000000
to F C *or* F D —— 10, 0920424. *Sec.* 36°.
So F G, 8.258 *&c.* —— 0, 9168902
to F C *or* F D 10.2078 *&c.* —— 1, 0089326, *as before.*

Or fifthly and laſtly, in the *Iſoskeles* C F D, where all the angles and the Baſe C D (as the ſide of the Pentagon) being known; the ſide of the *Iſoskeles* F C or F D ſought for, will be obtained by the moſt common Trigonometrical operation, thus;

As C F D, 72° ——— 9, 9782063. *S A.*
to C D, 12 ——— 1, 0791812.
So C D F, *or* D C F, 54° — 9, 9079576, *S A*
to C F *or* D F, 10.2078 *&c.* —— 1, 0089325, *as before.*

Now the Radius of the circumſcribing Circle, *viz.* F C or F D (to which A F is equall) being theſe five ſeverall ways found out, to be 10.21 *ferè*; the ſame with the Radius of the inſcribed Circle, *viz.* F G found before, 8.26 *ferè*, will make up the totall Diameter or Perpendicular of the *Pentagon*, *viz.* A G (which is the altitude thereof) to be 18.47 *ferè*; which according to the accompt of *Ramus*, and the other Authors before named, would be exactly 18. For that here you may obſerve by the way, how that in all ſuch ordinate Planes, as have their angles and ſides in an uneven or unequall number (and ſo the angles and the ſides ſeverally, are not exactly oppoſite one to another, that is, angle to angle, and ſide to ſide, but contrarily, (as the *Trigon*, *Pentagon*, *Hepragon*, *Hennea-gon*, and the like; the Diameter is compoſed of the ſemidiameters of the Circle circumſcribing

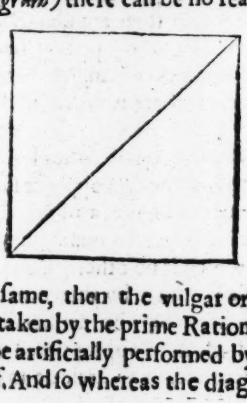
and

and inſcribed: And in all ſuch ordinate Planes, as have their angles and Sides in an even or equal number, (and ſo the angles and ſides ſeverally, are directly oppoſite one to another, that is, angle to angle, and ſide to ſide) as the *Tetra-gon, Hexa-gon, Octa-gon, Deca-gon*, and the like; the Diameter is no other then that of the circumſcribing Circle. And ſo by theſe Diameters may the areal contents of theſe Figures be obtained artificially as by their ſides, according as I noted at firſt: For ſo the artificial Line of meaſure for ſquaring of an ordinare *Pentagon* by its ſaid Diameter or Perpendicular, will be found (by the reaſon of the foregoing Theoreme, &c.) to be of the prime Rational Line in general, 1.1732 *ferè*. And from the premiſſes it appeares, that if the Diameter of a Circle be meaſured by any of the artificial Lines pertaining to the Diameters of the latter kind of ordinate Planes here mentioned, the Quadrat thereof ſhall be equal to the reſpective inſcribed Figure; where, by the Diameters, are meant their angular *Diameters*, or longer *Diagonals* paſſing between two extreamly oppoſite angles, through the Center of the Figure; whereas there is to be underſtood another *Diameter*, paſſing beween two oppoſite ſides, through the Center, (and to which is equall in the Hexagon the line paſſing between the ends of the two oppoſite ſides, and ſo ſubtending the Angle of the Figure, and which is more peculiarly and ſpecially called the *Diagonall-line*, or *Diagonie* of the ſame *Figure*) which is no other, then the *Diameter* of the inſcribed Circle; And ſo by this other *Diametrall* line, may the ſaid *Figures* be alſo artificially meaſured as before, by Lines of Meaſure appropriated thereunto; And therefore, if the *Diameter* of a Circle be taken thereby, the

The artificiall Line in general for the quadrate dimenſion of an ordinate Pentagon by it's Diameter or Perpendicular of altitude.

Quadrat thereof ſhal be equal to the reſpective circumſcribed Figure: And ſo alſo may theſe ordinate Polygonal or Polypleural Planes in general, be artificially meaſured by their ſhorter *Diagonal*-lines, which ſubtend their angles ſingly, (and are equal, neither to the diameters of their Circles circumſcribing, nor inſcribed, nor compoſed of both) & are more peculiarly called their *Diagonials* or diagonies; and which is in the *Hexagon*, the Side of its inſcribed Trigon; in the *Octagon*, the ſide of the inſcribed *Tetragon*; & in the *Decagon*, the ſide of the inſcribed *Pentagon*, &c. And which ſort of line is drawn and handled (amongſt others) in the next Pentagonal figure, ſet forth for the dimenſional proportions in a *Pentagon*. And ſo the artificial Line for the quadrate dimenſion of an ordinate *Pentagon* by its ſaid diagonie, or angular Subtenſe, will be of the prime Rational Line, (in the general Reaſon of Meaſure) 1.2336 *ferè*.

As for the ſecond (or intermediate) ordinate or regular Plane, namely, the *Tetragon*, (or rectangle *Iſopleuron*, or regular *Parallelogram*) there can be no readier way for the



meaſuring of the ſame, then the vulgar or natural way, by ſquaring its ſide, taken by the prime Rational Line: But elſe the ſame might be artificially performed by the diagony or diagonial thereof. And ſo whereas the diagony of the *Tetragon*,

Lon, is naturally double in power, to the ſide thereof, by the reaſon of *E.* 1. *p.* 47. and alſo *E.* 6. *p.* 31, before cited; here it will be made artificially, equal (immediatly in act, and ſo in power, according to Quantity diſcrete) to the Side, taken naturally, or by the natural Line of meaſure. And ſo the artificial Line of meaſure for this purpoſe I finde (according to the reaſon of the precedent Theoreme &c.) to be of the Rational Line in general, 1.4142 &c. *viz.*: $\sqrt{2}$. (which was noted for two ſeveral dimenſions before, namely one in the *Cone*, and the other of Triangles in general) which Line being divided as the former, and ſo the Diagonal of this Figure taken thereby, it will be found to agree with the Side thereof, taken by the prime or natural Rational Line; and ſo being ſquared, muſt needs produce the ſame ſuperficial Content: And therefore it appeares hence, that if the diameter of a Circle be taken by this Line, the Square thereof ſhall be the meaſure of the inſcribed Quadrat; whereas the Square of the diameter, is naturally, really, or geometrically, the circumſcribed Quadrat.

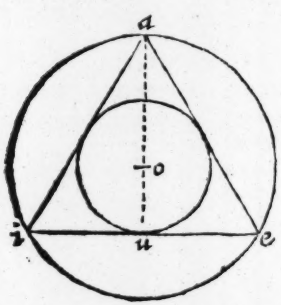
The dimenſion of a *Tetragon* by its Diagony; And the artificiall Line for performing the ſame.

And thus having demonſtrated the dimenſions of the three firſt ordinate Planes, geometrically, in every reſpect: we ſha next of all deliver the ſame purely arithmetically, by way of proportion, as we have extracted them in all the variety thereof that may be; ſome of which may be of ſingular uſe, not only for the more eaſie and ſpeedy ſuperficial dimenſion of theſe Figures ſimply, according to the natural Meaſure; but alſo for the ſolid dimension of the Pyramids raiſed or conſtituted upon them (and ſo conſequently of the foreſaid five plain, or rectiline regular Solids, after the natural and vulgar way of meaſuring them, which is the moſt difficult) as to the ſpeedy diſcovering of the ſemidiameters of the Circles

cles circumscribed to their Bases; and so thereby, and by the side of the Pyramid together, the Axis or Altitude of the same; as I shall shew in the next Section; like as I did before, for finding the Axis of a *Cone*. And first therefore for Linear Proportions (as I may term them) or Proportions of Linear dimension, in the ordinate or regular *Trigon*.

1 The Side (*a*, *e* or *a*, *i*) is to the Raie of the Circle,

- circumscribing (*a*. *o*.) } as 1. to { ·57735 $\sqrt{q\,\frac{1}{3}}$.
- inscribed (*o*. *u*.) } as 1. to { ·288675.
- And so to the diameter or Perpendicular (*a*, *u*.) } as 1. to { ·8660254 $\sqrt{q\,\frac{3}{4}}$ or ·75.
- And *vice-versâ*, the Perpendicular (*a*, *u*) is to the Side (*a*, *e*, or *a*, *i*) } as 1. to { 1.1547.



2 The

2 The Radius of the circumscribing Circle *(a, o)* is to the

Side *(a, e,* or *a i,)* Raie of the inscribed Circle *(o, u,)*	as 1. to	1.73205 $\sqrt{3}$.	
		0.5 *dupla.*	
And consequently to the Diameter or Perpendicular, *(a, u)*	as 1. to	1.5 *subsesquialtera.*	

3 The Radius of the inscribed Circle *(o, u)* is to the

Side *(a, e,* or *a, i)* Ray of the cirumscribing Circle, *(a, o,)*	as 1. to	3.4641	
		2. *subdupla.*	
And so to the Perpendicular or diameter *(a, u)*		3. *subtripla.*	

Whereby it appeares, that the Radius of the circumscribing or containing Circle, is $\frac{2}{3}$ of the Trigonal Perpendicular or diameter, and the Radius of the inscribed or contained Circle is $\frac{1}{3}$ of the same, and which is farther manifest by *E.* 14 *p.* 18.

Secondly, for superficiall Proportions in the ordinate Trigon; or Proportions of superficiall dimension:

4 The Quadrat of, the { Side / Diameter } is to the Trigon it selfe, as 1. to { .433012 / .57735. $\sqrt{\frac{1}{3}}$. }

Which latter, is the same with that of the side to the Raie of the circumscribing Circle.

And

And contrarily;

5 The totall Trigon is to the Quadrat of its { Side / Diameter, } as 1. to { 2.309401 / 1.73205. √q.3 }

And so consequently.

6 The { Side, / Diameter, } is to the side of the Quadrat equall to the Trigon, as 1. to { .658037. / .759836 *ferè*. }

Next for the like proportions in the ordinate *Pentagon*; and first in respect of Linear dimension;

Ray of the Circle { circumscribing (a.g.) / inscribed (g.h.) } as 1. to { .85065. / .68819. }

1. The side of the Pentagon (*a.b.*) is to the

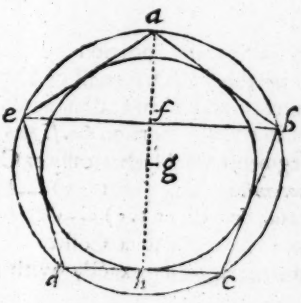
- And so to the *Diameter* or Perpendicular of the Pentagon (a. h.) as 1. to 1.53884.
- *Diagonall*, or the Subtense of the pentagonall angle (b.e.) * cutting the *Diameter* by extream and mean proportion) as 1. to 1.618034 *ferè*.

* *Flussat.* El. 16. p. 2.

4. The

4. The Radius of the ambient Circle (*a.g.*) is to the

- Side of the Pentagon, (*a.b.*) as 1. to 1.17557
- Ray of the inscribed Circle, (*g.h.*) as 1. to .809017
- (And so to the Pentagonall Diameter (*a.h.*) as 1. to 1.809017.)
- Pentagonall *D*iagonie, or angular Hypotenuse (*b.e.*) as 1. to 1.902113.



3. The Radius of the inscribed Circle (*g.b.*) is to the

- Side of the Pentagon, (*a.b.*) as 1. to 1.453085
- Ray of the ambient Circle, (*a.g.*) as 1. to 1.236068 (*ferè*.
- (And so to the *D*iameter or Perpendicular (*a.h.*) as 1. to 2.236068)
- Diagonal or Hypothenusal (*b.e.*) as 1. to 2.351141

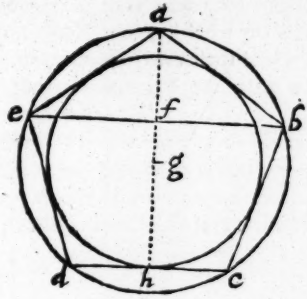
4 The

4. The Diameter of the Pentagon (*a.b.*) is to the

- Side (*a.b.*) as 1. to .649839.
- Ray of the Circle { circumſ. (*a.g.*) } as { 1. } to { .552786 / .44721 } { inſcrib. (*g.h.*) }
- (of which two circular Raies it is compoſed.)
- Pentagonall *Diagony* (*b.e.*) as 1 to 1.051462.

And to its greater portion or ſegment (*f. b.*) (being parted by the ſaid Diagonall or Hypothenuſall, according to extream and mean reaſon, as aforeſaid) as 1. to .618034, and to its leſſer portion (*a.f.*) as 1. to .381966: which two ſegments are Algebraically or Coſſically, the irrationall *Apotomies*, $\sqrt{1\frac{1}{4}}$ (or 1.25) $-\frac{1}{2}$ (or .5) for the greater ſegment; and $1\frac{1}{2}$ (or 1.5) $-\sqrt{1\frac{1}{4}}$ (or 1.25) for the leſſer ſegment; according to a Coſſicall invention of the ſaid two ſegments, agreeing exactly with the Trigonometricall.

5. The



5. The *Diagonall* or Hypotenuſall (b, e) is to the

Side, (a, b) as 1. to. 618034.

Ray of the Circle { circumſcrib. (a.g.) } as 1. to { .525731 / .425325 }
{ inſcribed, (g.h.) }

And ſo to the Diameter, (a, h) as 1. to .951056.

And to the greater ſegment thereof (f, h) (as being divided by the *Diagonall*, according to extream and mean reaſon, as aforeſaid) as 1. to .587785

And to the leſſer ſegment of that diviſion or partition, (a, f) as 1. to .363271.

Which two *Diametrall* ſegments, are Coſſically the irrationall Apotomies √ 1. 130635 &c. —o. 475528,

for the greater ſegment; and 1. 426584 &c. — √ 1 . 130635 &c. for the leſſer ſegment, according to a Coſſicall computation of theſe two ſegments, which we find to agree exactly with the Trigonometricall, as the former.

Which Pentagonall proportions before-going, (thoſe of the *Diagony* being ſecluded) would be according to the *Dimenſion* of the foregoing ordinate Pentagon, by *Ramus* &c. exactly rationall in the leaſt terms, thus;

1. The Side of the Pentagon to the
- Ray of the Circle
  - circumſcribing, as 6 to 5, *ſeſquiquinta*, (which is decimally irrationall, 1. to . 833333 infinitely, & defective.)
  - inſcribed, as 3 to 2, *ſeſquialtera*, (which is decimally irrational, 1. to . 66666 &c. infinitly, and defective.)
- And ſo to the *Diameter*, as 2 to 3, *ſubſeſquialtera* (which is decimally irrationall 1. to 1. 5, (or integrally, rationall, 2 to 1 5, defective.)

2. The

2. The Radius of the circumscribing Circle to the { Radius of the inscribed Circle, as 5 to 4, *sesquiquarta*, (which is decimally irrationall, 1. to .8 (or integrally, rationall, 10 to 8. deficient.

And consequently to the Pentagonall Diameter, as 5 to 9, *sub-superquadriquinta*, or *subsuperquadripartiens-quintas*, (which is decimally irrationall, 1. to 1. 8 (or integrally, rationall, 10 to 18 deficient.)

And so the Radius of the inscribed Circle, to the said *D*iameter, as 4 to 9, *subdupla-sesquiquarta*, (which is decimally irrationall, 1. to 2.25, (or integrally rationall, 100 to 225) and excessive or redundant.)

By which the reciprocall proportions may be had by inversion of the Termes; And here the proportion of the Pentagonall *D*iameter to the side, wilbe the same with that of the side to the Raie of the inscribed Circle, before noted, *viz.* 3 to 2 *sesquialtera*.

Then secondly in relation to superficiarie *D*imension, the Pentagonall proportions will be exactly, as followeth.

1. The Quadrat	Laterall	is to the Pentagon it selfe, as 1, to	1. 720477
	*D*iametral		0. 726543 *ferè*
	Diagonial		0. 657164 *ferè*

And contrariwise

2. The Pentagon is to the Quadrat	Laterall	as 1. to	.581234
	Diametral		1. 37638
	Diagonial		1. 52169

And ſo,

3. The { Side / Diameter / Diagonial } is to the Side of the Pentagonall Quadrat, as 1. to { 1.31167. / .85237. / .810656. }

Which three laſt Proportions are the moſt preciſe and proper Tetragoniſmal termes of an ordinate Pentagon; but chiefly the firſt of them.

And which ſuperficiall Proportions (ſecluding thoſe for the Diagony) would be, according to the former *Linear* Proportions, deduced from the foregoing Pentagonall dimenſion of *Ramus*, &c. in theſe Terms;

1. The Quadrat { *Laterall* / *Diametrall.* } to the Pentagō it ſelfe, as 1. to { 1. 666666 infinitely, & defective; being in vulgar terms, as 1. to $1\frac{2}{3}$, or $\frac{5}{3}$ *ſub-ſuper-bitertia*, or *ſub-ſuper-bipartiens-tertias.* / .740740, infinitely, & exceſſive; being more vulgarly, 1 to $\frac{20}{27}$, *ſuper-ſeptu-partiens-vigeſimas*; which is very neer *ſeſqui-tertia* in the leaſt terms, it being ſuperpartient $\frac{7}{20}$. }

Con-

Contrarily;

2. The Pentagō to the Quadrat { Laterall / Diametrall } as 1. to { .6, (or integrally, 1 0 to 6) exceſſive; being more vulgarly, 1 to $\frac{3}{5}$, *ſuper-bi-tertia, viz. ſuperpartient* $\frac{2}{3}$. / 1.35, (or integrally, 100 to 135,) defective; being in vulgar terms, as 1 to $1\frac{7}{25}$, *ſub-ſuper-ſeptu-pa-tiens-vigeſimas.* }

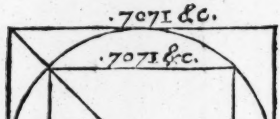
And ſo,

3. The Penta-gonall. { Side / Diam. } to the Side of the Pentagonall Quadrat, as 1. to { 1.29099, deficient. / .86066, redundant. }

But the former or Linear Proportions are erronious; Therefore, the latter or ſuperficiall.

Then as for the Proportions of dimenſion in the *Tetragon*; there be onely theſe two conſiderable; *viz.*

1. The Side is to the *Diagony* (or circumſcribing Circle's *Diameter*) as 1. to 1.4142, &c. *v.2.* (which is the generall Linear or Scalar Number, for Diagonial Quadrature, noted before)



And *Vice-verſâ*.

2. The Diagony is to the Side, as the diameter of a Circle to the ſide of it's inſcribed Quadrat, noted formerly in the dimenſion of a Circle; *viz.* 1. to .7071, &c. $\sqrt{\frac{1}{2}}$.

And ſo is the ſide to the ſemi-diagonie, (or the circumſcribing Circle's ſemi-diameter.)

And here, as the ſide of the *Tetragon*, is the *Tetrachord*, or the Chord tetragonall of the circumſcribing, comprehending, or containing Circle, (as ſubtending a Quadrant of it's Peripherie) So is the Diagony, the like Chord of the Circle deſcribed out of ſome angular point of the *Tetragon*, according to the ſide thereof, as the Radius; *viz.* the Circle deſcribed about the diagoniall Quadrat; the ſaid Diagony being then the Side of the *Tetragon* inſcribed; And ſo thoſe Circles are Tetrachordall.

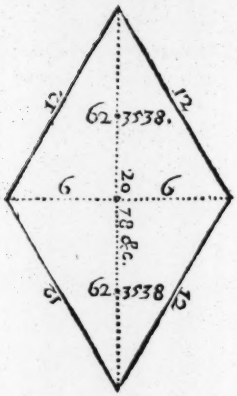
And here we may take in by the way, another kind of tetragonall Plane, very variable in reſpect of it's angles, but regular-like, (as I may term it) being equilaterall, though

though not equi-angular, called *Rhombus*, being the onely obliquangle equilaterall *Parallelogram*, and which therefore is a Quadrat, as it were, diſlocated in it's Terms, or compreſſed in the angles, as *Ramus* ſpeaketh *lib. Geomet.* 14. *el.* 7, 8, and ſo is a Triangulate, conſiſting as it were, of two equall and like iſoskelan Triangles, meeting upon one common baſe; And if the Triangles happen to be equilaterall, equiangular, or exactly regular; then will the *Rhombus* be artificially in the nature of an exact Quadrat, as that kind of Triangle is, and ſo all other regular Planes artificially are: and ſo will admit of the like artificiall, or quadrate dimenſion, by it's ſide, (or by either of the Diagonall-lines which may be drawn in it, and ſo by which it may be reſolved into 4 equall and like rectangle *Scalenons*; the leſſer or obtuſe angle- diagonall, which is the common baſe of the ſaid two *Iſopleurons*, being equall to the ſide of the *Rhombus*, as being a ſide of it's compoſing ordinate Triangle) according to an exact Quadrature. And the artificiall Line for this particular Rhomball Dimenſion, will be (according to the reaſon of the foregoing Theoreme, &c.) of the rationall Line in generall. 1. 0745, &c.

Concerning the dimenſion of a *Rhombus*.

Therefore for Examples-ſake, Suppoſe a *Rhombus* were given to be meaſured, whoſe ſide is found to be the ſame in meaſure with that of the foregoing equilater Triangle, which was put of the Rationall Line intirely, 12.00; and the Diagonall paſſing between it's two obtuſe angles, or ſubtending the two acute angles, is found to be equall with the ſide, whereby the *Rhomb* is thus biſected into, (or is compoſed, as it were of) two equall equilaterall Triangles, the Area of which Triangle was found naturally, 62. 3538.

and



and which therefore doubled, gives the true, naturall Area of the *Rhomb*, 124. 7076. Now the ſide of the *Rhomb*, 12. 00, being meaſured by it's proper and peculiar artificiall Line of quadrature, (centeſimally divided) will be found 11. 17 *ferè*, whoſe Quadrat is 124. 7689 *ferè*, for the Area of the *Rhombus*, agreeing very nearly with the true one; and which will be by the like Line of quadrature for an ordinate Trigon, 124. 8200 *ferè*, the ſide of the foreſaid Triangle being formerly found thereby, 7. 90 *ferè*, and ſo the Area, 62. 4100 *ferè*. But if the ſaid Line of Rhomball quadrature be milleſimally divided, then the ſide of the *Rhomb* will be found thereby 11. 167, which ſquared, yields 124. 701889 for the Rhomball area, which now wanteth of the true Area, *viz.* 124. 7076 $\frac{1}{8}$ (being the double of the foreſaid Triangle, *viz.* 62. 353829) onely about

about $\frac{1}{195}$ of the prime Rationall Line ſquared: as it exceeded the ſame before in the centeſimal operation, hardly ſo much as $\frac{1}{18}$ of the ſaid Line ſquared; and which Area will be by the foreſaid Line of Trigonall quadrature, (in a milleſimall partition) 124. 693628; the content of the foreſaid Triangle it ſelf, being found thereby, 62. 346814. And ſo the Area of this *Rhomb* produced by the Line of Trigonall quadrature, comes not ſo near the true Area, as that which is produced immediately by the Line of Rhomball quadrature it ſelf.

And by the artificiall Line for the dimenſion of Triangles in general, may the Area of any *Rhombus* be obtained, it's two diagonal-lines being meaſured thereby; for ſo the rectangle Parallelogram reſulting therefrom, ſhall be equall to the *Rhombus*; as the *Rectangle* of thoſe two lines, is naturally and really double to the *Rhomb*; or the *Rhomb* is half the *Rectangle* made of thoſe two lines, being meaſured by the naturall Line of meaſure; *viz.* the *Rectangle* from one whole Diagonall infolded with half the other. *And thus alſo may be artificially obtained the Area of any* Rhomboides *or* Trapezium.

SECT. II.

Setting forth the Dimenſion, both ſolid and ſuperficiall, of regular-baſed Pyramids in generall, and their Compounds: And demonſtrating the ſame particularly in the three firſt kinds of them.

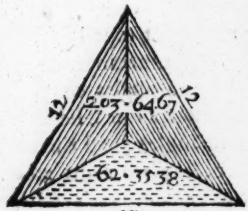
AND now having ſufficiently ſhewed our artificiall *Dimenſion* in the three firſt rectiline or angular ordinate Planes in particular; namely the *Trigon*, *Tetragon* and *Pentagon*, ſimply in themſelvs, (but chiefly the *Trigon* and *Pentagon*, as being in them, only requiſite) and ſo conſequently the like *Dimenſion* of all ordinate polygonall or polypleurall Planes whatſoever, by the ſame metricall reaſon: We ſhall next proceed to the like kind of dimenſion in them, as in order and relation to all Pyramidall Bodies, both prime or ſimple, and compound, (as I may ſo ſpeak) or Pyramids, and Pyramidates, as being their Baſes: or the dimenſion of theſe Solids, being founded, (as it were) conſtituted, or erected upon ſuch Planes; and ſo denominated from them accordingly, as I have ſaid before. And this I ſhall particularly ſhew in the three firſt ſorts of Pyramids, conſtituted or raiſed upon the three firſt Planes before named; and more eſpecially for

for that, as thoſe three Planes do concurre ſuperficially, to the compoſition, (or to the ſuperficiall compoſition) of the five famous ordinate Bodies, or rectiline regular Solids, as I ſaid before; namely, the *Trigon*, to the *Tetrahedron*, *Octahedron*, and *Icoſahedron*; the *Tetragon* to the *Hexahedron*, and the *Pentagon* to the *Dodecahedron*: ſo the three kinds of Pyramids erected or conſtituted upon them, do concurre in like manner ſolidly, to the compoſition, (or to the ſolid compoſition) of the ſaid five Bodies, as I ſhall ſhew particularly in each of them.

And ſeeing that of all the kinds of Pyramids (which may be as infinite in number, as the Figures, for their Baſes, upon which they are raiſed and conſtituted, and ſo from which they take their ſpecial denomination, as whether the ſame be trigonall, tetragonall, pentagonall, or howſoever polygonall, and ſo the reſpective Pyramids be denominated acccordingly) there is but one kind exactly ordinate or regular, and ſo is ſpecially and peculiarly, (for the excellency thereof) called *Tetraedron*, and by *Euclid*, ſimply by the name of *Pyramid*, in *E* 13. *p.* 13; it conſiſting of four equall ordinate *Trigons* compact together by ſolid angles (by *E* 11. *d.* 26) whih therefore are in number ſubtriple the plain, ſuperficial, or Trigonal angles conſtituting the ſame, (ſo that to the conſtitution of one ſolid angle, do here concurre three ſuperficiall angles; and therefore this ſolid angle is contained under two plain right angles preciſely, and ſo is $\frac{2}{3}$ of a ſolid right angle, as it's compoſing or baſiall angle is $\frac{2}{3}$ of a plain right angle) and ſo the angular lines, or ſharp edges, called the ſides of the *Pyramid*, (made by the connexion of the ſides of the ordinate trigonall Planes) are in number, ſubduple the trigonall ſides conſtituting the ſame) And ſo any one of the ſaid

Trigonall Planes may be put for the Baſe of this *Pyramid*, and thereupon the ſolid angle oppoſite thereunto, made by the inclination, connexion, or concurſion of the other three like Planes, (according to their verticall angles) ſhal be the top or verticall point of the ſame; between which, and the Center of the Baſe, ſhall be adjudged the perpendicular altitude, (or the Axis) thereof: We ſhall firſt therfore ſhew our artificial dimenſion (both ſolidly & ſuperficially) of the firſt kind of *Pyramid*, in the firſt of the foreſaid five regular Bodies, namely the *Tetrahedron*; and that by the Side of the Baſe, (which is the generall ſide of this Body) in anſwer to the foregoing quadrate dimenſion of an ordinate Trigon by it's ſide) and the Axis together; which we ſhall compare with the naturall or vulgar dimenſion, according as we have done in all the Figures beforegoing; whereby the ſame may withall be underſtood by ſuch as are yet to learn.

And therefore firſt for ſolid dimenſion; Let the ſide of a *Tetrahedron* be of the Rational Line in general, 12.00 (which is alſo the ſide of the baſe) ſo the trigonall Plane for the Baſe, wil be (as was found before in the dimenſion of an ordinate Trigon) 62.3538. Now the Axis, or altitude of the *Tetrahedrum* (and ſo of all right or iſoskelan Pyramids whatſoever, having regular baſes) is had geometrically (in a Triangular manner) as was formerly ſhewed



The ſolid dimenſion of a trigonal *Pyramid*.

shewed for the Axis of a Cone, according to *E.* 1. *p.* 47, &c. it being the greater containing side of the right angle in the rectangle Triangle made by the Axis of the said *Pyramid*, and the basiall Ray, (which is the ray of the Circle circumscribing the ordinate trigonall base) for the lesser containing side; and by the side of the *Pyramid*, being the *Hypotenuse* to the right angle, in the Center of the Base: so that the Axis of the *Pyramid* is the side of the residuall or differentiall Plane, (taken quadrately) between the Laterall Quadrat of the *Pyramid*, and the Radiall Quadrat of it's Base: which last Quadrat, (seeing that the side of the base, as being the side of an ordinate Trigon, is treble in power to the Ray of the Circle circumscribed to the base, by *E.* 13. *p.* 12) will be here found 48; by which the laterall Quadrat of the *Tetrahedrum viz.* 144, being diminished, there will remain the Quadrat of the Axis, 96; (*viz.* $q\,12 \_\_ q\,7$ *ferè*) whose Root, irrationall or ineffable, $9\frac{15}{19}$, or rather (according to a decimall extraction of the quadrate *Gnomon*) 9.7979, &c. (whereas the other is decimally, but 9.7894, &c.) is the true Axis or Altitude of this *Tetrahedron*: whereby it appeareth, that the Side of a *Tetrahedron* is potentially sesquialter the Axis of the same; and so *E* 14. *p.* 31. Or again, seing that the Side of the *Tetrahedron* is to the Diameter of the ambient Sphear, potentially sub-sesquialter, because the Diameter is potentially sesquialter the side, by *E* 13. *p.* 13, before cited; (and so the Diameter of the said Spheare is potentially quadruple-sesquialter to the semidiameter of the Circle circumscribing the Base of the said *Pyramid* inscribed,) therefore the Diameter of the said Spheare will come forth here potentially, 216; whose Root tetragonical, irrational or inexplicable, is vulgarly $14\frac{20}{39}$, which is by deci-

mall reſolution, 14.6896 &c. or the ſaid Root is more truly, by immediate decimal production of the gnonomical additament, or additionall *Gnomon*, 14.6969, &c. for the *Diameter* of the ambient Spheare; whoſe ſubtriple doubled, *viz.* 9.7979, &c. is the altitude of this ordinate trigonall or Tetrahedrall *Pyramid*, exactly as before; which then being conjunctly compounded with the ſubtriple of the baſe, *viz.* 20.7846, &c. or the whole baſe with the ſubtriple of that, *viz.* 3.26598, &c. there will reſult the ſolidity of the *Tetrahedron* (to cube-centeſimes of the Rationall Line, by a ſufficient extenſion or production of the two foreſaid terms of conſolidation) 203.646753 *ferè*. So that a Pyramid, is naturally the ſubtriple of a Priſme, having the ſame Baſe and altitude, by *E* 12. *p.* 7. as a Cone is the ſubtriple of a Cylinder, of equal baſe and altitude, by *E* 12. *p.* 10. and therefore if you triple the ſaid Tetrahedrum, you wil have a Priſm, of an ordinate trigonal baſe, according to the ſaid *E* 12. *p.* 7, being in content, 610.940259 *ferè*, and ſo of the ſame baſe and altitude with the Tetrahedrum.

And though a Cone do ſomewhat reſemble a Pyramid, and a Cylinder, a Priſme; yet a Cone cannot properly be called a Pyramid (as ſome do call it) nor a Cylinder a Priſme; they being not plain Solids, riſing from a rectiline or angular Baſe; but various, or variable gibbous Solids, riſing from a curviline, obliqueline, or circular Baſe; but yet a Cone may be underſtood to comprehend in it's ſelfe, any kind of Pyramid; and ſo a Cylinder to comprehend any kind of Priſme; their Baſes being the Circles circumſcribed to the Baſe of the Pyramid or Priſme, and ſo their whole Bodies circumſcribed to the whole *Pyramid* or *Priſme*, *viz.* their concave ſuperficies juſt touching

ing the angular lines, or the sides of the respective Pyramid or Prisme inscribed. And contrarily, may any Pyramid be understood to cõprehend or include a Cone, & any Prisme a Cylinder; the rectiline Planes of the Pyramid and Prism, just touching the convex Superficies of the inscribed Cone and Cylinder, (and so the angular or rectiline base of the Pyramid and Prism, compleatly circumscribed to the circular base of the Cone and Cylinder,) according to the nature of Mathematical Inscription and Circumscription, as you may see it fully set forth in *E*, 4. *d*. 1, 2, 3, 4, 5, 6, for Superficies; And *E*, 11. *d*. 31 and 32, for Solids.

But now to the main thing in hand, to wit, the solid dimension of the foregoing Pyramid, according to our artificial and compendious way (as before in the *Cone*) and here, by the side of the Base, & the Axis, or line of altitude; the artificial Line of Measure for which purpose, I find (according to the reason of the 3d. Theoreme, &c. following) to be of the prime or natural Rational Line in general, 1.9064, *fere*; which being duly set off therefrom; the side of the aforesaid Tetrahedrum's Base (put naturally 12. 00) will be found thereby, 6.29, (in a centesimall partition) whose Quadrat is 39.5641, for the artificiall base: And the Axis or Altitude of the Tetrahedrum, found geometrically before, 9.7979, &c. (which according to a centenarie solution of the unit, or of the Rationall Line, is 9.80 *fere*,) will be found by the foresaid Line, 5.14, for the artificiall Axis; which being wholly infolded with the whole Base, will produce the rectangle regular-based *Prisme*, or *Parallelipipedum*,

The artificiall Line for the solid dimension of a Trigonall *Pyramid* by the side of it's base and its Axis or Altitude together.

203.

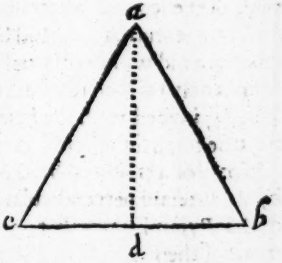
203.359474, for the artificial ſolidity of the *Tetrahedrum*; which differeth from the true, natural, ſolidity, *viz.* 203.646753, (by way of defect) not ſo much as $\frac{1}{3}$ of the prime Rationall Line cubed, or $\frac{1}{3}$ of a cube-unit. And by a further ſolution of the two Lines of Meaſure, (*viz.* the naturall and artificial Rationall Line) the difference will be found much leſſe, according to what I have ſaid, and alſo plainly demonſtrated in all the precedent *Dimensions*.

Concerning the ſuperficiall dimenſion of a trigonal *Pyramid*.

Now for the ſuperficiarie dimenſion of this kind of Pyramid; if it be exactly ordinate, as the *Tetrahedron*, then one of the Planes being had, the whole Superficies is easily had, by the quadruplication of that Plane: As the Plane of the foregoing Tetrahedrum, being found by the natural Dimension, 62.3538; the whole ſuperficies wil be 249.4152: And which may be artificially obtained by either of the Lines for the quadrate dimenſion of an ordinate *Trigon*; but moſt readily and properly, by that for the ſide. As the ſide of this Tetrahedrum being 12, wil be found by the *Line* of Lateral quadration of a *Trigon*, 7.90 *ferè* (as before the ſide of the *Trigon* ſimply) and ſo the Square thereof, 62.4100 *ferè*, for the Plane of the Tetrahedrum; and the quadruple of this, is 249.6400 *ferè*, for the total ſuperficies thereof; which exceedeth the former, or true ſuperficies, not ſo much as $\frac{1}{4}$ of a ſquare-unit, or Integer.

But if the Pyramid be not exactly ordinate; then ought the ſuperficies thereof to be conſidered firſt, and moſt properly, as that of a Cone, called *Conicum*, which I noted formerly in the *Dimenſion* thereof; to wit, without the Baſe; to which muſt afterwards be added the baſe, to make

make up the whole Superficies. For ſo indeed the true Pyramidall ſuperfice (what Pyramid ſoever it be) is moſt properly to be eſteemed; it being comprehended in a rectangle Plane ariſing from the ſemperimeter of the Baſe, and the perpendicular-line of the iſoskelan, or laterall triangular Plane of the Pyramid, which is the altitude of the ſaid Plane, (not the angular line between the Baſe and the top, called the ſide of the Pyramid, made by the meeting together of two of the Planes, according to their ſides, and ſo is no other then the ſide of the iſoskelan Plane) biſecting the ſame into two rectangle Triangles, upon the ſide of the baſe of the Pyramid (which ſide is alſo the baſe of the ſaid iſoskelan triangular Plane; the perpendicular falling thereupon from the top or verticall angle, by way of biſection or bipartition) As here the perpendicular *a, d* of the Triangle *a, b, c*, repreſenting the laterall Plane of the Pyramid, biſects the ſame into two rectangle Triangles, *a, d, b*, and *a, d, c*, right-angled at *d*, being the middle of the ſide of the ordinate Baſe of the Pyramid, *b, c*, which is alſo the baſe of the ſaid triangular Plane *a, b, c*; but the ſide of the Pyramid is the ſame with the ſide of the ſaid Plane (as noted even now) *viz. a, b*, or *a, c*: and the rectangle Plane made by that and the ſemperimeter of the baſe, will much exceed the Pyramidall ſuperfice before mentioned; which we may



conveniently demonſtrate in the foregoing Tetrahedrum; whoſe ſide, or the ſide of whoſe Plane, (w^ch, may here be repreſented by, *a*, *b*, or *a*, *c*,) being put naturally, 12; the perpendicular (wch may here be *a*, *d*) will be 10.3923, &c. (as was found formerly in the ordinate Trigon.) Now the ſide of this Pyramids baſe (*b*, *c*) being the ſame with the other, (*viz.* *a*, *b*, or *a*, *c*,) the perimeter of the Baſe will be 36, whoſe half, 18, being augmented, by the ſaid perpendicular, (or halfe the perpendicular, 5.19615 by the whole perimeter, 36) there will reſult the ſuperficies of this ordinate Tetrahedrall Pyramid, without the Baſe as it were, or of three of the Planes, 187.0614; to which being added the fourth equall Plane, for the more proper and peculiar baſe, as it were, of the *Pyramid*, *viz.* 62.3538; the aggregate will be 249.4152, for the totall ſuperficies, or ſurface of the Tetrahedrum, exactly as before, by the true, naturall dimenſion. But now the rectangle *Plane* made of the ſemiperimeter of the baſe, 18, and the ſide of the Tetrahedrum, 12, would be 216, for the ſuperficies of this *Pyramid* without the baſe (or of three of the equall *Planes* only) which exceeds the former, by 28.9386, and ſo being joyned with the baſe, 62.3538. would exceed the true totall ſuperficies accordingly.

Now for a triall of this dimenſion artificially, and that by the foreſaid perpendicular-line, and the Side of the baſe of the *Pyramid* together, whereby the rectangle *Plane* made of them, ſhall agree with the rectangle *Plane* made naturally of the ſemi-perimeter of the Baſe, and the ſaid hedrall perpendicular, for the true *Pyramidall* ſuperfice, *viz.* without the proper baſe: The artificiall Line of Meaſure ſpecially ſerving hereunto, I find (according to the reaſon of the 3^d. Theoreme, &c.) to be of the Ratio-

nall

nall Line in generall, deficiently, 0.8165 *ferè*, (which is Apotomally in Number, according to our generall reaſon of Meaſure, 1——.1835 *ferè*) which being ſet off from the ſaid Line, and the ſide of the foreſaid *Pyramid*'s baſe meaſured thereby (in a centeſimall partition) will be found, 14.70 *ferè*; and the *Perpendicular* being alſo meaſured by the ſame Line, will be found, 12.73. *ferè*; which two being multiplied together, there will reſult the rectangle *Parallelogram*, 187.1310 *ferè*, for the *Pyramidall* ſuperfice, before-mentioned (*viz.* without the Baſe) which exceedeth the like ſuperficies formerly found moſt truly, 187.0614, not fully ſo much, as will make in vulgar terms, $\frac{1}{14}$ of a ſquare-unit.

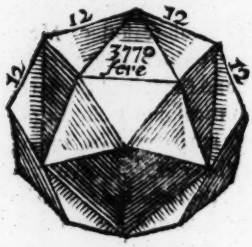
The artificial Line for the ſuperficial Dimenſion of a trigonall Pyramid, by the Side of its Baſe & the perpendicular-line of its laterall trigonall Plane, (or the altitude of the ſame) toge ther.

And thus much for the Dimenſion both ſolid and ſuperficiall of the firſt kind of *Pyramid* in generall, whether ordinate or inordinate, ſo as the Baſe be ordinate: And ſo in ſpeciall, of the firſt of the five ordinate plain Bodies, namely, the *Tetrahedron*; according to a *Pyramidall* dimenſion only.

As for the firſt kind of *Pyramidate*, called in generall *Priſma*, which I have mentioned before in the firſt *Part*; I ſhall afterwards ſpeak a *little* of the like dimenſion thereof, as is of a *Pyramid* it ſelfe.

Now for the ſecond kind of *Pyramidate*, called in generall a mixt *Polyhedron*, of which ſort are uſually reckoned three of the ſaid five ordinate Solids, to wit, the *Octahedron*, *Icoſahedron*, & *Dodecahedron*,

Cõcerning the Dimenſion of the *Icoſahedron* in a Pyramidal way.



ſeing that the ſecond of theſe is made up of the firſt kind of Pyramid (but not exactly ordinate,) in number 20, and equal and like, being comprehended under ſo many ordinate and equal Trigons, as their Baſes, by which they are only eminent, their whole Bodies beſides being latent, and meeting vertically in the Center of the Icoſahedrum, (or of its circumſcribing Spheare) which baſes or trigonal *P*lanes being compoſed together by ſolid angles, do therefore comprehend or contain the whole Icoſahedrum, according to *E.* 11. *d.* 29. and which ſolid angles (made by the connexion or concurſion of the ſaid *P*lanes or baſes, according to their angles) are, therefore in number ſubquintuple the plaine, ſuperficial, or trigonal angles conſtituting the ſame, (ſo that to the conſtituting of one ſolid angle, here do concurre five ſuperficial angles) unto all which from the Center of the Body, right lines being drawn, the whole Icoſahedrum is thereby divided into 20 equal and like trigonal *P*yramids, according to what I ſaid before: and the angular lines called the Sides (made by the inclination of the ſaid trigonal *P*lanes according to their

their ſides, and ſo equal with the ſame) are in number, halfe ſo many as the baſial or hedral ſides. Therefore I wil next briefly touch upon the dimenſion of this Body, how to perform the ſame, according to our artificial way of *Pyramidal* Dimenſion before demonſtrated; as it is vulgarly conceived to be compoſed of *Pyramids*: For the ſolidity of one of the ſaid compounding *Pyramids* being firſt obtained (which is the uſuall way of meaſuring this, and alſo the two other ordinate mixt *Polyhedrums*) the ſolidity of all the reſt, (and ſo conſequently of the total *Icoſahedrum*) is preſently had, by the vigecuplation only of that one *Pyramid*.

But now the whole difficulty of this Dimenſion of the Icoſahedrum, (and ſo of the other two foreſaid Bodies) conſiſts in the inveſtigation of the Axis, or line of altitude, of the compounding Pyramid (as it did before in the *Tetrahedron*) if the ſame be inquired geometrically (as Geometricians ſpeake) to wit, by firſt having the ſide of the ordinate Solid only, and ſo coming at length, after many tedious and troubleſome operations, both arithmetical and geometrical, to the ſaid perpendicular-line of altitude, (which is no other then the *Radius* of the Spheare inſcribed within the plain Solid) which way therefore I ſhal here pretermit, as being needleſs for me to demonſtrate; the ſame being ſhewed by divers practicall Authours, eſpecially *Ramus* and *Clavius* in Latine; and more abundantly in Engliſh, by our Countrey-man, M^r. *Digges* long ſince, in his learned Diſcourſe of geometrical Solids, annexed to his *Pantometria*, as a part thereof; to which Authours I therefore refer the diligent practizer for a full ſatisfaction in this poynt; my intention in this place, being only to bring in our new or artificial way of *Pyrami-*

dal *D*imenſion, in the aforeſaid ordinate Bodies, for the more eaſie and ſpeedy obtaining of their ſolidities in a *P*yramidal way, the *Hexahedron* being excepted. And therefore I ſhal here only ſhew, how, inſtrumentally or mechanically to get the ſaid *P*yramidall Axis or altitude, by getting firſt the altitude of the whole body, (which is no other then the total Dimetient of the inſcribed Spheare) being according to what I ſhewed before, for getting the altitudes of Cones and *P*yramids in general. Therefore, if from the ſuperiour *P*lane of the *Icoſahedrum*, being produced or extended, that is, from the inferiour or interiour ſuperfice of any *P*lane placed upon the uppermoſt *P*lane of the *Icoſahedrum*, a perpendicular be let down to the inferiour or oppoſite *P*lane or Baſe thereof, in like manner produced, that is, to the ſuperiour or exteriour ſuperfice of a *P*lane placed under the Icoſahedrum (and ſo, upon which the ſame lyeth) the ſaid perpendicular-line (being accurately meaſured) ſhall give the altitude of the whole Icoſahedrum, whoſe half ſhall be the Axis or Altitude of the compoſing *P*yramid ſought for.

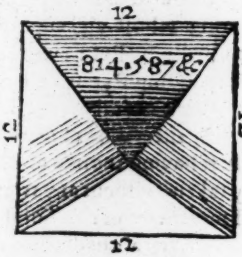
So that the great difficulty in the dimenſion of the Icoſahedrum, ariſing by the foreſaid geometrical inveſtigation of this *P*yramidall altitude, is by this means quite taken away, and the thing made very eaſie. Ane therefore whenſoever you would meaſure an *Icoſahedron* (or a *Dodecahedron*, for the ſame reaſon holds in it for the altitude) it is beſt to uſe this way; for ſo the ſolidity of the ſame wil then be obtained with little labour, eſpecially according to our foregoing artificial pyramidal Menſuration; For the ſaid Axis or altitude of the Icoſahedron's cõpoſing *P*yramid, being taken by the foreſaid artificial Line for the ſolid dimenſion of a Trigonal Pyramid, and the ſame be multi-

tiplied into the Quadrat of the ſide, taken by the ſame Line; the product ſhal be the ſolid content of the ſaid compounding Pyramid; whoſe vigecuple wilbe the ſolidity of the total Icoſahedrum. But how, readily to obtain the Axis of the compounding Pyramid, both of this, and alſo the other ordinate bodies (where need is) I ſhall afterwards ſhew among the dimenſional Proportions in theſe Bodies; by firſt having the ſide of the Body, and which is very eaſily taken, without any trouble at all.

And ſo the totall ſuperficies of this Solid, wil readily be had by the Line for the Lateral quadrature of an ordinate Trigon (according as I ſhewed before in the ſuperficies of a Tetrahedron) for the ſide thereof being meaſured thereby, its Quadrat wilbe the area of one of the baſes or *Flanes*, whoſe vigecuple wilbe the total Icoſahedral ſuperficies.

The Dimenſion of a tetragonall Pyramid, ſhewed in the *Octahedron*.

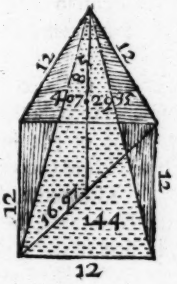
Now for the ſecond kind of *Pyramid*, to wit, the Tetragonal, upon a regular baſe, we ſhal next ſhew our artificial dimenſion thereof, both ſolid and ſuperficial, and firſt in the other trigonal ordinate mixt *Polyhedrum*, or the ſecond trigonal ordinate Body, called *Octahedron*, ſeing that the ſame is compoſed of two equal and like tetragonal *Pyramids* of equal altitudes (according to *E.14.p.16.*) meeting in their baſes, and ſo making one common baſe, which is a Quadrat deſcribed by the ſide of the *Octahedrum*, whereupon this regular Solid is comprehended under twice ſo many equal ordinate Trigons, as is the *Tetrahedrum*, being compoſed together by ſolid angles, which therefore are in number, ſubquadruple the ſuperficial or trigonal angles conſtituting the ſame, (ſo that four ſuper-



ſuperficial angles do here meet to make up one ſolid angle) And ſo this Body may alſo be cõſidered under another pyramidal compoſition, to wit, as compoſed of 8 equal and like trigonal *Pyramids*, (according to the number of its trigonal baſes, which ſhal alſo be the baſes of the ſaid *Pyramids*) concurring vertically in the Center thereof (or of its circumſcribing Spheare) like as the *Tetrahedron* (though that be an intire *Pyramid* of it ſelfe, and of the moſt ſimple kind of all, to wit, the Trigonal) may be conceived to be compoſed of 4 equal and like trigonal *Pyramids*, (according to its 4 baſes or *Planes*) meeting vertically in the Center of its body, or of its comprehending or containing Spheare. And ſo we ſhal here ſhew the artificial dimenſion of this Body in a pyramidal way, as being taken in the firſt pyramidal compoſition, by ſhewing the dimenſion of its tetragonal *Pyramid*; which we wil firſt perform by the natural way of its dimenſion for a confirmation of our artificial way, according as we have done in all the precedent Dimenſions.

And therefore firſt (for example-ſake) let the Side of the

the Baſe of a tetragonal Pyramid, as alſo the true ſide it ſelf of the *Pyramid*, be of the Rational Line in general, the ſame with that of the foregoing trigonal *Pyramid*, or the Tetrahedron, *viz.* 12.00: then the *Diagony* of the baſe, (which is alſo the *Diameter* of its circumſcribing Circle) wilbe 16.97; and thence, the Axis of the *Pyramid*



8.485, as being equal to the ſemi-diagony of the baſe, (or ſemidiameter of its circumſcribing Circle.) For ſeing firſt, that the *Diagony* of the Baſe, is double in power to the *Side* thereof, by *E*, 1. *p.* 47, &c. the power of the *Diagony* wilbe 288, whoſe Root irrational, 16.97, &c. wilbe the *Diagony* it ſelf; whoſe half therefore, 8.485, &c. is the ſemidiagony (or Radius of the circumſcribing Circle.) And then again, ſeing that the Axis, and baſial ſemidiagony, as being the two ſides about the right angle of the rectangle Triangle made by the ſaid two lines, and the ſide of the *Pyramid*, are both together but equal in power to the ſaid *Pyramidal Side*, as being the Hypotenuſal ſide of the ſaid Triangle, by *E*, 1. *p.* 47. before-cited; if therefore

the Lateral power of the *Pyramid*, *viz.* 144, be diminiſhed by the ſemi-diagonial power, *viz.* 72; (which is juſt half the lateral power,) there muſt needs remain the ſame for the power of the *Axis*; and ſo the Axis it ſelfe, the ſame with the baſial ſemi-diagony, *viz.* 8.485, &c. which (ſufficiently produced) being infolded with a trient of the Baſe, *viz.* 48; there wil reſult the true ſolidity of the *Pyramid*, (to cube-centeſmes) 407.293506: and ſo half the ſolidity of an *Octahedrum* compoſed of two ſuch *Pyramids*; which therefore doubled, wil give 814.587012 for the whole ſolidity of the Octahedrum. Now for the performing of this ſimple *Pyramidal* Dimenſion, by an artificial *Line* of Meaſure proper thereunto, which, I find (according to the reaſon of the 3d. Theoreme, &c. following) to be of the Rational Line in general, 1.4430, &c. The ſide of the Baſe of the foregoing tetragonal *Pyramid* put naturally 12.00, being meaſured thereby (under a centeſimal ſolution) wilbe found 8.32, whoſe Quadrat is 69.2224 for the artificial baſe: and the Axis of the *Pyramid*, being naturally, 8.48, wilbe found by the ſame Line, 5.88; which two artificial numbers of dimenſion being wholly multiplied together (that is, the whole baſe with the whole axis) there wil ariſe 407.027712, for the artificial ſolidity of the Pyramid; which wanteth of the true, or natural ſolidity, only $\frac{1}{4}$ *ferè*, of a cube-unit: and this then being doubled, wil give, 814.055424 for the artificial ſolidity of the Octahedrum, wanting of the true, natural ſolidity before declared, only about $\frac{1}{2}$ of a cube-unit, which is no very conſiderable matter, And if the ſaid artificial Line of Mea-

The artificiall Line for the ſolid dimenſion of a tetragonal Pyramid, by the ſide of it's Baſe, and it's Axis or Altitude together.

Measure be made 1000 parts, and then the Axis of the Pyramid, and the side of it's base, be measured thereby; they wil produce the solidity stil nearer the truth.

And if the solidity of an Octahedrum be immediatly requiree; then the total Axis or Diagony thereof (which is double to the Axis of the foresaid Pyramid, and so no other then the Diagony of it's base, or the Diameter of the Circle circumscribing the same, and also the Axis or Diameter of the Sphear circumscribing the Octahedron) being infolded with a trient of the componnding Pyramids base, (which is no other then the Octahedron's Lateral Quadrat, as was shewed before) or the whole base with a trient of the said Axis or Diagonial; there wil immediatly result the solidity of the Octahedrum: For so the true Axis or *Diagonial* of the foresaid Octahedrum, 16.97, &c. sufficiently produced by Radical extraction, or otherwise, *viz.* 16.97056274, &c. being infolded with a trient of the aforesaid pyramidal base, *viz.* 48. (or the whole base 144 with a trient of the axis, *viz.* 5.65685424, &c.) there, wil immediatly result the true total Octahedral solidity, 814.587012, as before. And so the total axis or *Diagony* of the Octahedrum, found by the former artificial *Line*, (in a centesimal solution) 11.77 *ferè*, being infolded with the foresaid total common base of the two compounding Pyramids, produced by the same *Line*, 69.2224; there wil also immediatly result the solidity of the Octahedrum, 814.747648 *ferè*, which comes much neerer the true solidity, then the former dimension; this differing therefrom (now by way of excesse) not so much as $\frac{1}{6}$ of a cube-unit, or integer, of the appointed measure; which Octahedral solidity is exactly quadruple to the foregoing Tetrahedral solidity, these two Bodyes having here one and the

ſame ſide in meaſure. And here therefore it appears in brief, that if the total Axis or Diagonial, and the Side, of an Octahedrum, be taken by the foreſaid artificial Line of meaſure for a Tetragonal Pyramid, and the Quadrat of the Side be augmented by the Axis; the reſulting rectangle regular-baſed oblong Priſm, or Parallelepipedum, ſhal compleatly contain the ſolidity of the Octahedrum.

As for the Diagonial, Axis, or Altitude of the *Octahedron*, the ſame may be alſo obtained inſtrumentally or mechanically, according to what I ſhewed in the *Tetrahedron*, and Pyramids in general; and alſo for the altitude of the *Icoſahedron*, and *Dodecahedron*; the altitude of this Body being conſidered according to a perpendicular-line comprehended between its two oppoſite angles, for as much as it is compoſed of two equal and like quadrangular Pyramids joyned together in their baſes, as I ſaid before.

The ſuperficiall dimenſion of a tetragonal Pyramid; And the artificiall Line for performing the ſame by the Side of the baſe, and the perpendicular line of the trigonall Plane together.

As for the ſuperficiarie dimenſion of this kind of Pyramid which now we have in hand, the ſame may be moſt readily performed in the ſame artificial manner as that of a trigonal Pyramid, to wit, by the ſide of the Baſe, and the perpendicular-line of it's triangular Plane together. And the artificial Line of Meaſure for this purpoſe, I find (according to the reaſon of the ſame Theoreme) to be of the Rational Line in general, 0.7071, &c. $\sqrt{\frac{1}{2}}$, which is Apotomally in Number, as 1.....2929 *ferè*) which Line being ſet off therefrom, and divided in a due manner, and then the two forenamed Lines of the Pyramid commenſurable by the ſame, be accor-

accordingly measured thereby, the product arising by their mutual multiplication, shal be the superficies of the Pyramid, (to wit, without the base) agreeing with that which is produced naturally, by the semiperimeter of the base, and the foresaid trigonal perpendicular multiplied together, according as I fully demonstrated before in the superficiary dimension of the trigonal Pyramid. As in the foregoing tetragonal Pyramid, the side of the base being 12, the perimeter thereof wil be 48, and so the semiperimeter 24: & the side of the Pyramid, or of its trigonal Plane, being the same with the side of the base, the perpendicular of the said Plane, wil be 10.39, &c. as was shewed in the superficiarie dimension of the *Tetrahedron*, and before that, in the dimension of the *Trigon* alone; which two multiplied together, (the said perpendicular being further produced, as formerly) there wil arise 249.4153, for the superficies of this Pyramid, without the base; agreeing with the total superficies of the foregoing Tetrahedrum, the Planes of that & this, being all one. Now the side of the base of this Pyramid (naturally 12.00) being measured by it's proper Line, for superficial measure, wil be found artificially, the same that the Diagony or Diameter of its base is naturally, *viz.* 16.97, &c. and the foresaid perpendicular wil be found by the same Line, (in a centesimal partition) 14.70 *ferè*; which two multiplied together, do produce 249.4590 *ferè*, for the Pyramidal superfice aforesaid; exceeding the true superfice, only so much as $\frac{1}{23}$ of a square-unit or integer, it being decimally, .0437, or 437 of 10000.

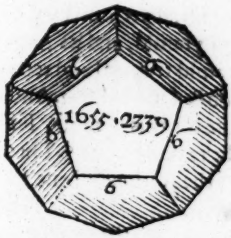
And thus may the superficies of an Octahedrum be obtained, it being only double the superficies of its compounding Pyramid: For so, the superficies of the forego-

Octahedrum, wil be by this latter or artificial Meaſure, 498.9180 *ferè*. And therefore if the ſide of an Octahedrum, and its hedral perpendicular, be taken by this artificial Line, and one of them be doubled; the Rectangle Plane made thereof, wil be the total ſuperficies of the Octahedrum: And ſo the hedral perpendicular of the foreſaid Octahedrum being taken by this Line, and doubled, wil be 29.39. which multiplyed by the ſide of the Octahedrum, found by the ſame Line, 16.97; the Plane produced therefrom, wil be, 498.7483, for the ſuperficies of the Octahedrum; which comes a little nearer the true ſuperficies, then the former; that exceeding the ſame only, .0874 *ferè*, which in vulgar terms, is not fully $\frac{1}{11}$; and this wanting thereof, but .0823, which in vulgar account, is hardly $\frac{1}{12}$; the true ſuperficies being 498.8306, (double to the ſuperficies of the foregoing Tetrahedrum:) ſo that the ſide and hedral perpendicular of an *Octahedron* being taken by the prime Rational Line, and doubled; the Plane ariſing from their mutual multiplication, ſhal be the true ſuperficies of the Octahedrum; being no other then that of the baſial ſemiperimeter, and the trigonal perpendicular of its tetragonal compounding Pyramid, doubled; As before, the double product of 24 and 10.39. &c. (or 12, and 20.78, &c.) being now the ſingle product of 24, and 20.78, &c. Or the ſide being quadrupled, and the ſaid perpendicular taken ſingle, ſhal together produce the ſame; being no other then that of the whole baſial perimeter of the foreſaid Pyramid, and the perpendicular of its triangular Plane together, for the double ſuperficies of the Pyramid; which is the ſuperficies of the Octahedrum: As heer the product of 48, and 10.39, &c. Or the ſuperficies of this Solid, may be had againe artificially by the Lines of

of quadrature pertaining to the *Trigon*, as I ſhewed before for the ſuperficies of a Tetrahedrum; for ſo, one of the Planes or baſes being had; the Octuple thereof ſhal be the total Octahedral ſuperficies.

Now for the third ſort of Pyramid, to wit, the pentagonal, upon an ordinate Baſe; I ſhal next briefly ſhew our artificial Dimenſion thereof, and that in the third ordinate mixt Polyhedron, or the pentagonal ordinate Body, namely the *Dodecahedron*; ſeing that the ſame is compoſed of this ſort of *Pyramid*, in number 12, equal and alike, which are comprehended under ſo many equall ordinate *Pentagons*, as being their Baſes, whereby they are only

The Dimenſion of a pentagonal Pyramid demonſtrated in the *Dodecahedron*.



eminent, their whole bodies beſides being hidden, & meeting vertically in the Center of the *Dodecahedrum*, (or of its ambient Spheare) as thoſe of the Icoſahedrum, beforementioned; the ſtructure, or fabrick of theſe two bodies being much alike: Which baſes therefore, or *Pentagonal Planes*, being compact together by ſolid angles, do comprehend

prehend the total *Dodecahedrum*, by *E*, 11. *d*. 28. and theſe ſolid angles (made by the inclination or connexion of the ſaid Baſes or *P*lanes according to their angles) are thereupon, numerally ſubtriple the plain, ſuperficial, or pentagonal angles compoſing the ſame, which are juſt ſo many as the trigonal angles of the *Icoſahedrum*, the ſides of theſe two Bodies, or the ſides of all their baſes together, being numerally the ſame; (ſo that three ſuperficiary angles do here meet together, for the compoſition or conſtitution of one ſolid angle, as in the *Tetrahedrum*; and this ſolid angle is the greateſt of any of the other Bodies, being contained or included by $3\frac{1}{5}$ plain right angles; and ſo is in quantity $1\frac{1}{5}$ ſolid right angle, as its compoſing, or bedral angle, is $1\frac{1}{5}$ plain or ſuperficial right angle.) And ſo wee ſhal here conſequently ſhew our artificial *D*imenſion of this ordinate Solid in a pyramidal way. And this *P*yramidal dimenſion I ſhal perform by the ſide of the baſe of the compounding *P*yramid, and its Axis together, as I did before in the *P*yramid trigonal and tetragonal, in reference to the Tetrahedron & Octahedron. And the *L*ine of meaſure for this purpoſe, I find (according to the reaſon of the 3d. Theoreme, &c.) to be of the Rationall *L*ine in generall, 1. 2036, &c. Therefore admitting the ſide of the baſe of a *Dodecahedron's* compounding Pyramide (which is alſo the ſide of the *Dodecahedrum* it ſelf) to be of the Rational *L*ine in general, 6.00; then the Axis or altitude of the ſaid *P*yramid (being the ſemi-altitude of the *Dodecahedrum*, or the ſemi-axis, or ſemi-dimetient of it's inſcribed Spheare) wil be found, (according to the proportion of the ſide of a *Dode-*

The artificiall Line for the ſolid dimenſion of a pentagonal Pyramid, by the ſide of it's baſe, and Axis together.

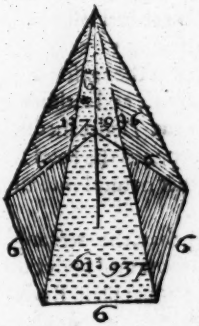
Dodecahedrum to the Axis or *Dimetient* of its inscribed Sphear, noted afterwards) to be 6.68; which two *Pyramidal* lines being measured by the foresaid artificial *Line*, mary solution) wil be found, the first, 4.98, for ial side of the *Pyramids* base, and the other, 5.55,

ramids artificial Axis; Now 4.98, being squa-s, 24.8004, for the artificial Base; which aug-y 5.55, yields 137.642220 for the artificial soli-he compounding Pyramid singly; and which ed by the number of the compounding Pyramids, 551.706640, for the solidity of *Dodecahedrum*; deed differs somwhat considerably (by way of om the true natural solidity, in respect of the total *Dodecahedrum*, though not of its compounding Pyramid alone. For the side of the *Dodecahedrum*, or of its Pyramids base, being naturally 6, the true area of the base, wil be found (according to the trigonometrical operations in the Pentagonal *Dimensions* beforegoing) 61.9371, &c.

prehend the total *Dodecahedrum*, by *E*, 11. *d*. 28. and theſe ſolid angles (made by the inclination or connexion of the ſaid Baſes or *P*lanes according to their angles) are thereupon, numerally ſubtriple the plain, ſuperficia tagonal angles compoſing the ſame, which are ju ny as the trigonal angles of the *Icoſahedrum*, th theſe two Bodies, or the ſides of all their baſes being numerally the ſame; (ſo that three ſuperfi gles do here meet together, for the compoſition ſtitution of one ſolid angle, as in the *Tetrahedr* this ſolid angle is the greateſt of any of the othe being contained or included by $3\frac{1}{7}$ plain right an ſo is in quantity $1\frac{1}{5}$ ſolid right angle, as its com hedral angle, is $1\frac{1}{7}$ plain or ſuperficial right angle. wee ſhal here conſequently ſhew our artificial *I* of this ordinate Solid in a pyramidal way. And ramidal dimenſion I ſhal perform by the ſide of t the compounding *P*yramid, and its Axis togethe before in the *P*yramid trigonal and tetragonal, in to the Tetrahedron & Octahedron. And the *L*in ſure for this purpoſe, I find (ac the reaſon of the 3d. Theoreme, 8 of the Rationall *L*ine in generall &c. Therefore: the ſide of the baſe of a *Dodec* compounding Pyramide (whi the ſide of the *Dodecahedrum* it ..., to be of the Rational *L*ine in general, 6.00; then the Axis or altitude of the ſaid *P*yramid (being the ſemi-altitude of the *Dodecahedrum*, or the ſemi-axis, or ſemi-dimetient of it's inſcribed Spheare) wil be found, (according to the proportion of the ſide of a *Dodt-*

The artificiall Line for the ſolid dimenſion of a pentagonal Pyramid, by the ſide of it's baſe, and Axis together.

Dodecahedrum to the Axis or *Dimetient* of its inscribed Sphear, noted afterwards) to be 6.68; which two *Pyramidal* lines being measured by the foresaid artificial *Line*, (in a centenary solution) wil be found, the first, 4.98, for the artificial side of the *Pyramids* base, and the other, 5.55,



for the Pyramids artificial Axis; Now 4.98, being squared, yields, 24.8004, for the artificial Base; which augmented by 5.55, yields 137.642220 for the artificial solidity of the compounding Pyramid singly; and which augmented by the number of the compounding Pyramids, yields, 1651.706640, for the solidity of *Dodecahedrum*; which indeed differs somwhat considerably (by way of defect) from the true natural solidity, in respect of the total *Dodecahedrum*, though not of its compounding Pyramid alone. For the side of the *Dodecahedrum*, or of its Pyramids base, being naturally 6, the true area of the base, wil be found (according to the trigonometrical operations in the Pentagonal *Dimensions* beforegoing) 61.9371, &c.

whoſe ſubtriple, 20.6457, &c, being infolded with the whole axis or altitude of the Pyramid, naturally 6.68,&c. there wil ariſe the true, natural ſolidity of the Pyramid, 137.936159*ferè*; which our meaſure wants of, not $\frac{1}{3}$ of a ſolid integer or unit; and the duodecuple hereof, 1655.233908*ferè*, is the true ſolidity of the *Dodecahedrum*. Or the Baſe and Axis of the Pyramid being wholly increaſed together, there wil reſult the Pentagonal Priſme, 413.808476, containing three of the compounding Pyramids, (according to the reaſon of *E*, 12. *p*. 7. before cited) being of equal baſe and altitude with the Pyramid.) and ſo a quarter of the *Dodecahedrum*; whoſe quadruple therefore, 1655.233904, is the total *Dodecahedrum*; which agrees with the other natural meaſure thereof, without any ſenſible difference.

Or again, ſeing that the rectangle Solid (*Priſma* or *Parallelepipedum*) contained under the Perpendicular from the Center of any regular plain Body, to any of it's baſes or Planes, and a trient of the total ſuperficies, comprehends the ſolidity of the whole body (according to what I ſhewed formerly in the dimenſion of a Sphear, for the producing of its ſolid content by the ſemidiameter, and a trient of the Superficies, the plain Solid ariſing therefrom, being equal to the ſpherical Solid: or wch is all one, the plain Solid made of the whole *D*iameter, and a ſextant of the ſpherical ſuperficies,) in as much as that which is contained under the ſaid Perpendicular, (which is here no other then the Axis of the compounding Pyramid, and the ſemi-axis of the inſcribed Sphear) and a trient of one of the baſes or Planes (which is the baſe of the compounding Pyramid, as aforeſaid) comprehends the ſolidity of one of the compounding Pyramids; and ſo conſequently that which is contained under

under the said Perpendicular, and a trient of all the bases or Planes together (as being the bases of all the compounding Pyramids together) must needs comprehend the solidity of all the Pyramids together, and so of the whole ordinate Body; (as the rectangle Plane contained under the perpendicular from the Center of any rectiline regular *Plane*, or superficial Figure, to any of it's sides, and the semi-perimeter of the same, comprehends the *Area* of the whole Figure (as I shewed formerly in the Pentagon, and which answereth to that of the dimension of a Circle, for the producing of it's *Area* by the semidiameter, and semiperiphery, the rectangle *P*lane or *P*arallelogram resulting therefrom (by their mutual implication) being equal to the Circular *P*lane: or which is all as one, the rectangle *P*lane made of the whole diameter & a quadrant of the *P*eriphery, or of the whole *P*eriphery & a quadrant of the diameter) in as much, as that which is contained under the said perpendicular, (which is no other then the perpendicular of altitude of the Figure's compounding *Trigon*, and the Radius of the inscribed Circle) and half the side, (as being, half the base of the said *Trigon*) comprehends the supercies, or *area*, of one of the compounding Trigons; and so, that which is contained under the said perpendicular and half the perimeter of the Figure (as being half of the bases of all the compounding Trigons together) must needs comprehend the superficial Content of all the said Trigons together, and consequently of the whole regular Figure it self,) Therefore the Base of the *Dodecahedrum* being 61.9371, &c. the total superficies thereof, wil be 743.2462 &c. whose subtriple, 247.7487, &c, being augmented by the foresaid perpendicular (or axis of the compounding *P*yramid) 6.68, &c. there wil result the total *D*odecahe-

dral solidity, 1655.2339, &c. exactly as before: which may be plainly seen by the subsequent Logarithmical operations, whereby these Pyramidal and Dodecahedral dimensions, are most readily and accurately performed.

Therefore first,

Side of the Dodecahedron, or of its compounding Pyramid's Base, 6.

Pentagonall Base, 61.9371 &c.	1,7919515
Subtriple. 20.6457 &c.	1,3148302. } A
Pyramidall Axis, 6.68 &c.	0,8248479. }
Pentagonall Pyramid. 127.936 &c.	2,1396781. } A
12	1,0791812. }
Dodecahedron, 1655.233 &c.	3,2188593.

Again, 2ly.

Pyramidall Base, 61.9371 &c.	1,7919515. } A
Axis, 6.68 &c.	0,8248479. }
Pentagonall Prisme, 413.808 &c.	2,6167994. } A
4	0,6020600. }
Dodecahedron, 1655.2339, &c. before *ferè*.	3,2188594. as be-

Or

Or again, 3^{17}.

Dodecahedrons base, or Hedrall area, 61.9371 &c.	1,7919515.	A
12	1.0791812.	A
Totall Superficies, 743.2462 &c.	2,8711327.	
Subtriple, 247.7487 &c.	2,3940115.	A
Dodecahedron's semi-altitude, (or Pyramids axis or altitude) or inscribed Sphear's Radius, 6.68 &c.	0,8248479.	A

Dodecahedron it self, 1655.2339 &c. 3, 2188594. exactly as before.

Which our Measure found by the foresaid artificial *Line* of *Pyramidal* consolidation, falleth short of indeed, about 3 integers, or units, and an half: But then if the said *Pyramidal Line* of Measure be increased in its parts, by a subdecuple solution of the former; the side of the aforesaid *Pyramids* base (naturally 6.000) wil be found thereby, 4.985, *ferè*, whose Quadrat is 24.850225 *ferè*, for the artificial base: and the Axis of the said *Pyramid* (naturally 6.681) wil be found thereby, 5.551 *ferè*; which two by a conjunct composition, wil produce 137.943598975 *ferè*, for the solidity of the Pyramid (which now exceeds the true solidity, being 137.936158730, hardly so much as $\frac{1}{134}$ of a solid integer or unit) whose duodecuple, 1655.323187700, is for the solidity of the *Dodecahedron*; which now differeth from the true solidity, being correspondently 1655.233904760 (by way of excesse) hardly $\frac{1}{11}$ of a solid integer or unit.

As

As for the ſuperficies of a Do decahedron, the ſame may be readily obtained by any of the three Lines of quadrature pertaining to an ordinate *Pentagon*, but that for the ſide is the moſt fit and proper (though all of them wil produce the ſame thing) for ſo the ſide of the Dodecahedron being taken thereby, the Square thereof ſhal be the baſial or hedral area, whoſe duodecuple wil be the total *Dodecahedral* ſuperficies. But a farre better way for the dimenſion both ſolid and ſuperficial of this, and the other plain regular Bodies, I ſhal ſhew in the next Section, which wil be wholly taken up about the ſaid five Bodies.

Concerning the ſuperficial dimenſion of a Pentagon all Pyramid; And withall, the artificiall Line for performing the ſame, by the ſide of the baſe, and the perpendicular of the trigonal Plane together.

As for the ſuperficial Dimenſion of a pentagonal *Pyramid*, ſeing it is but the ſame with that, which I have fully ſhewed in the two preceding Pyramids, both naturally and artificially, in the moſt ready manner that may be; to wit, artificially, by the ſide of the Baſe, and the perpendicular-line of the triangular or laterall Plane together: Therefore I ſhal not need to inſiſt upon the ſame in this laſt Pyramid here particularly handled, by way of exemplary illuſtration; But ſhal only give the artificial Line of meaſure for the performance thereof, as I find it to be from the natural *Line* of meaſure, or prime Rational *Line* in general, deficiently, 0.63245, &c. (that is Apotomally in number, from our general reaſon of Meaſure, as $1 - .36754$, *&c.* the ſaid artificial *Line* being $\sqrt{\frac{2}{5}}$ of the natural *Line*, in it's power quadratick.

And thus may the *Dimenſion* both ſolid and ſuperficiall of this and all other Pyramids, be performed artificia ly

cially by the other dimensional lines of their Bases (specified before in the dimension of those basial Figures simply, and by which we said, they might be also artificially, or quadratically measured;) as, the Diameters or *Perpendiculars*, and Diagonals, together with their Axes or Altitudes, for solid measure; and with the perpendicular-lines, or altitudes of their triangular *Planes*, for superficial measure: And which, though it be needles, that by the side of the Base, (with the Axis and trigonal *Perpendicular*) being most ready, and also most proper, according to what I noted in the beginning of this Section, in the basial Figures: Yet having for variety of Art in this kind, not spared the pains of extracting or eradicating the artificiall Lines serving thereunto; I thought it might not be amiss, to set them down here also; as they are from the natural *Line* of measure, or prime Rational Line in generall, in a decu-millenary solution; those for solid dimension, (as also the other beforegoing) being all of them, thereof redundantly; and those for superficial dimension, all of them deficiently.

The

The artificiall Line of Measure, is for the *Pyramid*,

- Trigonall,
- Tetragonal,
- Pentagonal,

in respect of the basial *Diameter*, conjunctly with the

- Axis, or Altitude of the Body,

1.7320$\sqrt{q3}$	Solid measure.
1.8172$\sqrt{c6}$	
1.6043.	

- Trigonall,
- Tetragonal,
- Pentagonal,

in respect of the basial *Diameter*, conjunctly with the

- Perpendic line, or altitude of the trigonal or lateral Plane.

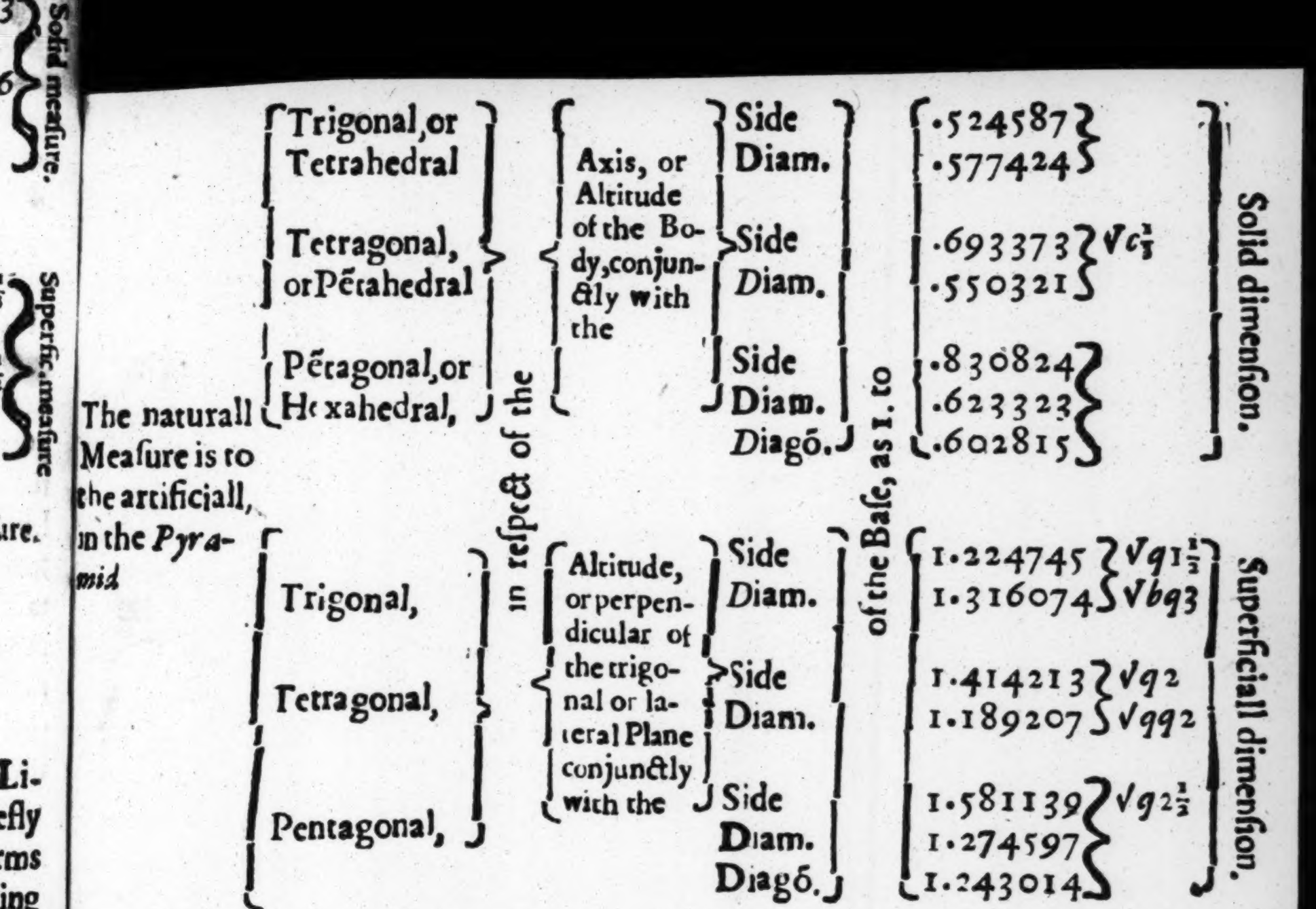
0.7598$\sqrt{qq\frac{1}{3}}$	Superficiall measure
0.8409$\sqrt{qq\frac{1}{2}}$	
0.7846 *ferè*.	

And for the Pentagonall Pyramid, in regard of the Basial Diagony with the

Axis, or Altitude,	1.6589 *ferè*.	Solid measure.
Trigonall perpendic.	0.8045 *ferè*.	Superficiall measure.

All which Pyramidal *Dimensions* beforegoing, in a Linear, instrumental, or geometrical way, I shal next briefly expresse in an Arithmetical way, in the proportional terms following, from an ample solution of the unit, (according as I did before in the Cone and Cylinder,) by which, the artificial Measure may be readily produced from the natural; or the naturall Measure be reduced to the artificiall. Therefore,

The



Place this between Page 136 and 137.

And what hath been here delivered concerning the artificial Dimenſion of *Pyramids* upon regular Baſes; the like is to be underſtood for the dimenſion (both ſolid and ſuperficiall) of *Priſms* upon the like baſes, by artificiall Lines of meaſure peculiarly appropriated and applied to them for that purpoſe, in regard either of any of the baſial-lines aforenamed, with the Axis or Altitude for ſolid meaſure, or with the ſame for ſuperficial meaſure, (ſeeing the Axis or Altitude and the ſide, is all one in right or upright Priſms, as in right Cylinders, which I ſhewed formerly.) But theſe I ſhal here paſſe by, leaving them to the induſtry of the ingenious Practitioner, that ſhal pleaſe to exerciſe himſelf therein; and the rather for that the artificial Lines of meaſure pertaining to the Pyramids, wil alſo ſerve (if need be) for the dimenſion of the correſponding Priſms, (as I ſhewed formerly between the *Cone* and *Cylinder*) ſeeing that a Priſm is only triple its correſpondent Pyramid, according to *E*, 12. *p*. 7. as a Cylinder is triple its correſpondent Cone, according to *E*, 12. *p*. 10, as I noted formerly; And as the proportion of the *Priſm* to the *Pyramid* is triple, for ſolidity; ſo it is double for ſuperficiety; the ſimilitude, and ſo the reaſon between a *Pyramid* and *Priſm*, being the ſame (in both *D*imenſions) with that between a Cone and Cylinder: And ſo the Lines of artificial *D*imenſion, pertaining to the trigonal or tetrahedral Pyramid, wil ſerve for the trigonal or pentahedral Priſm; And the *L*ines for the tetragonal or pentahedral Pyramid, will ſerve for the tetragonal, or hexahedral Priſm: And the Lines for the *P*entagonal, or hexahedral *P*yramid, wil ſerve for the pentagonal, or hep-

Concerning the dimenſion of Priſms.

tahedral *Priſm*; and ſo forward: For as the *Pyramid* begins *à Quaternario*, So the *Priſm*, *à Quinario*.

And by the ſame artificial manner of meaſuring (as by the natural) may be obtained the ſolid content of any oblique or inclined *Pyramid* or *Priſm* upon a regular baſe, as readily as of a right or upright one, according as I ſhewed formerly for oblique Cones and Cylinders; the true altitudes of theſe Bodies (not their Axes) being conſidered; ſeeing that every ſuch oblique kind of Body, is equal in ſolidity to the right or erect body, having the ſame (or being of equal) baſe and altitude; by the reaſon of *E*, 12. *p*. 5 and 6, and *p*. 11 and 14, and alſo *E*. 11, *p*. 30 and 31, as I noted formerly.

Concerning the dimenſion of oblique or inclined Pyramids & Priſms.

SECT.

SECT. III.

Exhibiting a more ſpeciall and peculiar artificiall way of meaſuring both ſolidly and ſuperficially, the four plain ordinate Bodies, or rectiline regular Solids before handled, then was before; & this, after the moſt exquiſite manner that may be. Together with the like artificiall dimenſion of the other like regular Body, ſeverall waies.

HAving now ſhewed our artificial Dimenſion ſeveral waies, in *Pyramids*, and their Compounds, or *Pyramidates*; and conſequently of the five fore-named ordinate plain Bodies, or rectiline regular Solids, in a *Pyramidal* way: (the *Hexahedron* excepted.) We ſhal next come to ſhew the dimenſion of the ſaid Bodies in a farre better way; and indeed, in the moſt excellent artificial, and compendious manner that can poſſibly be found out: and that for ſuperficial meaſure, with the ſame ſpeed, eaſe and exactnes, and

so in the very same manner, as that of the *Circle*, *Trigon*, and *Pentagon*, &c. And for solid measure, as that of a *Cube*, by the natural or vulgar way of measuring the same (which we shal also here shew by the like artificial way with the rest) And this only by squaring and cubing any one of their dimensional lines (as in the *Sphear*) by artificial Lines of measure convenient for the purpose: Which may indeed, I almost despaired of, in regard of the great difficulty which I found to be in the solid dimension of these Bodies, by the usual or natural way, according to their Pyramidal compositions, (the *Hexahedron* excepted) especially the the two last and greatest of them, namely the *Icosahedron* and *Dodecahedron*. But yet considering the excellency of them in their compositions, constitutions, and structures above any other solid Figures; from whence they are called by *Pappus*, and other of the *Greeks*, τετραγώνα ἰσογωνια, *i. e. ordinata bene ordinata*; and commonly the *Pythagorean, and Platonical Bodies*, as being first invented (as is generally supposed) by *Pythagoras*, and afterwards set forth briefly by * *Plato* in the composition and fabrick of the World, as in the *Heaven*, and the four *Elements*; and so are also called therefrom, the Cosmical or mundane Bodies; and so that in the knowledge and understanding of the natures, properties and affections thereof, lieth as great a difficulty, nicety, and curiosity of Geometry, as may be; in so much as that *Proclus* makes the singular and admirable end of the *Mathematiques*, to be in the knowledge of these five Bodies, in respect of their constitution, adscription, and comparation, collation, or application among themselves: And so thereupon our Countreyman *Billingsley*,

* Plato *in Timæo, de Anima Mundi, seu Natura.*

Billingsley, in his learned Annotations upon *Euclids* Elements in Englisſh, after the 25th. defin. of the 11th. book, ſpeaking of the dignity and excellency of the ſaid 5 Bodies; ſaith, that they are as it were, the end, and perfection of all Geometry, and for whoſe ſakes was written, whatſoever was written in Geometry: Therefore for the avoiding and removing of all difficulties in their *Dimenſion*s, and ſo the facilitating of the ſame, as to the obtaining of their ſolid & ſuperficial Capacities (eſpecially ſolid) as eaſily and exactly, as of any other Figure whatſoever, and that ſeveral wayes, both naturally and artificially; I reſolved (by Gods aſſiſtance) to proſecute my metrical conceits and invention herein, as far as in any other Figure.

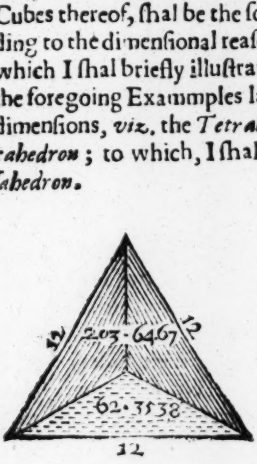
And ſeing that the ſides of theſe Bodies may more readily and accurately be taken by a Line of meaſure, then any other their lines of dimenſion adſcribed to them, (and indeed very accurately, without any trouble) as being their only natural lines of dimenſion, and ſo only apparent of of themſelves in them; (according to what I formerly ſaid for rectiline *Plane*s or Superficies in general: Therefore I ſhal here briefly demonſtrate their dimenſions both ſolid and ſuperficiall (by way of example) artificially by their ſides only; and withall, ſhal by the way, give artificial Lines for the like dimenſion of them, by ſome other of their dimenſional lines adſcribed to them. And firſt for ſolid dimenſion, the Lines for the Sides of theſe Bodies, (or for the cubick dimenſion of them by their Sides) I find, by the reaſon of the 2d. *Theoreme*, &c. following, to be of the prime Rational Line in general, (in a decu-millenary ſolution,) as followeth.

The artificial Line for solid Measure, or Line of Cubature is for the side of the

Tetrahedron, 2.0396 } redundant
Octahedron, 1.2849 }

Icosahedron, 0.7710 } deficient
Dodecahedr. 0.5072 }

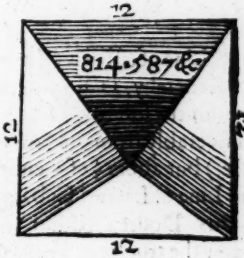
Which Lines being duly set off and divided, as the former, and so the Sides of these bodies measured thereby; the Cubes thereof, shal be the solid contents of the same, according to the dimensional reason of the prime Rational Line: which I shal briefly illustrate in three of these Bodies, by the foregoing Exammples laid down in their Pyramidal dimensions, *viz.* the *Tetrahedron*, *Octahedron* and *Dodecahedron*; to which, I shal here add the like for the *Icosahedron*.



Therefore first; the Side of the *Tetrahedrum* before handled, being put naturally, 12.00, the true solidity thereof, was there found, by the naturall Pyramidall Dimension, 203.646753, and by our artificial Pyramidall *Dimension*, 203.359474. Now the Line of Cubature

ture for the ſide of a *Tetrahedrum* being firſt put under a centenary ſolution only, and the ſide of the foreſaid Tetrahedrum meaſured thereby, the ſame wil be found 5.88, whoſe Cube is 203.297472 for the ſolidity of the Tetrahedrum; which wanteth of the true ſolidity, about as much as the foregoing artificial Pyramidal Dimenſion, *viz.* $\frac{1}{7}$ of cube-integer or unit only; which is near enough for any ordinary uſe: But however proceeding by a decuple of the former parts, in the Line of Cubation, *viz.* 1000; the ſolidity of this *Tetrahedrum* wil be found thereby, 203.608800,&c. which now wants of the true ſolidity, hardly ſo much as $\frac{1}{12}$ of a cube-integer; the ſide of the Tetrahedrum being now by this Line, 5.883.

II. The Side of the foregoing *Octahedron* being put naturally 12.00, the ſolid content thereof was found 814.587012, by the natural pyramidal Dimenſion, and 814.055424, by the artificial. Now the Line of cubical Dimenſion for the ſide of an *Octahedron*, being put in a centeſimal partition; the ſide of the aforeſaid Octahedrum, wil be found thereby 9.34, which cubed, gives 814.780504, for the ſolidity of the Octahedrum, which exceeds the true ſolidity, ſcarcely $\frac{1}{7}$ of a cube-integer or unit. And this very meaſure would be produced by the artificial Line for the ſolid dimenſion of a tetragonal Pyramid, by the diagony of its baſe and its Axis together: it being the ſame with the



Line

Line for cubing an Octahedron by its Axis or Diagonial, as I ſhal ſhew afterwards; and the Axis of this Body being double to the Axis of its compounding tetragonal Pyramid, and ſo equal to the Diagony of the ſaid Pyramids baſe, as I ſhewed formerly.

III. The ſide of the *Dodecahedron* formerly handled, being put naturally, 6.00; the ſolidity thereof, was found by the natural Pyram. dimenſion, 1655.233905 *ferè*; and by the like artificiall dimenſion (in a millenary ſolution of the proper Line of Meaſure) 1655.323188 *fere*. Now the ſide of this *Dodecahedrum* being taken by its Line of cubation (in a centeſimal ſolution) wil be found thereby, 11.83 *ferè*, whoſe Cube is, 1655.595487 *ferè*, for the ſolidity of the Dodecahedron, differing inconſiderably from the true ſolidity, being (by way of exceſſe,) but about $\frac{1}{3}$ of a cube-integer. But however if the ſaid Line of Lateral Cubature be made 1000, the ſolidity of this Dodecahedrum wil be produced thereby, much nearer the true one, *viz.* 1655.175675, &c. which now differeth from the true ſolidity (by way of defect) hardly $\frac{1}{17}$ of a cubique integer or unit: the ſide of the Dodecahedrum being now by this Line, 11.829.

IV. The ſide of an *Icoſahedron* being put naturally, 12.00, the ſolidity thereof wil be found by the natural Pyramidal Dimenſion, upon the very point of 3770: For the Axis of the compounding Pyramid, wil be found (by the proportion

portion of the ſide of an *Icoſahedrum* to the Axis of its inſcribed Sphear, or altitude of its own body, hereafter declared) to be 9.069, &c. which with a trient of the baſe (being the ſame with that of the foregoing Tetrahedrum and Octahedrum, *viz.* 62.3538, &c.) *viz.* 20.7846, &c. wil produce the ſolidity of the compounding Pyramid, 188.498398. (which by the proper artificial Pyramidal Line, wil be 188.325116,) whoſe vigecuple, 3769.967960, is the ſolidity of the *Icoſahedrum*: All which you may ſee moſt accurately produced by the ſeveral Logarithmetical operations or artificial Numerations following, according to thoſe formerly in the *Dodecahedron*. And therefore firſt.

Side of the Icoſahedrum, or of its Pyramid's Baſe	12.	
Trigonal Baſe, 62.3538, &c.	1,7948631	
Subtriple, 20.7846, &c.	1,3177419	A
Pyramid's Axis, 9.069, &c.	0,9575658	
Trigonal Pyramid, 188.498, &c.	2,2753077	A
20	1,3010300	
Icoſahedron, 3769.9679, &c.	3,5763377	

Again, 2ly.

Pyramidall Base, 62.3538, &c.	1,7948631	A
Axis, 9.069, &c.	0,9575658	
Trigonal Prism, or Pentahedr. 565.493, &c.	2,7524289	A
$6\frac{2}{3}$	0,8239088	
Icosahedron, 3769.9679, &c.	3,5763377, *as before.*	

Or again, 3ly.

Icosahedron's Base, 62.3538, &c.	1,7948631	A
20	1,3010300	
Totall Superficies, 1247.0765, &c.	3,0958931	
Subtriple, 415.69219, &c.	2,6187719	A
Icosahedron's semi-altitude, (or compounding Pyramid's altitude) or inscribed Sphear's semi-diameter 9.069, &c.	0,9575658	A
Icosahedron it self, 3769.9679, &c.	3,5763377, *exactly as before.*	

Or 4ly. (to come towards our present way of measuring here proposed) the same wil be most readily produced in a cubical manner, without any trouble of Calculation, or arithmetical and geometrical operation; and that first by comparing the *Cube* of the Icosahedrum's side with the Icosahedrum it self, according to the most exquisite terms of proportion, noted in the next Section; and this only by one simple composition of Numbers, from the artificial Numeration, thus;

Ico-

Icosahedron's Laterall Cube, 1728, 2.18169, &c.	3,2375437 / 0,3387941	A
Icosahedron it self, 3769.967, &c.	3,5763378, as before *ferè*.	

Or 5ly. and lastly; the same from thence, by finding the content of the *Cube* equal to the *Icosahedrum* according to the most exquisite Proportion of the side of the *Icosahedrum* to the side of that *Cube* or *Hexahedrum*, noted immediatly after the former Proportion; (to which doth answer exactly, our present artificial Mensuration, or organical, or mechanical Cubation) As,

Side of the Icosahedron, 12, 1.2969, &c.	1,0791812 / 0,1129314	A
Side of the equal Cube, or Hexahedron, 15.5636, &c.	1,1921126 / 3	M
Cube or Hexahedron, equall to the Icosahedron, 3769.967, &c.	3,5763378, *exactly as before.*	

Which several Logarithmical operations, do agree wel with that Algebraical or Cossical computation of Mr. *Diggs* in his forementioned discourse of geometrical Solids. *Probl.* 14. where having Cossically cast up the solid content of this very Icosahedrum, he saith at last, that the same being reduced into rational numbers, wil fal between 3769 and 3770.

And with these several artificial Numerations, wil be found to agree very nearly, our present artificial Mensuration: For the side of this *Icosahedrum* being measured by

its proper Cubatorie Line, (taken in a centenary solution) will be found thereby, 15.56 (as before, the side of its equal Cube, by the artificial Numeration) which cubed, yields, 3767.287616, for the solidity of the Icosahedrum, which indeed wanteth of the true solidity, between two and three integers of the appointed measure; but one example is not to be regarded: But however therefore, proceeding here in a more ample or numerous solution of the Lines of measure both natural and artificial, equally; the solidity of this Icosahedrum will be thus artificially produced very near the true content. As if the prime or natural Line be made 1000 parts, and so also the second, artificial, or Cubatorie Line; then the side of this Icosahedrum measured thereby, will be found 15.564 *ferè*, whose Cube is 3770.193726, &c. for the solidity of the Icosahedrum, which exceedeth the true Content, not so much as $\frac{1}{4}$ of the prime or natural Line, cubed; or $\frac{1}{4}$ of a cube-integer, or unit.

Concerning the superficial dimension of the 4 regular Solids beforegoing, by way of exact Quadrature.

Now for the like superficiary *Dimension* of these Bodies, the artificial Lines of quadrature for this purpose, I find (according to the reason of the first and 2d. Theoremes &c.) to be of the prime Rational Line in general (under the former solution) as followeth.

The

The artificiall Line for superficial measure, or Line of Quadrature, is for the Side of the

Tetrahedron, 0.7598	$\sqrt{99.\frac{1}{2}}$	
Octahedron, 0.5373 *ferè*		Deficiet
Icosahedron, 0.3398		
Dodecahedr. 0.2201 *ferè*		

By which Lines duly set off and divided, the sides of these Bodies being measured; their Quadrats shal be the total superficies; which I shal also briefly illustrate by help of the examples beforegoing in the Dimension of the *Trigon* and *Pentagon*, which I made use of formerly in the superficial Dimension of the *Tetrahedron* and *Octahedron* in a Pyramidal way.

Therefore first; the Side of the foregoing *Tetrahedrum* being naturally 12.00, the true totall Superficies thereof was formerly found, 249.4153, (which by the Line of quadrature pertaining to the side of an ordinate Trigon, was found, 249.6400 *ferè*.) Now the Line of quadrature peculiarly appropriated to the side of a *Tetrahedrum* (not having respect to the base or Plane simply, as being an ordinate Trigon) being centesimally divided; the side of this Tetrahedrum measured thereby, wil be found, 15.79, whose Quadrat is 249.3241, for the total superficies of the Tetrahedrum, which wanteth of the true superficies in

vulgar terms, only $\frac{1}{11}$ *ferè*, of a ſquare integer or unit, of the meaſure appointed.

II. The ſide of the foregoing *Octahedrum* being the ſame with that of the *Tetrahedrum*, *viz.* 12.00; the ſuperficial Content thereof wil be double to that of the Tetrahedrum, (as I ſhewed before) *viz.* 498 8306. Now the ſide of this Octahedrum being meaſured by its proper *Line* of quadration (in a centeſimal ſolution) wil be 22.33, whoſe Square is 498.6289, for the Octahedron's total Superficies, wanting of the true content, only $\frac{1}{7}$ of a ſquare-integer. Or the ſaid *Line* being made 1000, it wil give the Octahedron's ſide, 22.335 *ferè*, which ſquared, gives the Octahedron's ſuperficies, 498.852225 *ferè*, which exceeds the true Content, *viz.* 498. 830633 *ferè*, hardly $\frac{1}{45}$ (in vulgar terms) of a ſquare-unit.

III. The ſide of the former *Icoſahedrum* being the ſame with the ſide of the *Tetrahedrum* and *Octahedrum*; the ſuperficies thereof wil be quintuple the ſuperficies of the Tetrahedrum, and ſo double-ſeſquialter the ſuperficies of the Octahedrum, (according to what I have formerly ſpoken in the ſuperficial Compoſition of theſe Bodies) *viz.* 1247.0766 *ferè*: Now the ſide of this *Icoſahedrum*, being taken by its proper tetragoniſmal *Line*, under a centeſimal partition only, wil be found 35.31, whoſe Quadrat is 1246.7961, for the ſuperficies of the Icoſahedrum; which wants of the true ſuperficies, ſcarcely $\frac{1}{3}$ of a ſquare-integer. But yet the ſide of the Icoſahedrum being meaſured by its ſaid *Line* under a milleſimal ſolution, wil be found upon the point of 35.314, which quadrately, is 1247.078596 *ferè*, for the Icoſahedrum's ſuperficies, which now exceeds the true ſuperficies, (being 1247.076581) in vulgar terms, hardly $\frac{1}{2956}$ of a ſquare integer or unit.

IV. The

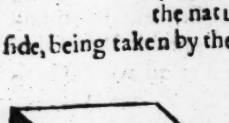
IV. The Side of the foregoing *Dodecahedrum*, being 6, the true totall ſuperficies thereof, was formerly found 743.2462, &c. Now the Line peculiarly appertaining to the ſide of a *Dodecahedrum*, for the quadrate dimenſion of its Superficies (not as relating to the Baſe or Plane thereof ſimply, as being an ordinate Pentagon) being firſt laid down under a centeſimal partition, and the side of this Dodecahedrum meaſured thereby, wil be found 27.26, whoſe Quadrat is 743.1076, for the total Superficies of the *Dodecahedrum*, which wanteth of the true ſuperficies, only about $\frac{1}{9}$ of a ſquare-integer or unit.

And ſo again, the side of a Dodecahedrum being double to the former, and ſo the ſame with the ſides of the three Bodies before going; the basial or hedral area, wil be the ſame with the area of the Pentagon formerly handled, *viz.* 247.7487, &c. and ſo the total ſuperficies (according to what I have ſaid of the composition of this Body) wil be upon the point of 2973, *viz.* 2972..9844, or more exactly, (by a further extension or production of the Pentagonal area) 2972.9849; which is quadruple the ſuperficies of the former Dodecahedrum: the baſe of this being quadruple the baſe of that. Now the side of this Dodecahedrum being meaſured by its foreſaid *Line* of Quadrature, wil be, 54.52; which ſquared, gives 2972.4304, for the Superficies of the *Dodecahedron*, which wanteth of the true Content only about $\frac{1}{2}$ of a ſquare integer: But however, proceeding in the partition of the *Line* of Quadrature, but one degree of parts further, *viz.* to 1000; the *Dodecahedron*'s ſuperficies wil be produced thereby, 2972.975625 (the ſide being 54.525) which now wanteth of the true ſuperficies, being 2972.984949 (in vulgar accompt) hardly $\frac{1}{100}$ of a quadrate unite or integer of the meaſure firſt aſſigned.

As

Concerning the dimenſion of the *Hexahedron*.

As for the other ordinate plain Body, namely the *Hexahedron*, (or orthogonial *Iſohedron*, or ordinate *Parallelepipedon*) the ſame cannot more readily be meaſured, (eſpecially for ſolid Meaſure) then by the natural way, which is by cubing it's ſide, being taken by the prime or naturall Rational Line:



And ſo almoſt as readily for ſuperficial alſo, by ſquaring its ſide, taken by the ſame Line; whoſe ſextuple wil be the the total ſuperficies: But elſe however, both theſe Dimensions may be performed artificially, either by the *Diagoniall*, *Diameter*, or *Axis* of its Body (which is no other then the axis or Dimetient of its circumſcribing or comprehending Sphear, and which in the Hexahedron cannot immediatly be taken, unleſſe the ſame be concave, and one ſide or baſe open) or elſe by the *Diagony* or Diameter of its Baſe (which is the ſame with the *D*iameter of the Circle circumſcribing the ſaid Baſe, as I ſhewed formerly in the Dimenſion of a tetragonal Pyramid, and before that, of a Tetragon it ſelf,, by its *D*iagony.) And ſo, whereas the *Diagoniall* or *Axis* of the *Hexahedron*, is naturally triple in potencie or poſſibility, to the ſide thereof, by *E*, 13.*p*.15, (as the *D*iagony or *D*iameter of its baſe is naturally double in the ſame manner, to the Side, by *E*, 1.*p*. 47, &c. which I ſhewed before,) here it becoms artificially equal (immediately in act, and ſo in power, quadrate and cubique) to the ſide taken naturally, or by the prime or natural Line of meaſure:

meaſure: And the artificial Line of meaſure for this purpoſe, in reference to ſolid meaſure, I find (according to the reaſon of the 2d. Theoreme, &c.) to be of the prime Rational Line in general, the ſame with that which was formerly found for the ſolid dimenſion of a trigonal Pyramid by it's baſiall *D*iameter or perpendicular; and its Axis together, *viz.* 1.7320, &c. $\sqrt{3}$. By which the *D*iagoniall or Axis of an Hexahedrum being taken, wil be found to agree with the Side thereof taken by the naturall Line of Meaſure, and ſo being cubed, muſt needs produce the ſame Content: Whereby it appears, that if the Axis or *D*iameter of a Sphear be taken by this Line, the Cube thereof ſhal be the inſcribed *Hexahedron*.

The ſolid dimenſion of an *Hexahedron* by its Axis or Diagoniall according to cubatiō: And the artificiall Line for performing the ſame.

II. Then for the ſuperficial dimenſion of this Body by its ſaid *Axis* or *Diagoniall*; I find the artificial Line for that purpoſe, to be the very ſame with that which was formerly found for the ſuperficial dimenſion of a tetragonal Pyramid, by the ſide of its baſe, and the perpendicular of its trigonall plane together, *viz.* 0.7071, $\sqrt{\frac{1}{2}}$: By which the ſaid Axis or *D*iagony being taken; the Square thereof ſhal be the total Superficies of the Hexahedrum: And therefore if the Diameter of a Sphear be taken by this Line, the Quadrat thereof ſhal be the inſcribed *Hexahedron's* ſuperficies.

The ſuperficiall dimenſion of the *Hexahedron* artificially by its Axis or Diagony, according to quadration.

III. As for the ſolid dimenſion of this Body artificially, by the Diagony of the baſe; the Line of meaſure ſerving thereunto, I find to be the very ſame with that which

The solid dimension of the *Hexahedron* by the Diagony or Diameter of its Base, according to Cubature, artificially.

was formerly noted for the dimension of a Tetragon by its *D*iagony, and also for the superficial dimension of a Cone by its whose basial periphery and side together, *viz.* 1.4142, $\sqrt{2}$: By which the basial *D*iagony being taken, it wil be found to agree with the side taken by the prime Rational Line; and so being cubed, must needs produce the same Hexahedral solidity.

IV. Then for the superficial dimension of this Solid, artificially, by its said basial *D*iagony or Diameter; the *L*ine of quadrate dimension for this purpose, I find (according to the reason of the 1 & 2d. Theoremes, &c.) to be of the Rational *L*ine in general, (defectively) 0.57735, &c. which is Apotomally in Number, 1——.42265, &c. the said artificial *L*ine being $\sqrt{\frac{1}{3}}$ of the natural *L*ine, taken in its quadratique power or capacity. By which *L*ine (first duly divided) the basiall or hedral *D*iagony being taken, its Quadrat shal be the total superficies of the *Hexahedron*.

The superficial dimenſiō of the *Hexhaedron* by its basial or hedral Diagony, according to quadrature artificially.

V. And so may the superficies of this Body, be wholly obtained in the like manner, by its Side (which may be termed the most precise and proper squaring of a Cube, as to it's superficiety, or superficial part; as also the two former waies by the *D*iagonial of the Base & of the total Hexahedron it self) and the *L*ine of quadrature convenient for this purpose, I find (by the reason of the foresaid Theoremes) to be of the prime rational *L*ine in general

The superficial dimension of the *Hexahedron* artificially by its Side, according to one exact quadrature.

ral (deficiently) o 4082, &c. the Apotomal ſegment, or parts of diminution, being .5917, &c. the Line it ſelf being $V\frac{1}{2}$ of the foreſaid Rational Line, in its power or capacity tetragonical. By which (firſt duly divided) the Side of the *Hexahedrum* being meaſured; its Quadrat ſhal be the total Hexahedral ſuperficies; Which way, as alſo that by the Diagony of the baſe, as they are more artificial and excellent in themſelves, ſo alſo more ready, for getting the ſuperficies of an Hexahedron, then the vulgar or natural way, by ſquaring the ſide taken by the natural Line of meaſure, and then ſextuplating that ſquare, as being the Baſe. And here it appears, that if the Diameter of a Sphear be taken by this laſt artificial *Line*, the Quadrat thereof ſhal be the total convexe ſuperficies of the circumſcribed *Hexahedron*.

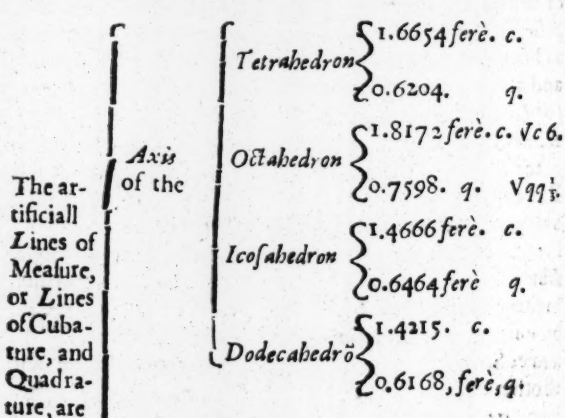
And this ordinate Body here laſt handled, may alſo be conceived or conſidered together with the other 4 beforegoing, under a Pyramidal cõpoſition & reſolutiõ (as al plain Solids generally are; even Pyramids thẽſelves; as I ſhew'd before in the firſt and moſt ſimple kind of all, to wit, the trigonal, and that in the *Tetrahedrum* it ſelf) according to its Baſes or Planes, which are to be underſtood, as the baſes of ſo many Pyramids equal and alike, by which they are only externally eminent or apparent, their whole bodies beſide, being internally latent, and ſo do meet in their vertical angles, in the center of the hexahedral Body; (or of its ambient Sphear,) This Body being compoſed ſuperficially or externally of, or contained internally under, ſix equal *Tetragons*, (according to *E*, 11. *d*. 25) compact together by ſolid (right) angles, which are in number, ſubtriple the plain, ſuperficial, or hedral (right) angles conſtituting or including the ſame, as in the *Tetrahedrum* and *Dodecahedrum*. And hereupon this regular *Solid* wil alſo

admit of a Pyramidal dimenſion, for the obtaining of its ſolid *Area*, like as the other; which wil be moſt eaſily perform'd, ſeing that the *Axis* or Altitude of its compounding tetragonal Pyramid, is equal to half the ſide (or altitude) of its body; and which conjunctly with a trient of the Baſe (or the whole Baſe with a trient of that) wil produce the Content of one of the compoſing Pyramids; whoſe ſextuple wil be the Content of all the Pyramids together, and ſo of the whole *Hexahedrum:* Or (more briefly) the ſaid Pyramidal Axis conjunctly with the whole Baſe, wil produce the Content of three of the ſaid Pyramids, for half the *Hexahedrum:* And therefore hence it is, that the ſide of this Body, (which is its altitude, and ſo double to the altitude of its composing Pyramid) being infolded with the Baſe (that is, cubed) produceth immediatly the Content of all the 6 ſuppoſed Pyramids together, for the total *Hexahedrum*.

And as the *Hexahedrum*; ſo alſo may the other four ordinate plain Bodies be artificially meaſured, both ſolidly and ſuperficially, (or cubically and quadrately) by their other ſeveral lines of dimension, beside their Sides, (as I noted before) as either by their *Axes*, *Diameters*, or *Diagonials*, which in the *Octahedron*, *Icoſahedron*, and *Dodecahedron*, are no other then the Axis or Diameter of their circumſcribing, comprehending or containing Sphear, as in the *Hexahedron*, (as I noted occasionally in them before) or by their *Altitudes*, (which in the *Dodecahedron* and *Icoſahedron*, are the ſame with the Diameter of their inſcribed Sphear; as alſo in the *Hexahedron*, it being there the ſame with the side, and in the *Octahedron*, is the ſame with its Diagoniall, or Axis, being alſo the circumſcribing Sphear's Axis, as I ſhewed formerly.) Or by their Basiall

or

or hedrall Diameters or perpendiculars; and in the *Dodecahedron*, by its basiall or hedral *D*iagony also, according as I shewed in the *D*imension of a pentagonal *Pyramid*, and also of a *Pentagon* it self simply. The artificial *L*ines, (or *L*ines of quadrature and Cubature) for some of which *D*imensions, that are most material, (though indeed none of them are absolutely needful, the *D*imensions of these Bodies being most easily and accurately performed by their Sides, as I have sufficiently shewed before) I shal here deliver, as I find them to be in relation to the prime Rational *L*ine in general, under a decumillenary solution, (according to the reason of the 2d. Theoreme, &c.) as followeth; beginning with solid dimension, as I have done in all the former Solids, as being the most considerable in these and all other solid Figures: the first of the two numbers belonging to each Body, noted with the letter *c.* expressing the quantity of the *L*ine for cubical or solid dimension, and the second number, noted with *q*, the quantity of the *L*ine for quadrate, or superficiary dimension.



By the first Section of wch Lines (being duly set off and divided) the *Axes* of these Bodies being taken, the Cubes and Squares thereof, shal be their several solidities and superficialities, according to the nature of the Line by which they are measured: whereby it appears, that if the *Axis* of a Sphear be taken by these Lines (except those which are for the *Tetrahedrum*) its Cube and Quadrat shal be the

the ſolidity and ſuperficiality of the inſcribed *Octahedrum*, *Icoſahedrum*, and *Dodecahedrum*, according to the reſpective Lines of meaſure, by which it is taken. And ſo the like may be done for the ſolidity and ſuperficiality of the inſcribed *Tetrahedron*, by the Diameter of its ambient Sphear: the Lines of meaſure for which purpoſe, I find to be of the Rational Line in general, 2.4980, for cubique dimenſion, and 0.9306, &c. for quadrate dimenſion.

By the ſecond Section of theſe Lines, the Altitude of an *Icoſahedrum* and *Dodecahedrum* being taken; their Cubes, and Quadrats, ſhal be the ſolid and ſuperficial Contents of their proper Bodies, according to the nature of the Line by which they are meaſured: And therefore, if the Diameter of a Sphear be taken by theſe Lines, the Cube and Quadrat thereof, ſhal be the ſolidity and ſuperficies of the circumſcribed *Icoſahedrum* and *Dodecahedrum*, according to the reſpective Line, by which it is meaſured.

And the like may be done in the *Tetrahedrum* and *Octahedrum* circumſcribed; and in all the five Bodies both inſcribed and circumſcribed, may the ſame be done alſo, by the Circumference of the greateſt or central Circle of the Sphear (circumſcribing and inſcribed) for finding their ſold and ſuperficial Contents: But the Lines of meaſure for theſe laſt-named Dimenſions, as alſo for the cubique and quadrate dimenſion, (or the cubing and ſquaring, as I may ſo term them) of the four Bodies here laſt handled, by their hedral Diameters or Perpendiculars, and hedrall Diagony alſo in the *Dodecahedrum*, (as I have already ſhewed in the *Hexahedrum*) I ſhal here omit, as needleſſe and ſuperfluous; and ſhal ſhew the chief of theſe Dimenſions, as alſo all the other beforegoing in theſe Bodies,

dies, which I have practically demonſtrated by way of example, in an arithmetical manner, from their ſeveral artificiall Lines of meaſure, (or Lines of *Cubature and Quadrature*) expreſſed only by Number; together firſt with the *L*inear dimenſions of theſe Bodies, (in regard of their foreſaid ſeveral lines of *D*imenſion, except thoſe here laſt of all named) leading thereunto; in a way of Proportion, after the moſt exquiſite manner that may be, as from the natural Meaſure, according as I have done in all the precedent dimenſions. But before I go to theſe, I think it very fit and expedient (for a concluſion of this Section) to give the practicall Reader to obſerve by the way, for the more eaſe and conveniency in this kind of Menſuration, or metricall practice, (or more artificial kind of practical Geometry) wch here we handle; & ſo to avoid multiplicity of artificiall metrical Lines or Scales, ariſing by the manifold particular dimenſions here conſidered and declared; what Lines of meaſure here already delivered and expreſſed by Number, do agree, either in the whole throughout, or in part only, in their meaſure or magnitude from the prime Rational Line, ſo farre as is generally needful for ordinary uſe; which is to centeſimal parts only of the Rational *L*ine from which they are taken, and which diviſion in that, for ſetting off the artificial Lines therefrom, and ſo alſo in the artificial *L*ines themſelvs for meaſuring thereby, is generally ſufficient for ordinary uſe, as I have both ſaid, and exemplarily ſhewed in moſt of the precedent *D*imenſions; it having conſiderably failed but in three of all the foregoing practical demonſtrative Examples, which was firſt, in the ſolid Content of the Sphear laſt handled, produced by the Line of Cubature pertaining to the *D*iameter: and ſecondly, in the ſolid content of the *D*odecahedrum

drum in the Pyramidal dimenſion, by the *Line* pertaining to a regular-baſed pentagonal Pyramid in general; where yet it did agree ſufficiently with the true, natural dimenſion, in the ſolidity of the compounding Pyramid it ſelf; and therefore not ſo conſiderable in the *Dodecahedrum*: and then in the ſolid content of the Icoſahedrum produced by the *Line* of Cubature belonging to its ſide: But two or three particular Examples are not to be regarded, in reſpect of the general; ſeing that the like artificial dimenſions of the ſame kind of Figures, may in other Examples hit right enough; the reaſon of theſe defections having been ſhewed ſufficiently at firſt, eſpecially in the laſt Section of the firſt Part. And therefore obſerve,

I. That the artificial *Line*, for the quadrate dimenſion or ſquaring of a *Circle*, by its *Diameter*, (expreſſed firſt of all by Number, from a decumilleſimal ſolution of the prime Rational *Line*, *viz.* 1.1284 *ferè*) and the *Line* for the cubique dimenſion, (or cubing) of a *Dodecahedron*, by its perpendicular-line of altitude (or inſcribed Spheare's dimetient) under the ſame ſolution of the Rational *Line*, *viz.* 1.1296) wil agree in part, *viz.* in a centeſimal ſolution of the Rational *Line*, being thereby 1.13 *ferè*, which I ſhewed in the dimenſion of *Circle*, to be the *Line* C D, according to the primary Line A B.

II. That the Line for the quadrate dimenſion of an ordinate *Pentagon* by its *Diameter* or *Perpendicular*, (expreſſed numerally, 1.1732 *ferè*) and the Line for the cubick dimenſion of an *Icoſahedron* by its perpendicular of altitude (or inſcribed Spheare's *Axis*) noted Numerally, 1.1654) wil agree ſufficiently in part, *viz.* in centeſimal parts of the Rational Line, being thereby, 1.17, the firſt compleat, the ſecond incompleat; which difference is not conſiderable.

III. The Line for the dimenſion of a *Tetragon* by its *Diagony*, (or circumſcribing Circle's Diameter) and for the artificial dimenſion of Triangles in general; and the Line for the ſolid dimenſion of an *Hexahedron* by its baſial or hedral *Diagony*; and alſo the Line for the ſuperficiary Dimenſion of a *Cone* by its Side and total baſial periphery together, do agree in the whole throughout, *viz.* 1.4142, &c. infinitely, being $\sqrt{2}$ of the intire prime Rational Line (as the unit or integer of Meaſure) in it's power quadratically: with which very nearly agrees the Line for the cubing of a *Dodecahedrum* by its Axis or Diagoniall, (or circumſcribing Sphear's *D*imetient) in centeſimal parts of the Rational Line, *viz.* 1.42 compleatly.

IV. The Line for the Quadrate ſuperficiall dimenſion of a Sphear, (or the quadration of the *Spherical)* by its Diameter or Axis; and the Line for the ſquare-like ſuperficiary dimenſion of a *Cylinder* (or the rectangular parallelogrammation of its Superficies) by its Side and Diameter togethet, do meet in the whole throughout, *viz.* 0.56418 &c, infinitely.

V. The Line for the quadrate ſuperficial dimenſion of a *Tetrahedron* (or quadration of its ſuperficies) by its *Side*; and for the like dimenſion of an *Octahedron* by its *Axis* or *Diagoniall* (or comprehending Spheare's Diameter) and for the rectangular ſuperficial dimenſion of a trigonal Pyramid, by the perpendicular of its baſe, and of its other or lateral triangular Plane together, do happen to be all one infinitly, in parts of the Rational Line, *viz.* 0.7598, &c. being $\sqrt{\frac{1}{3}}$ of the intire prime Rational Line, in its biquadratique potentiality, or capacity: with which agreeth in part, *viz.* to centeſimal parts, the Line of quadrature for the Side of a *Pentagon*, (noted formerly by Number,

0.7624

0.7624 *ferè*) *viz.* 0.76 compleat, which in the Dimenſion of a *Pentagon*, was demonſtrated by the Line P Q. in parts of the *Line* A B. So that here one and the ſame Line will ſerve thus farre, for theſe 4 ſeveral dimenſions. And very nearly with this Line, agrees the Line for the cubing of an *Icoſahedrum* by its ſide (noted formerly in parts of the Rational Line, 0.7710, &c.) taken centeſimally, *viz.* 0.77.

VI. The Line for the ſquare-like ſolidation (as I may term it) or rectangle-parallelepipedall dimenſion of a *trigonall Pyramid* by its *Axis* or altitude, and baſial Diameter or perpendicular together; and for the ſolid dimenſion of an *Hexahedron* by its *Axis* or *Diagoniall*, (or ambient Sphear's dimetient) do agree in the whole throughout, *viz.* 1.73205, &c, infinitely, being $\sqrt{3}$, of the integrall Rational *Line*, taken in its power zenzicall or tetragonicall.

VII. The Line for the rectangle-parallelipipedal Soli-dation of a *tetragonall Pyramid* by it's *Axis* or Altitude, and baſiall diagony or diameter tagether; and for the cubique dimenſion (or Cubation) of an *Octahedron* by its *Axis* or *Diagoniall*, (or containing Spheare's axis or diameter) are both one throughout, *viz.* 1.817, &c. infinitely being $\sqrt{6}$ of the prime Rational Line conſidered in its cubical capacity, or comprehenſibility.

VIII. The *Line* for the ſquare-like, or rectangle-ſuperficiary dimenſion of a *tetragonall Pyramid* by the ſide of its Baſe, and the perpendicular of its trigonal Plane together; and for the exact quadrate-ſuperficiary dimenſion of an *Hexahedron*, (or the quadration of its ſuperficies) by its *Axis* or *Diagoniall* (or circumſcribing Spheare's Diameter) do agree in the whole, *viz.* 0.7071, &c. being $\sqrt{\frac{1}{2}}$ in-

infinitely, of the prime Rational Line, in its power or capacity tetragonical.

IX. The *Line* for the rectangle-parallelepipedation, or parallelepipedal consolidation of a *pentagonall Pyramid* (as to an exact quadrate base) by the diagonal line or angular subtense of its Base, together with the Pyramid's *Axis* or altitude (noted formerly by number, 1.6589 *ferè*) and the *Line* for the exact Cubation of a *Tetrahedrum* by its *Axis* (noted in like manner, 1.6654 *ferè*) will sufficiently agree in part; as to centesmes of the Rational *Line*, *viz.* 1.66; this latter compleat, the former incompleat: but with so smal a difference, as that one and the same Line may thus farre serve indifferently for both these dimensions *viz.* 1.66.

X. The *Line* for the rectangle-superficiary dimension of a *pentagonall Pyramid* by the foresaid diagoniall of its Base, and the perpendicular of its trigonal Plane together, (*viz.* 0.8045 *ferè*) and the *Line* for the like dimension of a *Cone*, by the diameter of its base, and its side together, (0.7979 *ferè*) do agree in part, as to centesmes of the Rational *Line*, *viz.* 0.80. the first compleat, the second incompleat.

XI. The *Line* for the exact quadrate superficial dimension of a *Tetrahedrum*, by its *Axis*, (0.6204) and the *Line* for the like dimension of a *Dodecahedrum* by its *Axis* or *Diagoniall* (or ambient Sphear's dimetient) *viz.* 0.6168 *ferè*, will sufficiently accord, as to centesimal parts, *viz.* 0.62; the first compleat, the second incompleat.

XII. Lastly, the *Line* for cubing of a *Dodecahedron* by its side, (0.5072) and the *Line* for the squaring (as I may so term it) of an *Icosahedron*, as to its superficial part, by its perpendicular of altitude (or inscribed Sphears dimetient)

metient) *viz.* 0.5136) wil ſufficiently concurre in part, as to centeſimal parts of the Rational *Line*, *viz.* 0.51; the latter compleat, the former incompleat.

So that here you may ſee, how that of all the ſeveral geometrical dimenſions before particularly expreſſed, being in nũber 62, as requiring ſo many ſeveral lines of Meaſure, or Lines of quadrature, cubature, &c. the *Lines* for 28 of them, are contracted into 12, in reſpect of a centeſimal ſolution of the Rational *Line* from which they are taken, but no further, (according to theſe 12 Notes or obſervations) And the *Lines* for 15 of the ſame dimenſions, are contracted into 6, in regard of an infinite reſolution of the Rational *Line*, according to the 3. 4. 5. 6. 7. 8 Notes. And ſo with the other 34 dimenſions not here named, having their particular Lines of meaſure differing in the whole, (in reſpect of the fraction-part of the Rational Line, though not of the integral part) the artificial Lines for all the 62 dimensions aforeſaid, wil be contrived into 46, according to a centeſimal partition of the prime or natural Rational Line. And the like agreement of Lines as is here demonſtrated, may fall out between theſe and other the like Lines, for other Dimensions not here particularly expreſſed; and alſo between other Lines, which are none of them hereſet down: which I referre to the ingenious practiſer to consider, according as he may have occasion offered.

SECT.

SECT. IV.

Expreſſing the manifold Dimenſions in the five plain ordinate, or regular Bodies, Arithmetically, by way of Proportion, in the moſt exquiſite manner that may be.

AND ſo having in the Section immediatly beforeing, ſhewed the Dimenſion both ſolid and ſuperficial of the 5 plain ordinate (Pythagorean or Platonick) Bodies, according to our artificiall way of meaſuring, in the moſt exquiſite manner that may be: (or inſtrumentall Cubature and Quadrature,) We ſhal in this Section, lay down the ſame Dimenſions, with variety of others, in theſe Bodies, by Number, in the moſt exquiſite Terms of Proportion that may be; ſuch as have not yet been done (no more then the former artificial way of meaſuring the ſame) by any that I could ever meet with, or hear of: and which muſt needs very much exceed thoſe, tedious, obſcure, confuſed, Coſſical Terms which Mr. *Diggs* in his forementioned diſcourſe of theſe Bodies, hath Theorematically delivered; where yet, he hath left out the moſt material and uſefull ones, for the ready and ſpeedy diſcovering of their ſolid and ſuperficiall

ciall Contents, as being inſcribed in, andcircumſcribed to, a Sphear, in relation both to the *Axis* or Diameter, and the greateſt *Periphery* of the Sphear, circumſcribing and inſcribed.

And here I ſhal firſt begin with the *Linear* dimenſions of theſe Bodies, in all the variety thereof, according to the fore-named ſeveral Lines of dimenſion belonging to them, as uſually adſcribed to them for their *Dimenſions*, (as I did in the other ordinate Figures beſore-going, namely, the *Circle*, *Sphear*, *Trigon*, and *Pentagon*) and this, in relation to the firſt *Dimenſion* in Geometry, called in general, from the Greeks, *Euthymetrie*, or *Mecometrie*, and from the Latines *Longimetrie*; And which (in reſpect of the different kinds of Lines) I may call more generally from the Greeks, *Grammemetrie*; and from the Latines, *Linemetrie*.

Therefore,

Therefore,

1 The *Tetrahedron's Side*, is to its *Axis*, as 1. to .816497 *ferè*.

ambient Spheares *Axis*, as 1. to 1.224745 *ferè*.

V^2_3

alike.

alike.

2 The *Tetrahedron's Axis* is to its *Side*, as 1 to 1.224745 *ferè*.

ambient Spheares *Axis*, as 1. to 1.5, *sub-sesquialtera*.

3 The *Axis* of a Sphear, is to the inscribed *Tetrahedron's* *Side*, *Axis*, as 1 to .816497 *ferè*. .666667 *ferè*.

viz. .666666 infinitely, *sesquialtera*.

The

4. The *Side* of the { *Octahedron*, *Hexahedron*, *Icosahedron*, *Dodecahedron.* } is to its *Axis* or *Diagonial*, or circumscribing Sphear's Dimetient, as 1, to { $\sqrt{2}$. 1.414214 *ferè* / 1.73205. $\sqrt{3}$. / 1.902113. / 2.802517. }

Contrarily,

5. The *Axis*, Diagoniall, or angular Diameter (or the ambient Sphear's Dimetient) of the { *Octahedron*, *Hexahedron*, *Icosahedron*, *Dodecahedron.* } is to the *Side*, as 1 to { .707107 *ferè*. $\sqrt{\frac{1}{2}}$. / .57735. $\sqrt{\frac{1}{3}}$. / .525731. / .356822. }

6 The Diameter of a Sphear is to the Side of the circumscribed. { *Tetrahedron*, *Octahedron*, *Hexahedron*, *Icosahedron*, *Dodecahedron.* } as 1. to { 2.44949 *ferè*. / 1.224745 *supdupla*. / 1. *æqualis, seu una*. / 0.66158. / 0.449028 *ferè*. }

Contrariwise.

7. The Side of the	*Tetrahedron*,	is to the inscribed Sphear's Dimetient, as 1. to	.408248.
	Octahedron, *Hexahedron*.		.816497. *dupla*.
	Icosahedron,		1. *æqualis*, as before.
	Dodecahedron.		1.511522.
			2.227033. *ferè*.

And so the two last of these Proportions, are consequently of the Sides of those two Bodies to their Altitudes.

And by the 6th. Section of Proportions, as also by the 5th. Sect. of the like proportions, in relation to the correspondent Circumference of a Sphear, you may observe, how that a *Tetrahedrum* and an *Octahedrum* being circumscribed to one *Sphear*, the side of the *Tetrahedrum* wil be exactly double to the side of the *Octahedrum*: And so by the 7th Sect. beforegoing, you may observe contrarily, how that these two Bodies hauing one and the same side, the *D*iameter of the Sphear inscribed in the *Octahedrum*, wil be exactly double to the *D*iameter of the Sphear inscribed in the *Tetrahedrum*. And the like with these, you may also observe afterwards in the 5 and 6 Sections pertaining to the correspondent Circumference of a Sphear. And the same proportion wil the *Tetrahedrum* here hold to the *Octahedrum*, both for solid and superficiall dimension, as it doth for lateral dimension, as I shal shew afterwards.

8. *Tetrahedron's* Axis is to its inscribed Sphear's Axis, as 1. to .5 *dupla*. And therefore contrarily.

9. The Axis of a Sphear is to the Axis of its circumscribing *Tetrahedron*, as 1. to 2. *subdupla*.

10. The *Axis* of *Tetrahedron's* ambient or externall Sphear,

Sphear, is to the *Axis* of its inſcribed or internal Sphear, as 1, to .3333, &c. infinitely, *viz.* 3. to 1, *tripla*. And therefore converſly,

11. The Axis of *Tetrahedron's* inſcribed Sphear, is to the Axis of it's circumſcribing Sphear, as 1. to 3, *ſubtripla*.

12. The *Axis*, Diagoniall, or angular Diameter of the { *Hexahedron*, *Octahedron*, *Dodecahedron*, *Icoſahedron*. } Or their circumſcribing Sphear's Dimetient, is to their inſcribed Sphear's Dimetient, as 1. to { .57735. $\sqrt{\frac{1}{3}}$. / .794654. }

And ſo conſequently, the latter of theſe two Proportions, is to be underſtood of the *Axes* of thoſe two Bodies, to their *Altitudes*.

Converſly.

13. The Inſcribed Sphear's Dimetient, is to the circumſcribing Sphear's Dimetient, (or the *Axis*, or Diagoniall) of the { *Hexahedron*, *Octahedron*, *Dodecahedron*, *Icoſahedron*. } as 1. to { 1.73205, $\sqrt{3}$. / 1.25840 1 *ferè* }

And ſo the latter of theſe two Proportions, is of the *Altitudes* of thoſe two Bodies to their *Axes*.

By which two Sections of proportions in theſe 4 Bodies, and by the two laſt proportions in the *Tetrahedrum* next before-going, *viz.* Sect. 10 and 11. it appeareth, that theſe five regular Bodies, in reſpect of their Spherical inſcriptibility and circumſcriptibility, do require three ſeveral diſtinct Sphears, circumſcribing or containing, and inſcribed or contained: That is, they being all ſeverally inſcribed within one Sphear, cannot then alſo be exactly circumſcribed about (or cannot compleatly comprehend within them) one Sphear, but three ſeveral Sphears; whereof that which is inſcribed in the *Tetrahedrum* wil be the leaſt, and that inſcribed in the *Hexahedrum* and *Octahedrum*, wil be equal, and that which is inſcribed in the *Dodecahedrum* and *Icoſahedrum* wil be alſo as one, and the biggeſt of all.

And ſo again contrarily, theſe five Bodies being ſeverally circumſcribed about (or comprehending in them) one Spheare, cannot then again be exactly comprehended or contained of one Spheare, but muſt have three ſeverall comprehending, containing or including Sphears; of which, that for the *Tetrahedrum*, wil be the greateſt; that for the *Hexahedrum* and *Octahedrum* wil be alike; and that which is for the *Dodecahedrum* and *Icoſahedrum* wil alſo be of equal magnitude, and indeed the leaſt of all: (*See M.* Diggs *his Diſcourſe upon theſe Bodies*, Probl. 17.) And the like to theſe may be obſerved in theſe 5 bodies, as being inſcribed in, or circumſcribed about, one Sphear, in reſpect of the Circles circumſcribing their Baſes, the Diameters of theſe Circles and of the Bodie's circumſcribing and inſcribed Sphear, being compared together: And therefore

14 The

14. The Diameter of the Sphear circumscribing the { *Tetrahedron*, *Hexahedron*, *Octahedron*, *Dodecahedron*, *Icosahedron*. } is to the Diameter of the Circle circumscribing their Base, as 1. to, { .942809. / .816497 *ferè*. / .607062. }

And so the second and third of these Proportions, are also of the *Axes* or *Diagonies* of these 4 Bodies, to the diameters of their Base's ambient Circles: which in the *Hexahedron*, is of the corporal Diagony, to the basial or superficiall Diagony.

15. The Diameter of the Spheare inscribed in the { *Tetrahedron*, *Hexahedron*, *Octahedron*, *Dodecahedron*, *Icosahedron*. } is to the Diameter of the Circle circumscribing their base, as 1. to { 2.828427. / 1.414214 *ferè*. / 0.763932. }

And to the second of these Proportions, is consequently of the *Hexahedron's Side* to its Basiall *Diagony*.

The

The Converſe of theſe are,

16. The Diameter of the Circle circumſcribing the Baſe of the { *Tetrahedron*, *Hexahedron*, *Octahedron*, *Dodecahedron*, *Icoſahedron*. } is to the Diameter of their circumſcribing Sphear, as 1. to { 1.06066. / 1.224745. / 1.647278. }

And ſo the ſecond and third of theſe Proportions, are alſo of the diameters of the baſiall ambient Circles of theſe four bodies, to their *Axes* or *Diagonies:* which in the *Hexahedron*, is of the baſiall or hedrall diagony, to the totall corporall diagony.

17. The Diameter of the baſial or hedral ambient Circle of the { *Tetrahedron*, *Hexhaedron*, *Octahedron*, *Dodecahedron*, *Icoſahedron*. } is to their inſcribed Sphears Diameter, as 1. to { .353553. / .707107 *ſere* / 1.309016. }

And ſo the ſecond of theſe Proportions, is of the Hexahedron's baſial Diagony or Diameter, to its Side.

And the laſt of theſe Proportions, is of the baſiall ambient Circle's Diameter, to the Altitudes of thoſe two Bodies.

So

So that you may here ſee by theſe 4 Sections of proportions, how that theſe 5 bodies being deſcribed either within or about one Sphear, have only three ſeveral circumſcribing or containing Circles, for their Baſes; whereof, that which is for the *Tetrahedrum* is the largeſt; that for the *Hexahedrum* and *Octahedrum* are both one, and the next to it; and that which is for the *Dodecahedrum* and *Icoſahedrum* are alſo equal, and the leaſt of all; which *Euclid E.* 14. *p.* 5, and 21, ſpeaks of, only in reſpect of theſe bodie's inſcriptibility in one Sphear. As for the baſial or hedral inſcribed Circles of theſe 5 Bodies, whether inſcribed in, or circumſcribed about one Sphear; there is no ſuch parity or agreement amongſt them, but they are all different one from another.

And as for the proportions of the ſides of theſe Bodies to the Diameters of their hedral Circle's whether circumſcribing or inſcribed; and to their hedral Diameters or perpendiculars, &c. the ſame are to be had in the three ordinate Planes before handled, *viz.* the *Trigon*, *Tetragon* & *Pentagon:* thoſe for the *Tetrahedron*, *Octahedron* and *Icoſahedron*, out of the *Trigon*; thoſe for the *Hexahedron*, out of the *Tetragon*; and thoſe for the *Dodecahedron*, out of the *Pentagon*; But indeed, that of the *Hexahedron's* ſide to its hedral ambient Circle's *Dimetient* (or its hedral Diameter) *& è contra*; is alſo noted in the 15. and 17. Sections.

And thus much for the linear proportions, in reſpect of the ſeverall lines of *Dimenſion* in theſe Bodies, being conſidered both ſimply or abſolutely in themſelves, and alſo as being inſcribed and circumſcribed, and that in relation to the *Diameter* of the Sphear circumſcribing and in-

inscribed. And from these we shal proceed to the like proportions, in respect of the Circumference answering to the said Sphear's Diameter, which I have not yet found touched upon by any man, in any kind whatsoever. And therefore.

1. The Circumference of a Sphear's greatest, or Centrall Circle, is to the Side of the inscribed

Tetrahedron,	as 1. to	.259899	*ferè.*
Octahedron,		.225079.	
Hexahedron,		.183776.	
Icosahedron,		.167345.	
Dodecahedron.		.113580.	exactly.

Vice-versâ.

2. The Side of the

Tetrahedron,	is to the circumscribing Sphear's greatest circumference, as 1. to	3.847649.	
Octahedron,		4.442883.	
Hexahedron,		5.441398.	
Icosahedron,		5.975664.	
Dodecahedron		8.804369.	

3. The *Periphery* of a Sphear's greatest Circle, is to its inscribed *Tetrahedrum's Axis*, as 1. to .212206. And so contrarily.

4. The *Axis* of a *Tetrahedrum*, is to its ambient Sphear's greatest or true *Periphery*, as 1. to 4.712389.

5. The

5. The *Periphery* of a Spheares greatest Circle, is to the side of the ambient.

Tetrahedron,	as 1. to	.779697	*subdupla fere.*
Octahedron,		.389848.	*subdupla.*
Hexahedron,		the same, as the Circumference to the Diameter, the Side of this body being equall with the Spheares Diameter.	
Icosahedron,		.210589. *fere.*	I.
Dodecahedron.		.142930.	D.

6. The side of the

Contrariwise,

Tetrahedron,	is to the inscribed Spheares greatest *Periphery*, as 1. to	1.28255.	*duplum.*
Octahedron,		2.56510.	
Hexahedron,		the same as the diam. to the Circumf. for the reason aforesaid	H.
Icosahedron,		4.748589.	
Dodecahedron.		6.996435.	

7. The *Periphery* of a Sphear's largest Circle, is to the *Axis* of its ambient *Tetrahedrum*, as 1. to .63662.

And again converslly,

8. The *Axis* of a *Tetrahedrum* is to its inscribed Sphear's greatest *Periphery*, as 1. to 1.570796.

As for the proportions of the circumscribing Spheare's greatest or Diametral Circumference of these Bodies, to their inscribed Spheare's like Circumference, *& contrà*: And of the greatest Circumference of the Sphear both circumscri-

cumscribing and inscribed, to the Circumference of the basial or hedral circumscribing Circles of these Bodies, *& contrà:* they wil be the same with those which are already expressed between the *Diameters, viz.* first in respect of the Sphear circumscribing and inscribed, between themselves; and then of both these Sphears severally with the Circles circumscribing the bases of these Bodies, as being described either within or without one Sphear (or several Sphears of one magnitude.)

Having thus expressed the Linear proportions in these Bodies, or the proportions of linear dimension, as many as (and indeed many more then) are here absolutely needful for the measuring of them: I shal now come to shew the proportions both superficial or quadrate, and solid or cubique deduced from thence, and both these conjunctly (for brevity sake) in reference to the other two *Dimensions* in Geometrie, called from the Grecians, *Embadometrie* (& *Platometrie*) & *Stereometrie*; and from the Latins, *Planometrie* & *Solido-metrie:* beginning with the latter of these as being most worthy and most considerable in solid Figures, as I have said before, the superficial dimension in them being not so useful or material, and also much more easily obtained; especially in these five Solids; whether the same be done naturally or artificially; And therefore first in respect of these Bodies considered simply and absolutely by themselves, without any inscriptibility and circumscriptibility, either spherical or mutual; the cubical proportions noted by the letter *c*, and the quadrate proportions by the letter *q*, wil be as followeth; the bodies being placed in order according to their magnitudes increasing, both solidly and superficially.

1. The

1. The Lateral Cube and Quadrat of the

Tetrahedron,	is to the Bodies solidity and superficies, as 1. to	·11785113. *c.*	T.
		1·7320508. *q.*	
Octahedron,		·47140452. *c.*	O.
		3·464102 *ferè q.*	
Hexahedron.		1.000000. *c.*	H. *æqualis.*
		6.000000. *q.*	*sub-sextupla.*
Icosahedron,		2·181695. *c.*	I.
		8.660254. *q.*	
Dodecahedron.		7·663119. *c.*	D.
		20·645729. *q.*	

Hence,

2. The Side of the

Tetrahedron,	is to the Side of the Cube and Quadrat equall to the Bodies solidity and superficies, as 1. to	·49028. *c.*	T.
		1·316074. *q.*	
Octahedron,		·778346 *ferè. c.*	O.
		1·86121. *q.*	
Hexahedron,		2.44949. *q.*	H.
Icosahedron,		1·29697. *c.*	I.
		2·94283. *q.*	
Dodecahedron,		1·971523. *c*	D.
		4·543757. *q.*	

And hereby it appears, that these 5 Bodies having all the same side: the *Octahedrum* is quadruple to the *Tetrahedrum* in solidity, and double in superficiality: (& which I have shewed before in their dimensions) The *Hexahedrum* is bigger then both those together, both in solidity and superficiality: the *Icosahedrum* is greater then those 3 together, in solidity, but lesser in the superficies: and is herein quintuple to the *Tetrahedrum*, and so double-sesquialter to the *Octahedrum* (as I have also shewed before) And the *Dodecahedrum* is larger then all the other together, both in solid and superficial dimension.

3. The	*Cube*	of Tetrahedron's Axis is to its	*Solidity*	as 1. to	.21650635.
	Quadrat		*Superficies*		2.59807621

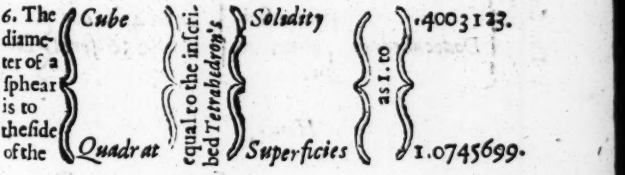
Hence,

4. The Axis of the *Tetrahedron* is to the side of the	*Cube*	equall to its	*Solidity*	as 1. to	.600468.
	Quadrat		*Superficies*		[illegible]611855 *ferè*

Then for the *Tetrahedrum* in reference to a Sphear, by way of inscriptibility therein, as to its Dimetient, it followeth.

5. The

Hence,



Next for the other 4 Bodies, both simply in themselves, and also in relation to their ambient *S*phear together, in respect of their common Axis or *D*imetient, it followeth.

7. The Cube and Quadrat of the *Axis*, or Diagonall of the

Octahedron,	or of the circumscribing Spheares Dimetient, is to the Bodies solidity and superficiality, as 1. to	.166667 *ferè c.* 1.7320508 *q.*	O.	
Hexahedron,		.19245009. *c.* 2.00000. *q. subdupla.*	H.	
Icosahedron.		.3170189. *c.* 2.393635. *q*	I.	
Dodecahedron,		.348145. *c.* 2.628656 *ferè.* *q.*	D.	

Hence,

8. The *Axis*, Diagonall, or angular Diameter of the

Octahedron,	or the circumscribing Spheares Axis or Diameter, is to the side of the Cube and Quadrat equall to the Bodies solidity & superficiality as 1. to	.550321. *c.* 1.316074. *q.*	O.	
Hexahedron,		.57735. *c.* the same as of the Spheares Diam. to the inscribed *Hexahedrons* side, and so of its own Axis or corporal Diagony, to its Side, noted before. 1.41421. *q.*	H.	
Icosahedron.		.68186. *c.* 1.54714 *q.*	I.	
Dodecahedron.		.703483 *ferè c.* 1.621313. *q.*	D.	

By

By which proportions, (as also by the 1. 2. and 5 Sections following of the like proportions, in relation to the greatest Periphery of a Sphear) it appears, that these five Bodies being all described within one & the same Sphear, do retain the same order and rank among themselves, in respect of their magnitudes or dimensions both solid and superficiall, as they do, being of one common, lateral dimension, but not in that proportion and difference of dimension either solid or superficiall; for that here the lateral *Dimension* being different in them all, and that by way of diminution from the least body to the greatest, the superficiall and solid dimension become thereby different in them all in like order, by way of augmentation; but nothing so much, as being all under the same lateral dimension. And hereby it also plainly appears,

I. That the *Hexahedrum* is triple the *Tetrahedrum* inscribed in the same Sphear, and so *E* 14. *p.* 32. And,

II. That the *Hexahedrum* is to the *Octahedrum* within the same Sphear, solidly, as it is superficially, according to *E* 14. *p.* 27; and also as the *Side* of the *Hexahedrum* is to the *Radius* of the Sphear by the same *Prop.* And.

III. That the *Octahedrum* is superficially sesquialter the *Tetrahedrum*, according to *E* 14. *p.* 14. and so (by the same *Prop.*) the *Tetrahedrum* is Basially or Hedrally, sesquitertian the *Octahedrum*. And,

IV. By the last named Sections of solid and superficial Proportions, together with the 3. and 5. Sections of Linear Proportions beforegoing, between the Diameter of a Sphear, and the sides of the 5 Bodies inscribed thererein; and also the first Section of Proportions, between the greatest Periphery of a Sphear, and the sides of the Bodies inscribed:

scribed; it appeareh, how that the *Octahedrum* is to the Triple of the *Tetrahedrum* inscribed in the same Sphear, as its side is to the side of the *Tetrahedrum*; and so *E*, 14. *p*. 22. And,

V. That the *Dodecahedrum* is to the *Icosahedrum* in the same Sphear, both solidly and superficially, as the *Hexahedrum* is to the *Icosahedrum* laterally, according to *E*, 14. *p*. 9 and 11.

Then for these Bodies in relation to a *S*phear, in respect of their circumscriptibility about the same, as to its Dimetient; and so for the *Hexahedrum*, *Dodecahedrum*, and *Icosahedrum*, simply and absolutely in themselves also, in respect of their Dimetients of altitude, being all one with their inscribed *S*phear's *D*imetient: their proportions both cubatory and quadratory, wil be as followeth: the Bodies being here placed in order according to their magnitudes both solid and superficial decreasing, which is according to their due hedral order, in respect of their denominations from the numbers of their bases or hedral Plates. And therefore,

9. The

9. The *Diametrall* Cube and Quadrat of a *Sphear*, is to the solidity & superficies of its circumscribing

Tetrahedron,	as 1. to	1.7320508. *c.* 10.3923048. *q.*	T
Hexahedron.		the same as its Lateral Cube & Quadrat.	H
Octahedron,		0.8660254. *c.* 5.1961524. *q.*	O
Dodecahedron.		0.693786. *c.* 4.162719. *q.*	D
Icosahedron,		0.631757 *c.* 3.79054 *q.*	I

And so the 4 last of these Proportions, are consequently, of the Cubes and Quadrats of the *Altitudes* of those two Bodies, to the solid and superficiall capacities of the Bodies themselves.

10. The *Dimetient* of a *Sphear*, is to the side of the Cube and Quadrat equall to the solidity and superficies of the ambient.

Tetrahedron,	as 1. to	1.200937. *c.* / 3.223710. *q.*	T
Hexahedron,		the same as in *Sect.* 2. the side of this Body agreeing with the inscribed Sphears dimetient.	H
Octahedron,		0.953193. *c.* / 2.279507. *q.*	O
Dodecahedron,		0.885269. *c.* / 2.040274. *q.*	D
Icosahedron,		0.858058. *c.* / 1.946932. *q.*	I

And so the 4 last of these proportions, are of the Altitudes of those two Bodies, to the sides of the Cubes and Quadrats, equal to their solidities and superficieties.

Whereby, as also by the 3, and 7 Sections of Proportions following, it plainly appears, how that these 5 Bodies being all described about one Sphear, the *Tetrahedrum* is the biggest of all, both in solid and superficiall dimension, and is exactly double to the *Octahedrum* in both these dimensions (as it was shewed before to be in its lateral dimension) and very near as bigg as the *Hexahedrum* and *Octahedrum* both together, in respect of both di-

dimenſions: And the *Icoſahedrum* is the leaſt of all in both theſe dimenſions; which is almoſt contrary to the former courſe holden in theſe Bodies, being deſcribed within one Sphear.

Now for the like Proportions, in relation to the Circumference of a Sphear's greateſt Circle; and firſt, in reſpect of theſe Bodies ſpherall inſcriptibility; it followeth,

1. The Cube and Quadrat of the greateſt or centrall Circumference of a Sphear, is to the ſolid & ſuperficiall capacity of the inſcribed

Tetrahedron.	as 1. to	.0020689369. *c.* .1169956. *q.*	T
Octahedron,		.0053752557. *c.* .1754934. *q.*	O
Hexahedron,		.0062068169. *c.* .202642367. *q.*	H
Icoſahedron.		.01022434. *c.* .2425259. *q.*	I
Dodecahedron,		.0112212. *c.* .266338. *q.*	D

There-

Therefore.

2. The circumference of a Sphear's greatest Circle, is to the side of the Cube and Quadrat equall to the solidand superficiall content of the inscribed

Tetrahedron.	as 1. 00	.127423.	*c.*	T
		.342046.	*q.*	
Octahedron,		.175173.	*ferè. c.*	O
		.418919.	*q.*	
Hexahedron,		.183776.	*c.* (*before for the side of the Hexahedron inscribed.*)	
		.450158.	*q.* (*before for the side of the Hexahedron inscribed.*)	
Icosahedron.		.2170427.	*c.*	I
		.492469.	*q.*	
Dodecahedron.		.223925.	*c.*	D
		.51608	*ferè q.*	

Then for the proportions cubatorie and quadratarie, in relation to a Sphear's greatest, Diametral, or true Peririphery, in respect of these Bodies Spherall circumscriptibility, it followeth,

3. The

3. The greatest Peripheriall Cube and Quadrat of a Sphear, is to the solidity & superficiality of its compre-pending, containing, or including

Tetrahedron,	as 1. to	.055861296.*c.* 1.0529606.*q.*	*T*
Hexahedron,		.032251534.*c.* .6079271. *q.*	H
Octahedron,		.027930648.*c.* .526480314.*q.*	O
Dodecahedron.		.02237567. *c.* .4217715. *q.*	D
Icosahedron,		.02037514. *c.* .3840622. *q.*	I

Where-

Whereupon.

4. The greatest, or Diametrall Periphery of a Sphear, is to the Side of the Cube and Quadrat equall to the solidity & superficiality of the circumscribing or cõprehending

Tetrahedron,		.3812701. *c.* 1.0261387. *q.*	T
Hexahedron	as 1. to	.318309. *c.* the same as of the circumf. to the Diam. the side of this *Body* agreeing with the Sphears diameter. .7796968. *q.*--	H
Octahedron.		.30340798. *c.* .7255896. *q.*	O
Dodecahedrõ		.2817898. *c.* .649439. *q.*	D
Icosahedron.		.2731284. *c.* .619727. *q.*	I

Lastly, for these Bodies, and a Globe or Sphear compared wholly together, both solidly and superficially; and that according, both to their inscription and circumscription, the Proportions wilbe as followeth, (the first or uppermost of the two Numbers belonging to each Body in the two first Sections, being for solid comparison, and the second for superficiall.)

5. A Globe or Spheat, is to the inscribed

Tetrahedron.	Solidly and superficially together, as 1. to	·12251753 } T ·3675526 *fere.* }
Octahedron		·318309886. agreeing with divers of the former proportions. ·551328895.
Hexahedron		·3675526 *fere.* agreeing with the superficiall comparison in the *Tetrahedron.* ·63661977.
Icosahedron		·6054614. } I ·76191789. }
Dodecahedrõ		·6649087. } D ·8367272. }

Con-

Contrariwise.

6. The

Tetrahedron.	Is to the circumscribing Globe or Sphear, solidly and superficially together, as 1, to	8.162097 2.720699	T
Octahedron.		3.14159265, agreeing with many of the foregoing proportions. 1.81379936	O
Hexahedron		2.720699. agreeing with the superficiall comparison in the *Tetrahedron* 1.5707963	H
Icosahedron.		1.6516327 *ferè.* 1.312477	I
Dodecahedrõ		1.503966. 1.195133.	D

7. The

Par

7. A Globe or Sphear, is to its ambient

7. A Globe or Sphear, is to its ambient

Tetrahedron	both solidly and superficially together (in one and the same reason) as 1. to	3.30797337.
Hexahedron		1.9098593. the same as of a Sphear to the Cube of its Axis, noted formerly; the side of this Body agreeing with the Axis of the Sphear. H
Octahedron		1.65398668. *subduplum Tetrahedri.*
Dodecahedrō		1.323034.
Icosahedron		1.206567.

Conversly,

8. The	*Tetrahedron*	is to the inscribed Globe or Sphear, both solidly and superficially together in one, as 1. to	·30229989	
	Hexahedron		·52359877, the same as of the Cube of a *Sphear's* Axis, to the Sphear it selfe; the Cube of the Axis being here the ambient *Hexahedrum*.	H
	Octahedron		·604599788. double to the Sphear inscribed in the *Tetrahedrum*.	
	Dodecahedrõ		·7546973.	
	Icosahedron		·8287974.	

By the two last of which *Sections* of solid and superficiall Proportions or comparisons between the 5 ordinate Bodies and a *Globe* or *Sphear*, it appeareth; that any of the said Bodies being circumscribed about a *Sphear*, the solid and superficiall capacity thereof, will be one and the same numerally, (or in the number of a given Measure) *viz.* where the said two severall Dimensions of the inscribed *Sphear* are alike in number: That is, two like Bodies exactly

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exactly encompassing or environing two severall, distinct *S*phear's, whereof the solidity of the one, & the superficiety of the other, are numerally alike; there wil the solid capacity of one of the like ambient Bodies, and the superficiall capacity of the other, be also alike in the number of measure. And so likewise, two distinct *S*phears being inscribed in two several like Bodies, whereof the solid measure of the one, and the superficial measure of the other, are numerally the same; there wil the solidity of one of the inscribed *S*phears, and the superficiety of the other, be also numerally the same: and which I have not found to be observed by any before.

Many more Proportions might here have been raised, if they were needfull; as namely of the Bodies among themselves in respect of their mutuall inscription and circumscription; and those also which are the converse of many of the former, to wit, the Proportions of these Bodies solidly and superficially, to the Cubes and Quadrats of their *S*ides, and of their Axes and Altitudes; and so of the *D*iameter and greatest Circumference of their circumscribing and inscribed *S*phears, whereby the sides, Axes and Altitudes of these Bodies, and so the Axes or *D*imetients, and greatest Peripheries, of their circumscribing and inscribed *S*phears, might be obtained, by having the solidities and superficialities of the Bodies only; and that after one Radical extraction, quadrate or cubiq; according as I delivered in the *Circle* and *Sphear*, for the obtaining of their *D*iameters and Peripheries, by their superficial and solid Contents; and so in the ordinate *Trigon* for the finding of its side, and *D*iametral or perpendicular line; and in the ordinate *Pentagon*, for its side and *D*iametrall or perpendicular, and *D*iagonal-line, by their areall

or superficial Contents: But that these kind of proportions are not so usefull, being indeed more of curiosity then of use, sith the superficial and solid Contents of Figures are more usually inquired out by their sides, *Diameters*, and other their Lines of *Dimension*, then these lines are by their superficiall and solid Contents; for that the thing chiefly aymed at in the dimension of all Figures, is their superficiall and solid Contents (and in solid Figures, chiefly the solid Content, as I said before) which must be obtained by their foresaid lines of *Dimension*.

And thus having handled the five famous ordinate (*Pythagorean* or *Platonick*) Bodies, or the angular, or rectilineal regular Solids both geometrically, in an instrumentall manner (according to our artificial way of measuring) and also mechanically, in the most exquisite proportional manner that may be, and that chiefly in reference to their solid and and superficiall dimensions: I shal next come to the second theorematicall Proposition before-mentioned, in which, our more particular or special reason of our artificial instrumentary dimensions (or mechanicall Cubature and Quadrature) of these Bodies, like as first of a *Sphear*, (as to the obtaining of the severall artificiall *Lines* of measure for performing the same) is contained.

THE.

THEOR. II.

Explicating particularly the foregoing artificiall Lines for the cubick dimenſion, (or Cubing) of a Sphear; from our particular or ſpeciall Reaſon of Dimenſion: And conſequently, the Lines for the like dimenſion of all the other regular Solids.

IF the *Axis* or *Dimetient* of a *Sphear*, equal to the *Cube* of the Parts of the Rational Line, be had; The ſame ſhal be the correſpondent *Line* of Cubature, according to the ſaid Parts. And the reaſon of that to the congrual Cubatorie Line, in reference to the whole intire Meaſure, wil be as the reaſon of the Parts to the Whole; which is as the reaſon of their Cubes. And the like for the correſpondent *Periphery*, or the Circumference of the greateſt Circle.

AND the like in both theſe, for the ſuperficiary (or quadrate) dimenſion of a *Sphear* (or quadration of the *Spherical*) only reſpect being here had to the quadrate parts of the Rational *Line*, as there to the cubique: and ſo indeed this may properly enough be referred to the firſt Theoreme, the reaſon here, being the ſame with that.

And the ſame reaſon holdeth in all the five fore-named ordinate plain or angular Bodies, both for ſolid or cubique, and for ſuperficiary or quadrate dimenſion, whether ſimply in themſelves, or in relation to a Sphear, by way of inſcription and circumſcription, and that by any of their dimenſional lines formerly named, as their ſides, and Axes or *Diagonials*, or angular *Diameters* and *Dimetients* of al-

altitude: And ſo by their circumſcribing and inſcribed *Sphear's Dimetient*, and alſo greateſt Circumference, and other lines of dimenſion, according to the ſeveral *Dimenſions* and dimenſional Proportions beforegoing: And alſo in reſpect of their relations one to another, by way of mutual inſcriptibility and circumſcriptibility.

SECT. V.

Shewing the Dimenſion of all exactly ordinate, or regular ſolid Bodies, artificially, for Gravity or Weight, as is for ſolid Meaſure: And demonſtrating the ſame particularly, in the firſt ordinate Solid here handled, namely a Sphear.

AND the like reaſon of *Dimenſion* to that before-going, wil hold in a *Sphear*, and the five plain regular (Platonick) Bodies, for gravity, or Quantity ponderall (according to any Metal & Weight aſſigned) as for ſolid magnitude, or Quantity dimenſional; there being generally the ſame mathematicall reaſon of theſe two *Quantities*, in ſo much, as that they are uſually by Mathematicians compared together in ſeveral kinds of Bodies: Or divers kinds of bodies are compared together among themſelves in this twofold reaſon of Quantity; to wit, *Magnitude* or dimenſion, and *Gravity*, or Ponderoſity

roſity, as you may ſee in *Archimedes*, *Ghetaldus*, and others: And ſo theſe two do proportionally anſwer each other, inſo much, as that one may be deduced from the other; as Gravity from Magnitude, and Magnitude from Gravity; or Weight from ſolid Meaſure, and *contrà*: And therefore the gravity or ponderoſity of each one of the foreſaid regular ſolid bodies taken in ſome certain magnitude or bigneſſe, being firſt known according to ſome certain Metall, Weight, and Meaſure appointed; there may be artificiall *Lines* of meaſure extracted for every ſeverall kind of Body, according to the ſaid particular Metall, Weight and Meaſure, (and that according to the foreſaid ſeverall *Lines* of Dimenſion in thoſe Bodies, by which they have been ſhewed to be artificially meaſured) ſo as that any one of the ſaid dimenſional lines of each Body in any magnitude whatſoever, being meaſured by its proper artificial *Line* or Scale for this purpoſe, & cubed, the ſame ſhal be the weight of the metalline Body propoſed: And which wil thereupon hold in a Sphear, and all the 5 plain regular Bodies, not only as conſidered ſimply and abſolutely in themſelves alone, but alſo as in relation one to another, by way of inſcription and circumſcription; and ſo in the ſaid 5 bodies, not only in the ſaid relations to a Sphear, as being inſcribed in, or circumſcribed to the ſame, but alſo mutually one to another, as was ſaid before for ſolid *(and ſuperficiall)* meaſure: And ſo the Diameter or Circumference of a Sphear, being taken by their proper, reſpective artificial Lines of Meaſure for this purpoſe (according to any certain Body, Metall, Weight, and Meaſore aſſigned) the ſeveral Cubes thereof, ſhal be the weight of the reſpective Body inſcribed, or circumſcribed, according to which is propoſed; And the firſt of theſe, is the

ſame

ſame (in reſpect of a Sphear's *D*iameter) with the artificiall dimenſion of the four greateſt of the ſaid 5 regular plain bodies, by their *Axes*, *Diagonies* or angular Dimetients, (theſe being all one with their circumſcribing Sphear's Dimetient, as I have ſhewed before) And the latter agrees with the like dimenſion of the three laſt of thoſe bodies, by their *Altitudes*, (being the ſame with their inſcribed Sphear's Dimetient; and is alſo in the *Hexahedron*, the ſame with its *Side*, as I have likewiſe ſhewed before) And ſo the reaſon of the artificial *L*ines for this cubical dimenſion of the foreſaid Bodies, for weight, as for ſolid meaſure, may be partly referred to the foregoing 2d. Theoreme; the difference being, that reſpect muſt be here had to the (compounding, denominate) parts ſimply, of the *Weight* propoſed, in ſuch manner, as is there to the cubical parts of the *Meaſure* propoſed; So that theſe artificial *L*ines may be eaſily produced therefrom; And therefore I ſhall not need here to raiſe a particular Theoreme upon the ſame. All which might be performed alſo Arithmetically by way of Proportion, from the natural Meaſure appointed, according as all the former Dimenſions: And both theſe waies I ſhal here particularly demonſtrate in the firſt regular *Solid* beforegoing, to wit, a *Sphear*, or *Globe*, ſimply in it ſelf, as comming moſt in uſe; and that in the moſt uſual and uſefull Metall for this purpoſe. But firſt I wil give the Proportions or compariſons of all the uſuall or principall (or commonly received) kinds of Metals, according to the experiments and obſervations of *Marinus Ghetaldus*, in his *Archimedes promotus* (who is generally ſuppoſed to have come the neareſt to the truth herein, of any man that hath ever yet written hereof) according as they are there delivered by him in the ſecond Table of that Tractate, next after

Theo-

after *Theor.* 9 or *prop.* 17: which Mr. *Gunter* in the 5th. chap. of the 3d. Book of his *Sector*, hath expreſſed in the ſame terms, but more decimally, and that in whole numbers, by changing the natural or vulgar fractions of thoſe numbers into decimal, and ſo expreſſing his natural mixt or heterogeneall numbers, in whole numbers, after a decimall manner, putting the firſt number 10000, whereas *Ghetaldus* puts it but 100; which I have here collected orderly into this Table following, in proportion direct and reciprocall, in reſpect of the equal Magnitudes and gravities of like Bodies of different Metals.

In like Bodies of ſeverall metals and equall magnitude, having the weight of the one, to find the weight of the reſt.					In like Bodies of ſeverall metals and equall weight, having the magnitude of the one, to find the magnitud of the reſt. The converſe of the former.
	Gold.	10000	3895	☉	
	Quick-ſilver.	7143	5453	☿	
	Lead.	6053	6435	♄	
	Silver.	5439	7161	☾	
	Braſſe.	4737	8222	♀	
	Iron.	4210	9250	♂	
	Tinne.	3895	10000	♃	

Which metalline Proportions or compariſons, Mr. *Oughtred* in his foreſaid Book of the *Circles of Proportion*, *Part.* 1. chap. 10. hath expreſſed in the Terms following, being deduced from *Ghetaldus* his firſt Table of Compariſons (if I much miſtake not) which is from *Prob.* 5, and 6, or *Prop.* 12, and 13, of that Treatiſe.

Gold ——	3990	*Braſſe* ——	1890
Quickſilver	2850	*Iron* ——	1680
Lead ——	2415	*Tinne* ——	1554
Silver ——	2030		

Here, note that the number for *Silver* should not be 2030 but 2170; which might be so set down by the negligence or oversight of the Printer, though I find it not noted by Mr. *Oughtred's* Translatour, who set forth his Book in English, among the typographicall errours collected by him in the end of that Book; but else all the other Numbers do correspond with the Numbers of the former Table.

And for a further explanation and use of both these proportional Tables, I referr the Reader to the forenamed several Authors, in the places forecited (though I shall partly shew it afterwards my self) And now of these Metals, seing that *Iron* is most used in the Body wch here we intend chiefly to treat of and handle, namely a *Globe* or *Sphear*, and that chiefly in reference to Bullet, especially the greater, or Cannon-bullet (for the use of Gunners) which is commonly made of the foresaid metall, (and which now a daies is too much used among us) We wil therefore here shew its cubicall dimension in the said Metall, according to its proper kind of *Weight* with us, wch is *Avoirdupois*-weight, beginning first with the *S*phear's *D*iameter. And the Line of equall parts for this purpose (according to the common esteem of the gravity or weight of an iron-Sphear or Bullet with us, at a certain magnitude, in a certain Measure given, expressed by & by) I find by the reason of the second Theoreme beforegoing, in respect of the proper composing, denominate parts of the integer of the weight proposed, (or according to our general reason of Measure, in respect of the Integer of the weight it self; the same reasons holding here for weight, that were formerly expressed for Measure, according to what I said before) to be of the common denominate parts of a Foot with us, (in a centesimal solution) 1.92. Now because we cannot distinctly divide the *L*ine of measure first given for this purpose, *viz.*

viz. an *Inch* or *Pollicar*, (as the prime Rational Line) into 100 parts, (as is requiſite to do) according to the ſimple, vulgar, or natural diviſion of a right Line: Therefore we wil here divide it by the artificial way, commonly called the *Diagonal* or decimal way; according to the rectangle *Parallelogram*, A B C D, whoſe length A C or *BD* being the *Inch* divided into 10 equal parts, and ſo its breadth A B or C D (taken at pleaſure) divided in like manner: the whole *Inch* wil be divided equally into 100 parts, according to the ſmal quadrangles or parallelogrãs, made by the ſeveral lines of diviſion, drawn direct according to the length of the Parallelogram, and oblique or diagonal, according to the breadth; and theſe latter lines may alſo be drawn exactly rectangular or tranverſe to the former lines. And to this Parallelogram is annexed another, *viz.* C D E F, whoſe length C E or D F, is A C, or B *D*, 1.92, for the Line of ponderal Cubation, (as I may term it) or of cubical gravity, proper and peculiar to the *D*iameter of a Sphear or Bullet of Iron, in the foreſaid kind of weight, according to the common Tenent here

following: which being likewiſe divided as the prime Line, according to the rectangle Parallelogram C D EF, and ſo the Diameter of an iron Sphear or Bullet be taken thereby; the Cube thereof ſhal be the weight of the bullet in its foreſaid weight *Avoirdupois*, according to the *Integer* of weight, *viz.* the *Pound*. As for example: An iron-bullet of 4 inches the *D*iameter, is commonly ſaid with us, to weigh 9^{li}. *avoirdupois*. Now for a trial of this by our Cubatorie Line of weight, or *L*ine of cubical gravity; I meaſure the diameter of this Bullet or *S*phear (being of the *L*ine A C, 4.00) by the *L*ine C E, or the diagonal Scale C *D* E F and find it to be thereof, 2.08, for the artificiall Diameter, whoſe Cube is 8.998912^{li}. for the weight of the bullet, which reduced into the proper parts of the fore-ſaid kind of weight, the ſame will be 8 pounds, 15 ounces, 15 drams, and very near $\frac{1}{4}$ of a dram, ſo that it wants of 9li. but about $\frac{1}{4}$ of a dram, which in ſuch a thing is as nothing. But becauſe the diameter of ſuch a Sphear, cannot be firſt meaſured, but muſt be had by means of the Circumference, firſt obtained by a Line of meaſure, (unles there be a pair of Callaper Compaſſes ready at hand) whereupon a Line for cubing the Sphear by its Circumference, muſt needs be generally more convenient for uſe, then that for the Diameter, according as I noted formerly in the generall dimenſion of a Sphear: Therefore, I will here alſo deliver a Line of weight for cubing an iron-Sphear by the Circumference, which (by the reaſons afore ſaid) I find to be of the *L*ine A C, 6.04, (*viz.* 6.04 inches) which being divided either diagonally into 100 parts, as the Line A C, or C E, or rather only according to the length thereof ſimply, ſeing it may ſufficiently bear it: and ſo the Circumference of an Iron-*S*phear or Bullet be

be taken thereby; the Cube thereof ſhal be the weight of the Bullet.

As ſuppoſe here again the former bullet of 9li. *avoirdupois*, whoſe diameter being put 4.00 inches, the Circumference wil be 12.57 inches *ferè*, which by its proper Line of Cubature, wil be found 2.08, as the *Diameter* before by its proper *Line*, and therefore by Cubation, muſt needs produce the ſame ponderoſity as before.

But here note, that an Iron-Sphear of 4 inches the diametral magnitude in *Ghetaldus* his meaſure, wil weigh with him, 12li. 2. ounc. 1 ſcrup. 14 gr. and $\frac{2}{37}$ of a grain; for ſo I find it to be by the former proportional numbers, by comparing this Sphear with a Sphear of Tinne of the ſame diametral magnitude, according as *Ghetaldus* ſheweth in his foreſaid book, and from him Mr. *Oughtred* in his fore-named book: and which is alſo expreſly noted by *Ghetaldus* in a Table, wherein he hath ſet down the weights of a *Sphear*, in all the foreſaid Metals, from $\frac{1}{4}$ of an inch the diametral magnitude, to 12 inches, or the whole Foot, proceeding all along by quarters of inches. But now 9li. *avoirdupois* is in our Troy-weight, by *Aſſize* or *Goldſmiths* weight (according to the commonly received proportion of the *Pound-avoirdupois* to the *Pound-troy*, 60 to 73) but 10$\frac{57}{60}$ pounds, or 10.95li. exactly; which is 10 pounds, 11$\frac{2}{5}$ or 11.4 ounces; or according to the common diviſion of the ounce-troy by penny-weights, is 10li. 11 oun. and 8 penny-weights exactly.

Now *Ghetaldus* divides a Pound into 12 ounces; an ounce into 24 ſcruples; and a ſcruple into 24 grains: ſo that his ounce weigheth 576 grains, and his pound, 6912 grains: And we divide our Pound-troy, into 12 ounces (as he doth his pound) but the ounce-troy we uſually di-

vide into 20 penny-weights, and a penny-weight into 24 grains, ſo that our ounce-troy weigheth but 480 grains, and conſequently our Pound-troy 5760 gr. All which ſheweth, that either his *Weight*, or *Meaſure*, or rather both of them, do differ from ours, as I ſhal further ſhew: As for the *Meaſure* which hee uſeth, he ſaith it to be the ancient *Romane Foot*, divided into 12 *unciæ*, or inches as ours is, and which by his deſcription and delineation thereof, in his forementioned book, ſeemeth to differ but very little from our Engliſh Foot, (if any thing at all) and that deficiently, and which Mr *Oughtred* in his forenamed book alſo obſerveth: But Mr. *John Greaves* ſometimes Profeſſour of Geometry in *Greſham Colledg London*, and now Profeſſour of Aſtronomy in the Univerſity of *Oxford*, hath in his Diſcourſe of the *Roman Foot*, &c. (publiſhed by him in Engliſh, *Anno* 1647; and deduced, not only from divers Authours, but alſo from his own obſervations and experiments, which in that his learned diſcourſe he ſeemeth to have made with great pains and induſtry, in his travails in forraign parts, but eſpecially in *Italy*, and there at *Rome*) more clearly expreſſed the Dimenſion or magnitude of the true Romane Foot; having done the ſame not onely linearly (by the half thereof, as *Ghetaldus* hath done) but alſo Numerally, in comparing it with the ſtandard meaſures of *England*, and divers other Nations: For the draughts or delineations thereof in Books, cannot give us the true length, in reſpect of the divers accidents happening to the paper whereon it is imprinted, and eſpecially the contraction or ſhrinking of it after the impreſſion, (as both theſe authours do give us to obſerve) which while it was moiſt in the Preſſe

Concerning the magnitude of the Romane Foot.

Preſſe, received the true Meaſure, but afterwards being dried, loſeth ſomewhat thereof; and ſo *Ghetaldus* in the laſt page of his foreſaid book, giveth us to note particularly in his draught there of half the Roman Foot, and how much is there to be added to it, to make up the true meaſure. For although Mr. *Greaves* doth conclude upon the ſame Meaſure of the Roman Foot, which (he ſaith) *Ghetaldus* doth, (for the trueſt Meaſure) yet if we compare their draughts or delineations of the half Foot, together, (in their foreſaid books) we ſhal find that of Mr. *Greaves*, to be ſhorter then that of *Ghetaldus*, by almoſt $\frac{1}{3}$ of *Ghetaldus* his Inch, as it is there ſet out; or $\frac{1}{10}$ of our Engliſh inch exactly. And ſuch like diſagreement is to be found among other Authours in their delineations or draughts of the ſame Roman Foot, for the reaſon aforeſaid.

Now the *Foot* which the ſaid Mr. *Greaves* (among ſuch a diverſity of opinions concerning the true Roman Foote, as are to be found, and ſo many Feet as are taken to be Romane) pitcheth upon for the moſt genuine and true Roman Foot, (being led not only by the authorities of *Ghetaldus*, and divers other learned and judicious men, (as he ſaith) but alſo by his own obſervations and experience) is that which is commonly called by writers, *Pes Colotianus*, from the place where it is (or ſometime was) to be found, namely, *in hortis Colotianis* in *Rome*, upon the Monument of *Coſſutius* (which now he ſaith to be remov'd thence) which *Foot* he comparing with our Engliſh Foot, which he took from the iron-Yard, or Standard of 3 Feet, at the *Guild-hall* in *London* (for there is no ſingle Foot-ſtandard) findeth to be 967 ſuch parts, as the Engliſh Foot contains 1000; and ſo the Engliſh Foot to contain 1034.13, ſuch as the ſaid

The Roman and Engliſh Foot compared together.

ſaid Roman Foot contains 1000: whereby this Romane Foot ſhould be 11.6 (or 11.604 exactly) ſuch parts as our Engliſh Foot is 12; *viz.* 11.6 (or 11.604) inches, and ſo wanting of the Engliſh Foot, only 0.4 inch, or 0.396 inch exactly. But there are two other Romane Feet reckoned by him, which (by his account) do come neater to our Engliſh Foot; the firſt whereof is that on the Monument of *Statilius*, *in hortis Vaticanis* in *Rome*, which he obſerved to be 972 ſuch parts as the Engliſh Foot is 1000, (and to be 1005.17 of the *Pes Colotianus*, being 1000) whereby this Foot wil be 11.7 *ferè*, of the Engliſh Foot, being 12, *viz.* 11.7 inches *ferè*, it being 11.66; which wanteth of the Engliſh Foot, but, 0.34 inch. And the other Foot is that of *Villalpandus*, deduced from the *Congius* of *Veſpaſian* in *Rome*, which he ſaith to be 986 parts of our Engliſh Foot containining 1000, (and 1019.65, of *Pes Colotianus*, being 1000) and ſo is 11.8 of the Engliſh Foot being 12, *viz.* 11.8 inches, which wants of the whole Engliſh Foot, only 0.2 inch. But the ancient Greek Foot doth by his obſervation more nearly agree with our Engliſh Foot, then this laſt Roman Foot, being (by his collation) 1007.29, of the Engliſh Foot 1000, which is hardly .09 of an inch above a Foot Engliſh, it being 12.087 inches engliſh.

As for the *Weights* uſed by *Ghetaldus* in his forenamed Treatiſe, which he ſaith to be the Weights uſed in his time (which is not very long ſince) they are ſurely the Roman weights (for he lived at *Rome* as I take it, when he wrote that book) which have continued the ſame for many ages without alteration, as ſome writers do atteſt: And which Mr. *Greaves* in his *Diſcourſe of the Denarius* (annexed to that of the *Roman Foot*) which he puts as an undeniable principle

principle and foundation from whence the weights of the Ancients may be deduced, as the Roman Foot for the Principle of their Meaſures) having collated with the Troy-weights from our Engliſh Standard for *Gold* and *Silver*, by grains thereof, ſaith, that the Roman Pound both ancient and moderne containeth 5256 ſuch grains, and ſo the Roman Ounce both ancient and modern, 438 of the ſame grains, the Troy-pound containing 5760 grains (as I noted before) and ſo the Troy-ounce, 480: whereupon the * Romane Pound and Ounce ſhould be (in the leaſt terms) but $\frac{73}{80}$ of our Engliſh pound and Ounce Troy-weight, and ſo the proportion of the Roman pound & ounce to our Troy-pound & ounce (for the converſion or commutation of the Roman weight to our Troy-weight) as 80 to 73 (which is the commonly received proportion of the Ounce-*avoirdupois* to the Ouuce-*troy*, for the like converſion, as I ſhal ſhew afterwards, and which is decimally, as 10000 to 9125 exactly.) As for the diviſion of the Romane ounce immediately by ſcruples, in number XXIV

The Roman Weight, and our Troy or Gold-ſmiths Weight, compared together.

* *Thomaſius* in the end of his Dictionary, reducing the weights and meaſures, &c. of the Auncients, to thoſe in uſe with the Engliſh nation; ſaith, that the Roman Pound is 10 oun. and an half Troy: and ſo the Roman ounce is 3 quarters and an half of an ounce-troy. By which accompt, the Roman pound wil contain but 5040 engliſh graines, or ſuch as our troy-pound contains, 5760, which comes ſhort of Mr. *Greaves* his accompt by 216 graines And ſo the Romane Ounce will containe but 420 ſuch graines, which falleth ſhort of Maſter *Greaves* his obſervation, by 18 grains. But herein wee rather give credit to the later and exacter obſervations and experiments of Mr. *Greaves*. But yet how our Troy-weight may have been altered ſince *Thomaſius* his time, I know not.

which *Ghetaldus* uſeth, Mr *Greaves* ſpeaketh not of it, but only by drams, in number VIII, as the *Ounce* is commonly divided by the Phyſitians of all Countreys, and the phyſicall or medicinal weights which we uſe in *England*, are the ſame with the Troy-weights for *Gold* and *Silver*; only the Ounce-troy is commonly by our Goldſmiths divided into 20 penny-weights (as I ſhewed before) a penny-weight conſiſting of 24 grains: and the *Ounce* by Phyſitians is univerſally divided into 8 drachmes; a *Drachme* into 3 ſcruples; and a *ſcruple* (by an evil cuſtome received in ſhops) into 20 graines, which ought to have (according to the ancient cuſtome) 24 grains, and ſo be equal with a penny-weight; ſo that the Number of grains in the medicinall Pound and Ounce, is the ſame with that in the Troy: For the *ſcruple* being 20 grains, the *Dram* wil be 60, the *Ounce* 480, and ſo the *Pound* 5760, as before-noted: whereas elſe the *Scruple* being made 24 grains, (as anciently it was) the *dram* would be with us, 72 gr. the *Ounce* 576, and ſo the *Pound* 6912 gr. as *Ghetaldus* hath it; but the number of grains contained in the Roman Pound or ounce, with the Romanes themſelves, Mr. *Greaves* ſheweth not, whereby we might find what the difference is between their grain and ours: But collating the number of grains contained in the Pound or Ounce uſed by *Ghetaldus* (which we take to be the ſame which Mr. *Greaves* noteth for the Roman-pound and Ounce, both ancient and modern) with the number of grains from the Engliſh-*Standard* for *Gold* and *Silver*, contained in the ſaid Roman-pound and Ounce, (which we ſhewed even now from Mr. *Greaves*) we ſhal find the *Grain* (ſuch as the Roman-pound contains

Morellus in cap. 1. Prolegom. ad compoſit. medicament.

tains 6912, and the Ounce 576, according to *Ghetaldus*, as we lately ſhewed) to be (in the leaſt terms) but $\frac{73}{96}$ of the Engliſh grain, (ſuch as the Romane pound contains 5256, and the Romane Ounce 438, according to the obſervations and experiments of Mr. *Greaves*) So that the Romane-gram ſhould be to the Troy grain, from the Engliſh-Standard (for the converſion of Romane grains to our Troy-grains) as 96 to 73; & ſo cõſequently the Roman-weights in generall (as pounds and ounces) reduced into the proper grains, wil be to our Troy-weight in graines, accordingly.

And according to theſe collations and proportions of theſe two *Weights* the one to the other; the former iron-*S*pheare weighing with *Ghetaldus* 12 li. 2. oun. 1 ſcrup. $14\frac{2}{37}$ gr. or $8413\frac{2}{37}$ graines Roman (according to his meaſure of the *S*pheare's *D*iameter by inches of the Roman Foot) will be in Troy-weight from the Engliſh *S*tandard, 11 li. 1 oun. 2 drams, $16\frac{104}{111}$ gr. (or according to the vulgar *D*iviſion of the ounce-troy by penny-weights, 11 li. 1 oun. 5 penny-weights, and $16\frac{104}{111}$ gr.) or $63976\frac{104}{111}$ gr. which is very neer 63977 gr. For firſt, I ſay,

As 80 to 73, *So* 12 li. 2 oun. 1 ſcrup. 14 gr. Roman, (*viz.* 12 li. 2 oun. 38 gr.) or $146\frac{38}{576}$ oun. Roman, to $133\frac{6571}{23040}$ ounces-troy; which is 133 oun. and 137 gr. *ferè*: and theſe reduced into libral weight, are 11 li. 1 oun. 2 dr. or 5 p. w. and 17 gr. *ferè*, as before.

Then ſecondly; As 96 to 73, So are $8413\frac{2}{37}$ gr. Roman, to $63976\frac{104}{111}$ gr. Troy, as before.

Which exceeds the former troy-weight of this Spheare (deduced from its avoirdupois-weight, according to the common proportion, 60 to 73) *viz.* 10 li. 11 oun. 3 drams, and 12 gr. (or 10 li. 11 oun. and 8 penny-weight, as before)

or 63072 gr. by 1 oun. 7 dr. and 5 gr. or 1 oun. 17 p. w. and 17 gr. or 905 gr. which difference in the Troy-weight here, may ariſe not only from the difference between *Ghetaldus* his Weight and Meaſure, and ours, but alſo from ſome difference in the Metal it ſelf, which I ſhal ſpeak of afterwards. Now if we ſhall reduce his meaſure of the Spheare's dimetient, being $\frac{4}{12}$, or 4 inches of the Roman Foot, (which we ſhewed before, to be the *Pes Colotianus*, as being moſt approved of by him, for the true ancient Roman *Foot*) to inches of our Engliſh Foot (according to Mr. *Greaves* his foreſaid collation of theſe two Feet together) the ſaid Spheare's *Dimetient* will be leſſe then 4 inches of the Engliſh Foot, *viz.* but 3.868 inches-engliſh (the whole Engliſh Foot, or 12 inches-Engliſh, being 12 [illegible] inches-Romane, according to Mr. *Greaves* his proportion of the Colotian Foot to the Engliſh Foot, 1000 to [illegible], or 967 to 1000, before noted) and ſo the formet weight of the *Sphear*, 12 li. 2 oun. 1 ſcrup. $14\frac{2}{137}$ gr. with *Ghetaldus*, or 11 li. 1 oun. 2 dr. 17 gr. *ferè*, with us in Troy-weight, (which according to the common proportion of the Troy *libral* weight, to the avoirdupois libral weight, 73 to 60, is 9 li. 2 oun. and 1.057 dr. avoirdupois) anſwering to the Spheare's diametrall magnitude of 4 inches upon the Roman Foot, will anſwer to 3.868 inches upon the engliſh Foot. Or againe; if we ſhall reduce our engliſh meaſure of the ſaid Spheare's dimetient, 4 inches (and ſo commonly holden to weigh 9 li. avoirdupois, which is 10.95 li. troy, as we ſhewed before) to Roman meaſure in inches, we ſhall find the ſame (according to the formet Pedal Collations) to be more then 4 Roman inches, *viz.* 4.136 inches, and the gravity of an iron Sphear of this *Diameter*, will be in *Ghetaldus* his weight, 93044.86 graines,

or

or 13 li. 5 oun. 4 dr. 20.86 gr. (or according to *Ghetaldus* his *Diviſion* of the ounce) 13 li. 5 oun. 12 ſcrup. and 21 gr. *ferè)* which is in our Troy-weight (according to the former Collations of theſe two weights) 70752.86 gr. or 12 li, 3 oun. 3 dr. 12.86 gr. (or by the common diviſion of the ounce-troy by penny-weights, 12 li. 3 oun. 8. p. w. and 0.86 gr.) * For ſeing that *S*pheares of the ſame kind, are among themſelves in gravity, as the Cubes of their Dimetients are in magnitude: Therefore the weight of the iron-*S*pheare, whoſe *Dia*meter is 4 inches Romane-meaſure, or 3.8 68 inches engliſh-meaſure, being found as before; the weight of the other Spheare of the ſame kind, whoſe diameter is 4 inches engliſh-meaſure, or 4.136 inches Roman-meaſure, will be found alſo as before: Or more readily, by having the weight of ſuch a Spheare, whoſe Diameter is one inch, which by *Ghetaldus* his Meaſure and weight, is $1314\frac{22}{37}$ graines. or 2 oun. 6 ſcrup. $18\frac{22}{37}$ gr. and ſo in our troy-weight, $999\frac{71}{111}$ grains, or 2 oun. 1 p. w. and 16 gr. which weight therefore will anſwer to an iron-Spheare whoſe *D*iameter is 0.967 inch, engliſh-meaſure, for this anſwers to one inch-Roman; and ſo the weight of ſuch a Spheare who'e Diameter is one inch-engliſh (which is 1.034 inch-romane) ſhould be by this accompt, 1105.51 graines-Engliſh, or 2 oun. 2 dr. 25.5 gr. or 2 oun. 6 p. w. and 1.5 gr. troy (which are in Roman weight, 1453.83 gr. or 2 oun. 4 dr. 13.8 gr. or by *Ghetaldus* his diviſion of the ounce; 2 oun. 12 ſcrup. and 13.8 gr.)

* *Archimed.* promot. theor. 9. prop. 17.

Or (here briefly to ſhew the uſe of the former proportional Numbers for Metals among themſelves) in reſpect of two Sphearesof different kindsof Metals, and like magnitude) the ſame weight of the foregoing Spheare of Iron,

of 4 inches engliſh-meaſure, or 4.136 inches Roman-meaſure, the diameter maybe produced as before, by the weight of a Sphear of any other Metal firſt had, being of the ſame magnitude: As for example, a *Sphear* of Tinne, (for ſo *Ghetaldus* commonly deduceth the weights of other metalline Sphears from a ſtanneal Sphear, or a Sphear of Tinne) whoſe diametrall magnitude is 4 inches-engliſh, or 4.136 inches-Romane, I find to weigh by *Ghetaldus*, 86066.5 grains (or 11^{li}. 5 oun. 10 ſcrup. and 2.5 gr.) which is with us in Troy-weight, 65446.4 gr. (or 11^{li}.4 oun. 2 dr. 46.4 gr. or 11^{li}. 4 oun. 6 p.w. 22.4 gr.) Now therefore, according to the foreſaid proportionall Numbers for *Tinne* and *Iron*, I ſay; As 3895 to 4210 (which is with *Ghetaldus* in his fore-mentioned ſecond Table of the Compariſon of divers kinds of Bodies in gravity and magnitude, as 38$\frac{18}{19}$ to 42$\frac{2}{19}$) or more accurately (the former termes being incompleat and unabſolute) As 1554 to 1680 (which is with *Ghetaldus* in his foreſaid firſt Table of the like Compariſons, as 1 to 1$\frac{3}{37}$, that is, as 37 to 40, and which is decimally, as 1. to 1.081,081, infinitly; or integrally, 1000 to 1081, or 1000,000 to 1081,081 compleatly) So is 86066.5 gr. (or 12^{li}. 5 oun. 10 ſcr. 2.5 gr.) the *Sphear* of Tinne in *Ghetaldus his* weight, or 65446.4 gr. (*viz.* 11^{li}. 4 oun. 2 dr. 46.4 gr.) the ſame Sphear in our Troy-weight, to 93044.86 gr. (or 13^{li}. 5 oun. 12. ſcr. 21 gr.) the Sphear of Iron in *Ghetaldus* his weight; or to 70752.86 gr. (*viz.* 12li. 3 oun. 3 dr. 12.86 gr.) the ſame Sphear in our troy-weight, as formerly: which being converted into *Avoirdupois*-weight, (according to the former terms of proportion between theſe two *Weights*) wil be 10.096^{li}. *viz.* 10^{li}. 1 oun. 8.576 dr. according to the common diviſion of the *Pound-avoirdupois* into 16 ounces,

ounces, and ſo of the *ounce* into 16 drams, and under which diviſion or parts of weight they go not; this kind of weight ſerving with us for al kind of coarſer or groſſer Commodities, it being the common Mercatory weight, (and called by a French name, *Avoirdupois*) and ſo for the weighing of all Metals, but *Gold* and *Silver*, to which only the Troy-weight is aſſigned: Neither indeed is there any need of ſuch exactneſſe in the weight of a bullet or any other Body of Iron, or other like baſe Metall, as to graines, but only to ſatisfie Art it ſelf in the curioſity thereof.

But now whether the common Tenent of an iron-bullet of 4 inches the diameter, to weigh juſt 9^{li}. auoirdupois, were deduced from any certain and exact experiment or no, is a queſtion here to be made; Nor was it my purpoſe to trie the ſame, having not conveniences and accomodations thereunto: but only having the true weight of a Sphear or Bullet of Iron of any magnitude, in a certain meaſure given (as *Inches*) thereby to ſhew a way for the exact and moſt ſpeedy obtaining of the weight of a Sphear or bullet of any magnitude whatſoever, which I have partly declared & demonſtrated already, and ſhal more by & by: And looking in ſeverall books of Gunnery, wherein are ſet down the weights of iron-bullets (or round ſhot, as they call it) fitted to all the uſual pieces of Ordnance with us in *England*, according to the diameters of their bore, mouths, or concavities, (abating uſually $\frac{1}{4}$ of an inch of the Gun's ſaid diameter, for the diameter of the bullet) I found in one of them, the weight aſſigned to an iron-bullet of 4 inches diameter or craſſitude, to be juſt 9^{li}. and in another book this weight aſſigned to a bullet of 4 $\frac{1}{2}$ inches the diameter and ſo in other books, other weights aſſigned to a bullet

of

of the ſame magnitude: And therefore finding ſuch a diſcrepancie among our Maſters of Gunnery in this thing, ſo that I could not diſcover from them any certainty in the weight of an iron-Sphear or bullet, at a certain magnitude or craſſitude: I repaired one day in *Auguſt* 1648, unto Mr. *John Reynolds*, one of the Clarks of the *Mint* in the Tower of *London*, (and ſometime Aſſay-maſter at *Goldſmiths-hall*) a man much noted by artiſts for his induſtry and ingenuity in the Mathematiques; (and ſo indeed I find him to be) who very courteouſly entertaining me at his houſe in the Tower, with diſcourſe about many experiments made by him in the Mathematicks, and ſhewing the ſame to me (as he had once done in ſome of them, long before) among wch were thoſe cõcerning the weights of Metals, and their proportions one to another: I deſired of him to be reſolved in this point of art concerning the weight of a Sphear or bullet of Iron, according to ſome certain magnitude or craſſitude, knowing him to have all accomodations fit for the finding out of the ſame; and he thereupon produced me an experiment made by him not long before, upon a large bullet, which he then ſhewed me; but I obſerved the ſame to be very unfit to ground an experiment upon, being not only much ruſty, but alſo having ſeverall holes and cavities therein, (which ſeemed to be chiefly from the antiquity of it) which might wel hinder the finding of the true weight, according to its diametrall magnitude, and which himſelfe then doubted much of: whereupon I importuned him for another the like experiment, which might be exact; And ſo both for his own ſatisfaction, and mine alſo, he ſoon after, got another bullet, which was very ſound & ſolid, and clear enough from ruſt, and alſo as round every way,

way, as could well be imagined; whoſe *Diameter* we thereupon took by a pair of Callaper Compaſſes (conceiving it to be a ſurer way to find the ſame, then by the Circumference) as preciſely as poſſibly we could, and meaſuring the ſame upon a line of inches, found it to be 4.8 inches; and then for my further ſatisfaction, I took the Circumference of the bullet with a thred, by means of the ſmall creaſe or Circle which encompaſſed the bullet exactly in the middle, being made by the Mould in which it was caſt, and meaſuring the thred upon the ſaid line of inches, we found it to be 15.25 inches, which by Cyclo-perimetricall proportionality, gives the diameter as before; ſo that we might wel conclude, the magnitude of this bullet to be rightly found by us. Then for the weight thereof, we tried the ſame with all poſſible preciſeneſſe, both by *Avoirdupois*-weights, and *Troy*-weights, and found it to weigh, $15^{li}.\ 12\frac{1}{4}$ oun. avoirdupoiz; and $19^{li}.\ 1\frac{25}{31}$ oun. Troy, *viz.* $19^{li}.$ 1 oun. p.w. 15 gr. According to which experiment, an iron-bullet (made of caſt iron, ſuch as bullets are uſually made of, and which weigheth much lighter then forged iron) of 4 inches the diameter, wil weigh upon the point of $9^{li}.$ and 2 oun. avoirdepois, which differs not much from the common Tenent.

And ſo according to this our experiment (to which we wil adhere for the finding out of the weight of any *S*phear or bullet made of caſt-iron) the artificial *L*ine of meaſure, or cubatory Line of gravity, for the ſpeedy diſcovering of the weight of any Sphear or bullet whatſoever, made of caſt-iron, by the *D*iameter or Circumference thereof, as was formerly ſhewed, wil be 1.91 inch, for the *D*iameter, and 6.01 inches for the Circumference; and theſe in reſpect of libral weight, as the two former *L*ines, *viz.*

1.92 inch, and 6.04 inches, deduced from the former common Tenent of the weight of an iron bullet of 4 inches diameter, which differ but very little one from another, as they are here ſet, from a centeſimal diviſion of the *Inch*. Now theſe two latter Lines being divided as the former, and the Diameter and Circumference of the foreſaid Sphear or bullet meaſured thereby, the ſame will be found ſeverally, to be 2.09 *ferè* (which by the other Lines were 2.08) which cubed, yields 9.129329 *ferè*. for the weight of the bullet, which is 9*li*. 2 oun. 1 dr. averdepoiz, differing from the true weight (by way of exceſſe) only 1 dr. which is not conſiderable.

But now what ſort of Iron, *Marinus Ghetaldus* in his experiments upon the ſame, for the weight thereof, and ſo its proportion to other Metals, meaneth, whereby a Sphear of 4 inches diameter, engliſh-meaſure, (or 4.136 inches Roman-meaſure) ſhould weigh according to him, 10.096*li*. avoirdupois (as being deduced from 12.283*li*. troy and that from 13.461*li*. Roman) we know not, if Mr. *Greaves* his foreſaid collations and compariſons of the Romane weight and Meaſure (uſed by *Ghetaldus*) with the Engliſh, be true, (as we are willing to beleeve they are) and alſo the foreſaid proportion of the Troy-pound-weight to the Avoirdupois-pound-weight, as it is commonly holden, *viz*. 73 to 60; although the aforenamed Mr. *Reynolds* will have it to be, as 17 to 14; deducing the ſame from a generall Maxime, and undeniable Principle (as he ſaith) that 136 pounds-Troy, and 112 pounds *Avoirdupois* are equilibral, or equivalent in weight, which are in the leaſt terms of proportionality, 17 and 14; and which indeed I find by experience

The Troy and Avoirdupois Pound compared together.

to

to be the truer, as I ſhall afterwards ſhew: And this is to be underſtood properly (as the Terms here ſtand, from the greater to the leſſe) for the reduction of Troy-pound-weight (or Troy-pounds) to Avoirdupois-pound-weight (or Avoirdupois-pounds) in aſmuch as 73 *li.* Troy, make but 60 *li.* Avoirdupois; or rather 17 *li.* troy make but 14 *li.* avoirdupois: and ſo the Avoirdupois Pound, is (in the leaſt terms) $\frac{73}{60}$, or rather $\frac{17}{14}$ the Troy-Pound; whereas elſe, if we will preciſely compare the *Pound-troy* ſimply, as the leſſe, with the *Pound-avoirdupois*, as the greater; then muſt the Terms be rather taken the contrary way: And thereupon the *Pound-troy* will be to the *Pound-avoirdupois*, as 60 to 73, or rather, 14 to 17: And ſo the Pound-troy will be (in the leaſt terms) $\frac{60}{73}$, or rather $\frac{14}{17}$ of the Pound-avoirdupois. And which ſeveral Terms of proportion, though they ſeem to differ but very litle in the Reaſon thereof it ſelfe; yet may the difference of the weights of things, produced ſeverally thereby (by way of converſion or reduction of one kind of weight to the other) be many times conſiderable; and the more, the greater that the quantity of the weight ſo reduced, is; as I ſhall plainly ſhew afterwards: And ſo I will here firſt compare theſe two ſeverall Proportions together (and that according to the common acception of the Terms, from the *Pound-troy* to the *Pound-avoirdupois*, *viz.* the greater Term as antecedent, to the leſſe, as conſequent, and ſo the reaſons of the Terms will be of the greater inequality; and the rationality of the firſt, or common Terms, will be (by prolation, from the *Parabole* of the ſaid Terms, or Quantity of the Reaſon) *ſuper-tredecupartiens-ſexageſimas*, *viz.* ſuperpartient $\frac{13}{60}$: and of the other Terms, *ſupertripartiens quartas-decimas*, *viz.* ſuperpartient $\frac{3}{14}$, and ſo the difference of the Reaſons, or the differen-

tiall Reason, but that of 1. to 420, *viz.* $\frac{1}{420}$, according to a plain or simple subduction, or differencing of *Reasons*, by reduction of them to one common or conjunct *Consequent*, which imitates the subduction of Fractionall Numbers, by reduction to one common or conjunct *Denominator*; and which is the most genuine, proper, and rational subduction, or discrimination of *Reasons* (both as *Clavius*, *Ramus* and divers other of the best Authours do teach) thus;

Ram. Arithm. l. 2. c. 2 & 3. & Lat. Schon. Et Clav. de Propert. compost. in fine 9 Elem. Eucl.

		511 . 510	1
Anteced.	73 . 17	1022 . 1020	2
Conseq.	60 . 14	840	840
Common or conjunct Conseq. in the least term by mediation.		420	420

Or more plainly thus, after the manner of Fractions.

	dif. 1	
compound anteced.	511 . 510	*by mediation.*
	dif. 2	
compound anteced.	1022 . 1020	*or terms cognominall.*
Antecedents.	73 X 17	
Consequents.	60 X 14	
Common conseq.	840	
Common conseq.	420	*in the least term by mediation.*

and

And which may alſo be ſeen by the like operation in the ſuperpartient termes only, thus;

	dif. 1.
Numeratours cognominall	91 , 90 *by mediation.*
	dif. 2
Numeratours cognominall.	182 . 180
Superpartient termes	$\left\{\frac{13}{60}\right. \times \frac{2}{14}$
Common Denominatour.	840
Common Denom.	420 *by mediation.*

And not according to that operation of Reaſons or Proportions, which ſome do falſly and improperly call the ſubduction or ſubtraction of Reaſons (as *Clavius* alſo ſaith in the place fore-cited) being indeed the true diviſion or Reſolution of Reaſons, and which is as the diviſion of Fractions; whereby they make the quotall Reaſon to be the differentiall or reſiduall Reaſon; and which would then be here by prolation, of the greater inequality, *viz.* $1\frac{1}{110}$, according to the termes $\frac{111}{110}$ (whereas the other or true differentiall Reaſon, is of the leſſe inequality, by very much) but approaching very nigh unto a Reaſon of equality, or unity, or a Reaſon ſingular and individual; and this latter is alſo unexpreſſible or indenominate as the former, and is had by a conjunct compoſition only of the alternate or heterologall terms, thus.

Anteced.	73. · 17	1022	511	}	*Quotient.*
Conſeq.	60 · 14	1020	510	}	

Or more plainly thus,

$$\frac{511}{1022} \Big| \frac{510}{1020}$$

$$73 \times 17$$
$$60 \quad 14$$

Or yet more plainly and readily, by permutation of the termes of the dividing or reſolving Reaſon, in reſpect of their places; for ſo the work of Reſolution wil be changed into a Compoſition of the termes, according to the ordinary Multiplication of Fractions, thus,

$$\frac{511}{1022}$$

$$\frac{73 \cdot 14}{60 \cdot 17}$$

$$\frac{1020}{510}$$

And

And the latter of theſe Proportions between the *Troy* and *Avoirdupois-pound*, makes the Pound Troy to be more of the Pound Avoirdupois, or to come nearer the ſame in quantity, then the other or common Proportion doth; as may be more plainly diſcerned, by reducing the ſaid ſeverall Proportions into decimal Termes, for ſo, the Proportion of the *Pound-troy*, to the *Pound-avoirdupois* from the Termes of 73 to 60, wil be, as 1.0000. to .8219, and from the terms of 17 to 14, as 1.0000 to .8235. Or by taking the Terms the contrary way, (as for the converſion or reduction of Avoirdupois-pounds to Troy-pounds) the *Pound-avoirdupois* wil be to the *Pound-troy* from the termes of 60 to 73, decimally, as 1.00000, &c. to 1.21666, &c. infinitely; and from the Terms of 14 to 17, as 1.00000, &c. to 1.21428, &c. and ſo here the Pound Avoirdupois is by theſe latter Terms, leſſe of the Pound Troy, (and ſo comes nearer the ſame) then by the other Terms. And the Reaſon of theſe latter Terms (as they here ſtand, from the leſſe to the greater) wil be greater then the Reaſon of the firſt or common Terms (according to the exact comparing of *Proportions* or *Reaſons* together, as you may ſee in the fore-cited places of *Ramus* and *Clavius*) in regard that the two ſeverall *Reaſons* being reduced to one common Conſequent (as aforeſaid) the new, compound, or correſpondent Antecedent of the latter Proportion, wil be greater then the like Term of the firſt; or the foreſaid Antecedent of the latter Proportion, wil be more of the common Conſequent, then the like Antecedent of the firſt Proportion (as in the former operations, the contrary happened, where the Terms of Proportion were put the contrary way, *viz.* from the greater Term as antecedent, to the leſſe, as Conſequent) as you may

may here plainly see by these severall subsequent operations.

		2 *dif.*		
Anteced. 60 . 14	1020	. 1022	}	*comp. anteced.*
Conseq. 73 . 17	1241		}	*common conseq.*

Or more plainly thus, in a Fractionall manner.

2

1020 | 1022

Anteced. 60 X 14 *Differ.*

Conseq. 73 X 17

Com. conseq. 1241 *in the least terms.*

Or again, the same wil appear by a contrary operation, which is by reducing the two severall Proportions (as the terms be here put) to one common Antecedent; for so the new, compound, or correspondent Consequent of the latter Proportion or Reason, wil be lesse then the like Term of the first or common Proportion, (and thereby the Reason of these latter Terms, wil accordingly be greater then that of the other Terms (according to the fore-cited Authours) as you may plainly see by this next operation following; the compound Antecedents of the operations next beforegoing, being here changed into the like Consequents.

Com.

Com. Anteced.	420	*by mediation*
Common	840	*Antecedent*
Anteced.	60 X 14	
Conſeq.	73 X 17	
New or Compound	1022 | 1020	*Conſequents*
Comp. Conſeq.	511 | 510	*by mediation.*

And here the difference of the *Reaſons* (or the differentiall *Reaſon*) wil be that of 2 to 1241, *viz.* $\frac{2}{1241}$, as you may ſee in the firſt of theſe two operations before-going.

And now according to theſe latter termer termes of Proportion of the Pound-troy to the Pound-avoirdupois, *viz.* 17 to 14; the foreſaid Sphear of Iron wil be in avoirdupois weight, 10.1158*li*. which (by converſion of the fraction-part into the proper parts of this weight) is 10*li*. 1 oun. 13.6 dr. whereas by the common terms, it was 10*li*. 1 oun. 8.6 dr. and ſo the difference of weight, 5 dr. exactly.

But I conceive (as in all probability and reaſon I ſhould) the Iron uſed by *Ghetaldus* in his experiments, to be the finer ſort of iron, or forged Iron, which weigheth heavier then caſt, coarſe, or droſſie iron, the proportion of weight between them, being (as I have deduced it from the experiments of Mr. *Reynolds*; and partly of my ſelf alſo, upon this Metal) in general, ſuch as is between 1000000 and 951832 arguing from forged Iron to caſt iron, as from the more to the leſſe: and ſo contrarily from caſt-iron to forged iron, as from the leſſer to the greater, the proportion of weight, wil be ſuch as of 1000000 to 1050605: For the ſaid Mr. *Reynolds* found

The Proportion between forged iron and caſt-iron.

the weight of a cube-inch of fine Iron, which had been kept in the Treaſury of the Tower of *London*, ever ſince King *Henry* the 7th. his time or longer (and that very neatly in a velvet-Caſe) to be 4.169 ounces Troy, *viz.* 4 oun. 3 p.w. & 9.12 gr. troy. which is 0.3474li. troy: And from the former Bullet of caſt-iron of 4.8 inches the Diameter, which we found to weigh 19 pound, 1 oun. 15 penny-weight, and 15. grains troy, or 229. ounces, 15 p.w. and 15 gr. (which is $229\frac{25}{32}$ ounces troy) we gather the weight of one cube-inch of caſt-iron to be 3.968 oun. troy; which is 0.33068li. troy. And ſo according to this experiment, a Sphear or bullet of one inch *Diameter*, made of fine, or forged Iron, wil weigh 1.18288 oun. troy, which is 0.1819li. troy: and ſo a Sphear or bullet of 4 inches Diameter, made of the ſame metal, wil weigh 11.642 poundstroy (or 11li.7 oun. 5 dr. and 38.17 gr. or 11li.7 oun. 14 p.w. and 2.17 gr.) which being compared with the weight of an iron-Sphear of 4 inches *Diameter* (engliſh-meaſure) decuced from *Ghetaldus* before into our Troy-weight, *viz.* 12.283li. &c. troy, or 12li. 3 oun. 3 dr. 12.86 gr. or 12li. 3 oun. 8 p.w. and 0.86 gr.) it will be found to want thereof, 7 oun. 5 dr. 34.11 gr. or 7 oun. 13 p.w. and 22.11 gr. (according to the difference between 12.283483*li.* and 11.642044*li.* being 0.641339*li.*) And ſo the weight of a *S*phear of fine Iron of 1 inch diameter, here found 2.18288 oun. troy, *viz.* 2 oun. 1 dr. 27.78 gr. or 2 oun. 3 p.w. 15.78 gr. being compared with the weight of a ferreall or iron-Sphear of the ſame *Diameter*, deduced formerly from *Ghetaldus* into our troy-weight, *viz.* 2.30315 oun. or 2 oun. 2 dr. 25.5 gr. or 2 oun. 6. p.w. and 1.5 gr. it wil be found to want thereof 57.7 gr. *viz.* 2 p.w. 9.7 g. which operations agreeing ſo nearly one with another

ther, it is thereby manifeſt, that the Iron meant by *Ghetaldus*, is the finer ſort of Iron, or purely forged iron; But that we cannot make his experiments and ours exactly to agree in a Sphear of this Metall; may happen not only in reſpect of the difference between his weight and meaſure and ours, and the uncertainty of the proportions between them, whereby the one might be exactly reduced to the other; but alſo in reſpect of the difference between the Metal uſed by him and us; for that all Iron (or other Metal) of the like ſort, is not always of the ſame gravity or ponderoſity preciſely becauſe all is not of a like fineſſe or coarſneſſe, and ſo that which is fineſt, wil ſtill be heavieſt.

Now *Ghetaldus* beginning to find out the weights of metalline Spheares, according to the uſuall known Weights with him, found that no diligence or induſtry of man could make a Spheare ſo exact as it ought to be, and therefore he procured a *Cylinder* to be made, and that of Tinne, equall in height to the *D*iameter of its Baſe (which were of a certaine magnitude in inches of the Roman Foot) for that this might be turned in a *L*athe much exacter, and more eaſily then a Spheare: and hereby found, that a Cylinder made of Tinne, being of one inch or $\frac{1}{12}$ of the Roman Foot in its craſſitude and altitude, did weigh 3 oun. 4 ſcrup. which reduced into grai nes of his weight, is 1824 gr. whoſe $\frac{2}{3}$ being 1216 gr. is the weight or gravity of a *S*pheare of the ſame Metall, whoſe diameter is equal with the diameter or height of the Cylinder; according to the demonſtrations of *Archimedes*, *lib.* 1. *de Sph. and Cyl. prop.* 32. It being there ſhewed by him, that a *Cylinder*, whoſe Baſe is equall to the greateſt Circle in a Spheare, and its altitude equal to the diameter of the Spheare (or of the ſaid Circle) is ſeſquialter the ſaid *S*pheare. And ſo *Ghetaldus* having found the

weight of one Spheare of Tinne (at a certaine magnitude) from thence he deduceth the weight of any other Spheare, and that not only of the ſame Metall, but alſo of any other metall, by the proportions of Tinne to the other Metals (in like Bodies of equal magnitude) found out by him at firſt.

Now the foreſaid weight of 1216 gr. (or 2 oun. 2 ſcr. 16 gr.) for a Spheare of Tinne of 1 inch the diameter with *Ghetaldus*, will anſwer to the like kind of Spheare whoſe diameter is 0.967 inch from the Engliſh Foot, (according Mr. *Greaves* his foreſaid compariſon of the Roman Foot with the Engliſh) which being converted into our Troy-weight (according to Mr. *Greaves* his foreſaid compariſon of the Romane weight with our Troy-weight) is 924$\frac{3}{5}$ gr. (*viz.* 1 oun. 7 dr. 24$\frac{3}{5}$ gr. or 1 oun. 18 p.w. 12$\frac{3}{5}$ gr.) and hence we gather the weight of a ſtanneall Spheare, whoſe diameter is 1 inch, or $\frac{1}{19}$ of the Engliſh Foot, to be 1022.6 gr. engliſh, (*viz.* 2 oun. 1 dr. 2.6 gr. or 2 oun. 2 p.w. 14.6 gr.) whereby we may eaſily obtaine the weight of any other Spheare of the ſame Metall, and alſo of any Spheare of any the other metals, by means of the foregoing proportionall Numbers between Tinne and thoſe other Metals: For ſo a Sphear or Bullet of *Gold* (ſuppoſed fine) of one inch diameter Engliſh-meaſure, will be found, (according to the proportion of 3895 to 10000, which is with *Ghetaldus* in his ſecond Table of the compariſon of ſeveral ſorts of Bodies in gravity and magnitude, 38$\frac{18}{19}$ to 100) or rather (the former termes being uncompleat) of 1554 to 3990 (which is with *Ghetaldus* in his firſt Table of the compariſon of the ſame Bodies in gravity and magnitude, 1 to 2$\frac{21}{37}$, and that's as 37 to 95 in the leaſt rationall and abſolute termes) to weigh 2625.6 gr. engliſh, (which is 5.47 oun. troy, *viz.* 5$\frac{1}{2}$ *ferè*, being 5 ou. 3 dr. 45.6 gr. or 5 oun. 9 p.w. 9.6 gr.

9.6 gr.) And ſo the like Body of fine *Silver*, of the ſame magnitude, will be found, (according to the proportion of 3895 to 5439, which is with *Ghetaldus* in his foreſaid ſecond Table of the Compariſon of ſeverall Bodies in gravity and magnitude, $38\frac{18}{19}$ to $54\frac{22}{57}$) or more accurately (the former termes being not abſolute or compleat) of 1554 to 2170. (which is with *Ghetaldus* is his forementioned firſt Table of the like Compariſons, 1 to $1\frac{44}{111}$, and that is as 111 to 155, in the leaſt termes rationall and abſolute, and which are by decimall numeration, as 1. to 1.396, 396 infinitly; or integrally, 1000 to 1396, or 1000, 000 to 1396, 396 exactly) to weigh 1427.95 gr. engliſh (*viz.* 2 oun. 7 dr. 47.95 gr. troy, or 2 oun. 19 p.w. 11.95 gr.) In like manner will be deduced the weight of Iron and the other baſer ſort of Metals, from Tinne (in Bodies of like form and magnitude) But becauſe the other Metals beſides *Gold* and *Silver*, as uſually weighed with us, by the common Mercatory weight aforeſaid, called *Avoirdupois*. Therefore for the more immediate and ſpeedy obtaining of their weights from Tinne (in ſphericall bodies) it is beſt to have the weight of the foregoing Stanneall Spheare of one inch diameter, *viz.* 1022$\frac{3}{7}$ gr. engliſh, or 2.1304 oun. troy, (*viz.* 2 oun. 2 p.w. 14.6 gr. as before) reduced into avoirdupois unciall weight, and that decimally, whereby the weight of the like kind of Spheare of any other magnitude may be readily obtained; and ſo conſequently the weight of any other metalline Spheare therefrom, in its proper weight of *Avoirdupois*. Which ſaid weight therfore of 2.1304 oun. troy, will be in avoirdupois unciall weight (according to the cõmonly received proportiõ of the Troy ounce-weight to the avoirdupois ounce-weight, 73 to 80) 2.3347 oun. (*viz.* 2 oun. 5.355 dr.) But according to

Mr. *Reynolds* his proportion of theſe two weights the one to the other, which is, as 51 to 56 (being deduced from his proportion of the Troy-pound-weight to the Avoirdupois-pound-weight noted before, and which proportions of weight are the neareſt and trueſt that may be, as I ſhall ſtraightway ſhew) the ſame ſpheare will be 2.33928 oun. avoirdupois (*viz.* 2 oun. 5.428 dr.) which exceedeth the former weight only .073 of a dram, which in this kind of weight is altogether inconſiderable.

Or for the ready obtaining of a Sphear of Tinne, of any magnitude, (and ſo of any other Metal therefrom) in libral-weight, it is convenient to have the foreſaid Sphear of Tinne of 1 inch diameter, in libral-weight, both Troy and Avoirdupois, and that decimally; which wil here be in Troy-weight, o.17753*li.* and in Avoirdupois-weight, (which is here chiefly to be regarded) according to the commonly received proportion of the troy-weight to the avoirdupois weight) o.1459189*li.* and according the other (and better) proportion, o.146205*li.* the difference of which from the other (by way of exceſſe) being hardly, o.0003*li.* *viz.* 3 parts of a pound-avoirdupois, divided into 10000 equal parts; Which difference of weight, although it be here of little or no value (as alſo that in the operation immediatly preceding) in regard of the ſmalneſſe of the Body here handled, whereby the difference between the common Proportions of the Troy-weight to the Avoirdupois-weight, and the other proportions, may ſeem to be very ſmall, and inconſiderable: Yet if we go to to reduce a metalline (or other) Body of a greater magnitude or dimenſion, out of one of theſe Weights into the other (according to the ſaid ſeverall proportions beforegoing) we ſhal find the difference of weight (in one and the ſame kind) to be

ſtil

ſtil greater, and the greater or weightier that the Body is, the greater will be the difference of the weight produced by the ſaid different termes of proportion, in reſpect both of libraall and unciall weight, inſomuch as that the difference of weight will many times be conſiderable: As in the foregoing Sphear of iron, of 4 inches the diameter, (engliſh-meaſure) whoſe weight being found from *Ghetaldus*, to be 13.461*li.* Romane, and from that, to be in Troy weight (according to Mr. *Greaves* his collations of theſe two kinds of weight the one with the other) 12.283*li.* and then the ſame converted into Avoirdupois weight, according to the two ſeverall proportions between theſe two weights; the difference of weight was there found to be 5 drams avoirdepois, which is almoſt $\frac{1}{3}$ of an ounce. And greater differences of weight (in this reſpect) then this, I ſhall ſhew by and by, and alſo more afterwards.

The Troy and Avoirdupois Ounce compared together; as alſo the ſeverall Proportions aſſigned between them.

Now therefore, for the proportion between the *Troy*-ounce, and the Avoirdupois-ounce according to the Terms 51 and 56 (deduced from the foreſaid Maxime or Principle of libraall weight, that 136*li.* Troy, and 112*li.* Avoirdupois are equilibral, or equiponderant, and ſo accordingly from thence, 1632 oun. Troy and 1792 oun. Avoirdupois, which are in the leaſt terms of Proportion, 51 and 56) the ſame being compared with the foreſaid common termes, 73 and 80, wil ſeem indeed to differ but little therefrom in the Reaſon it ſelf, and that deficiently, as the terms here ſtand, from the leſſe to the greater, like as the correſpondent Terms before for libraall weight, did from the common terms, being taken from the greater term to the

Thomaſius in the fore-cited place of his Dictionary, ſaith, that the Goldſmiths (or Troy) Ounce hath to the Avoirdupois-Ounce, proportion ſ. quintadecimal, So that (to wit) 11 ounces troy are exactly equall to 12 ounces avoirdupois; and which (he ſaith) he proved by a moſt juſt and exact Ballance. But this Proportion is ſomewhat greater then either of the other two here noted, & makes the ounce Averd. to be more of the ounce Troy then either of thoſe two.

Mr. *Wingate* in his *Arithm.l.* 1 *c.*1. *Sect.* 24. ſaith, that the avoirdupois-Pound is compoſed of 14 oun. troy, and 12 p.w. *viz.* 14.6 oun. troy; which is according to the commonly received Proportions between the Avoirpois and Troy-weight: But according to the other proportions, it will be moſt truly but 14 oun. 11 p.w. and $10\frac{2}{7}$ gr. troy.

the leſſe, *viz* 7 to 60, and 17 to 14: the Reaſon of the common terms, as they here ſtand from the leſſe as the Antecedent, to the greater, as the Conſequent, being by prolation, of the leſſe inequality, or inequality of the leſſe, *ſub-ſuper-ſeptuapartiens ſeptuageſimas-tertias, viz.* ſub-ſuper-partient $\frac{3}{73}$; and of the other Terms, (ſtanding correſpondently) *ſub-ſuper-quintuapartiens quinquageſimaprimas viz.* ſub-ſuperpartient $\frac{5}{51}$; and ſo the difference of the *Reaſons*, or the differentiall or reſiduall Reaſon (according to the plain, ſimple and proper ſubduction of Reaſons before ſhewed) that of 1 to 560, *vix.* $\frac{1}{560}$; And this, for the converſion or reduction of Troy-uncial weight to the like Avoirdupois-weight; and ſo the Ounce-troy wil be (in the leaſt terms) $\frac{80}{73}$, or more truly, $\frac{56}{51}$ the Ounce-avoirdupois: Or elſe, the *Terms*, in this poſition, from the leſſe to the greater, might ſeem rather to be taken contrarily (according to an exact compariſon) for the proportion of the Avoirdupois-ounce, (as the leſſe) to the troy-onnce (as the greater) for ſo the Avoirdupois-ounce, wil be $\frac{73}{80}$, or rather $\frac{51}{56}$ of the *Troy*-ounce. And theſe latter *Terms* of

Pro-

proportion, do make the Ounce Troy more to exceed the Ounce-avoirdupois, then the firſt or common Terms do, and ſo make the Ounce-avoirdupois to come ſhort of the Ounce-troy (or be leſſe of the ſame) accordingly; as may more eaſily be ſeen, by reducing of the foreſaid Terms of proportion into decimall termes; for ſo, the proportion of 73 to 80, will be as 1.00000 to 1.09589, and of 51 to 56, as 1.00000 to 1.09804 *ferè*: And contrarily; by permutation of the Terms, the proportion of 80 to 73, will be decimally, as 1.0000 to .9125 exactly (as I ſhewed formerly, upon another the like occaſion,) and of 56 to 51, as 1.0000 &c. to .9107 &c.

The two foregoing ſeveral Proportions between the Troy and Avoirdupois Weights compared, and examined experimentally, by the Ballance, in the foregoing Iron Bullet.

And therefore, now to ſhew (by the way) the difference between the foreſaid ſeverall proportions of the Troy and Avoirdupoiz weights both librall and unciall, the one to the other, by comparing them with the weight of ſome Body, taken both by Troy and Avoirdupois-weights, and that eſpecially in one and the ſame Ballance, and ſo converting it out of the one kind of weight into the other, by the ſaid two ſeverall kinds of proportions between them, whereby may be known which of theſe are the neerer and truer, as moſt agreeing with the Ballance it ſelfe: We will here take the foregoing Cannon-bullet of caſt-iron, of 4.8 inches the diameter, whoſe weight we found (as I have noted before) in the Tower of *London*, by an exact Ballance, with Weights both Troy and Avoirdupois, to be 19*li.* 1 oun. 15 p.w. 15 gr. Troy (which is 19*li.* and $1\frac{15}{32}$ oun. or wholly in librall-weight, $19\frac{15}{256}$ or 19.1484375 *li.* exactly) and 15*li.* $12\frac{1}{4}$ oun.

$12\frac{1}{4}$ oun. Avoirdupois (which is wholly in libral-weight, $15\frac{42}{64}$ or 15.765625 *li.* compleatly) Now if we shall convert the said Bullet from its Troy-weight to Avoirdupois-weight, according to the common proportion of the Troy-libral-weight to the Avoirdupois-libral-weight, *viz.* 73 to 60, we shall find the same to be (in the least termes) $15\frac{1725}{2336}$ or 15.738 44 *li.* Avoirdupoiz, which wanteth of the true weight from the Ballance, 0.027 18 *li.* which is by reduction or conversion into the proper denominate, compounding parts of this weight 0.oun. 6.958 drams avoirdupois, which is upon the point of 7 dr. and that's almost halfe an ounce avoirdupois. But according to the other proportion of the Troy libral-weight to the Avoirdupois libral weight, *viz.* 17 to 14, the said bullet will be found to weigh (in the least terms) $15\frac{837}{1088}$ or 15.7693 *li.* Avoirdupoiz, which exceedeth the weight, not fully one dram, being but 0.94 dr. as will appeare by reduction of the fractionall termes into the proper denominate parts of this weight, and which is not considerable.

Againe; if we will reduce the said Bullet taken wholly in Troy-unciall-weight, being $229\frac{25}{32}$ or 229.781 25 oun. exactly, into Avoirdupois unciall-weight, according to the said severall proportions of the one to the other, we shall find the very same differences necessarily to happen: for according to the common proportion of the Troy-unciall-weight to the Avoirdupois, *viz.* 73 to 80, it will be found (in the least termes) $251\frac{110}{146}$ or 251.815 oun. avoirdupoiz, which wanteth of the true or ballance-weight, (being 252.25 oun. *Jo.*435 oun. which by reduction, gives 6.96 drams as before: But according to the other proportion of the Troy unciall-weight to the Avoirdupoiz, *viz.* 51 to 56, it will be found $252\frac{28}{68}$ oun. or 252.3088 oun. avoirdupoiz,

poiz, which differeth from the ballance-weight (by way of exceſſe) only 0.0588 oun. which is by reduction, 0.94 dr. as before.

And ſo again contrarily, if we work from the Avoirdupoiz-weight to the Troy-weight, we ſhal find the like proportional differences of weight accordingly. For firſt if we convert the foreſaid bullet, out of its true avoirdupois-weight from the Ballance, $15\frac{43}{64}$li. or 15.7656, &c. into Troy-weight, according to the vulgarly received proportion of 60 to 73, for libral-weight (being the converſe of the former) we ſhal find the ſame to be (in the leaſt termes) $19\frac{697}{3843}$, or 19.1815104li. Troy, which differeth from the true weight of the Ballance ($19\frac{19}{128}$ or 19.1484375li.) by way of exceſſe, 0.0330729li. which is by converſion into the proper, compounding denominate parts of this weight o.oun. 3 dr. 10.5 gr. troy, or o.oun. 7 p.w. 22.5 gr. But according to the other proportion of 14 to 17, it wil be found (in the leaſt termes) $19\frac{129}{896}$ or 19.1439732li. Troy, which differeth from the true weight (by way of defect) only 0.0044643li. which is by a continuall reduction into the leaſt parts of weight, but 25.7 gr. or 1 p.w. and 1.7 gr. which is of little or no value in this thing.

And ſo likewiſe if we reduce the ſaid Bullet out of its true avoirdupois weight taken wholly in ounces (according to the Ballance) being $252\frac{1}{4}$ or 252.25 oun. into Troy weight by ounces; firſt, according to the common proportion of 80 to 73, for unciall-weight (being the converſe of the former) we ſhall find the ſame to be (in the leaſt terms $230\frac{57}{315}$ oun. troy, or 230.178125 compleatly, which exceedeth the true weight, $229\frac{25}{32}$ oun. troy, or 229.78125 exactly, by 0.396875 oun. troy exactly, which is by reduction into the leaſt parts of weight, 3 dr. 10.5 gr. or 7 p.w. 22.5 gr.

22.5 gr. as before from libral weight: But according to the proportion of 56 to 51, it wil be found to be (in the leaſt terms) $229\frac{63}{314}$ oun. troy, or 229.727678, which is deficient from the true weight, only 0.053572 oun. which by the like reduction, is but 1.p.w. and 1.7 gr. as before from libral weight.

By which operations it is ſufficiently evident, that theſe latter Proportions between the Troy and Avoirdupoiz weights, are the truer, (and indeed the neareſt and trueſt that may be found) and which I ſhall (upon the like occaſion) further confirm afterwards by a double experiment from the weight of a liquid body, in the meaſuring of Veſſels.

And thus much by the way concerning Weight and Meaſure in generall, in reference to the work here in hand, (being the like artificial *Dimenſion* of metalline regular Bodies, for the ſpeedy diſcovering of their gravities or weights, and more particularly of a ferreall or iron *Sphear*, as was formerly of a Sphear in generall for ſolid meaſure) being induced thereunto by *Ghetaldus* in his foreſaid work of Metals, in which he diſſereth from us in both theſe, as we have abundantly ſhewed.

And ſo theſe operations beforegoing in the particular metalline Sphears aforenamed, for the weight thereof, are from the experiments of M. *Ghetaldus*, & reduced from his weight and meaſure to ours, according to the obſervations and experiments of our Countreyman Mr. *Greaves* upon the ſame, and his collations of them together, as aforeſaid; And alſo in one of them, from my own experience, according to our Engliſh weights and meaſure; with which we muſt reſt contented, til ſome other experiments be produced, both in theſe, and alſo in the other Metals, from

from our Engliſh weight and meaſure, and which we may expect from Mr. *Reynolds* aforeſaid, who hath taken great care and pains, and uſed much induſtry therein.

As for the weights of Metals compared in *S*phears of one and the ſame magnitude, ſet down in the latter part of Mr. *Ponds* Almanack, in Troy-weight (where alſo are noted the foreſaid common proportions between Troy and *Avoirdupois* weights) they are arcording to the experiments of *Gheraldus*, being deduced from his proportional or comparative numbers, into the parts of Troy-weight, though not very preciſely; which therefore I have here put moſt correct and exact thus; ſuppoſing (with him) firſt a Sphear of Gold, to weigh juſt one pound-troy, and from thence, the other metalline Sphears of the ſame magnitude, to weigh accordingly, as followeth.

		oun.	*p.w.*	*gr.*	*mi.*		*gr.*	
1.	G.	12.	00.	00.	00.		5760.	☉. 1
2.	Q-S.	8.	11.	10.	6.	*ferè*	$4114\frac{2}{19}$	☿. 2
3.	I.	7.	5.	6.	6.		$3486\frac{6}{19}$	♄. 3
4.	S.	6.	10.	12.	12.		$3132\frac{12}{19}$	☾. 4
5.	B.	5.	13.	16.	8.		$2728\frac{8}{19}$	♀. 5
6.	I.	5.	1.	1.	5.		$2425\frac{5}{19}$	♂. 6
7.	T.	4.	13.	11.	7.		$2243\frac{7}{19}$	♃. 7

But now to return a little to our foregoing work of the artificiall dimenſion, or diametral and Circumferentiall cubation of a Sphear of caſt-iron, for the weight thereof, which as we ſhewed before by the Integer of weight it ſelf (or by libral weight) *So* we will next ſhew how to perform the ſame by the compoſing, denominate parts thereof immediatly, *viz.* ounces avoirdupoiz (or uncial weight)

And the *Line* or *Scale* of equal parts for this purpoſe, in reſpect both of the foreſaid common Tenent of the weight of an iron-Sphear or bullet, and alſo our own experiment, I finde (according to the reaſon of the precedent ſecond Theoreme, and alſo our generall reaſon) to be for the Diameter of ſuch a Sphear, (as to a centeſimal partition of the meaſure given) 0.76 inch, which is but very little above $\frac{3}{4}$ of an inch: and the Line for the Circumference, according to the common Tenent, to be 2.40 inches *ferè*, and according to our experiment, 2.39 inches *ferè*, (ſo that here alſo one and the ſame Line of meaſure may indifferently ſerve in both) which two Lines being divided as the former, and then the Diameter or Circumference be taken by their proper reſpective *Line* or *Scale*, and cubed, the ſame ſhal be the weight of the Sphear or Bullet in ounces and decimal parts immediatly: For ſo the Diameter of the foreſaid bullet of 9*li.* and 2 oun. *ferè* avoirdupois, (*viz.* 9.123626*li. ferè*) being 4 inches, and the Circumference, 12.57 *ferè*, wil be found each of them, by their proper cubatorie Line or *Scale*, for unciall gravity, (being made 100 parts) to be 5.27 *ferè*, which cubed, yields 146.363183 *ferè*, for the weight of the Bullet in ounces, which exceedeth the true weight being 145.978009 oun. by 0.385174 oun. *ferè*, which by converſion or reduction, yieldeth about 6 drams, and which in a thing of this nature is not conſiderable: But yet if we wil ſtand more preciſely uppon the weight of this Bullet, if then we divide the two foreſaid artificiall Lines of meaſure for this purpoſe into more parts, as 1000 (for the naturall Line, or the Inch being ſo divided, the artificial Line wil be thereof, for the Diameter, 0.760 *ferè*, and for the Circumference, 2.387 *ferè*) & ſo meaſure the Diameter & Circumference of this Sphear or bullet thereby, we ſhal find the ſame

ſame to be ſeverally, 5.265, which cubed, gives 145.946-984625 ounces for the weight of the bullet, which wants of the true weight aforeſaid, hardly half a dram, which in this kind of weight is as near as need be deſired.

And ſo again in the other Bullet of 15*li.* & $12\frac{1}{3}$ oun. avoirdupoiz (or 252.25 ounces) weighed by us; if the Diameter or Circumf. noted formerly, be taken by theſe Lines under a centeſimall diviſion, they wil be found each of them to be 6.32 *ferè*, whoſe Cube is 252.435968 oun. *ferè*, which exceedeth the true weight, only 0.185968 oun. which by converſion, gives near upon 3 drams, and which is not conſiderable: But being meaſured by the ſame *L*ines under a milleſimall partition, they wil be found each of them, 6.318, which cubed, affords, 252.196389, &c. ounces; which now wants of the true weight, not fully one dram being but 0.86 dr.

And ſo alſo if the Diameter or Circumference of this Bullet be taken by their proper reſpective Lines of meaſure for librall weight beforegoing (*viz.* 1.91 inch, for the Diameter, and 6.01 inches, for the Circumference) the ſame wil be found ſeverally (according to a centeſimal partition of the Lines) 2.51 *ferè*, whoſe Cube is 15.813-251 *ferè* for the weight of the bullet, (which by reduction is 15*li.* 13 oun. and 0.19 dr. avoirdupois) exceeding the true librall weight, *viz.* 15.765625*li.* (or 15*li.* and 12.25 oun.) only 0.047626*li. ferè*, which by converſion into the proper parts of this weight, gives about $\frac{3}{4}$ of an ounce, *viz.* 12.19 dr. But being meaſured by the ſaid *L*ines in a milleſimal partition, (for the natural Line, or the *Inch*, being 1000 parts, the artificiall Line wil be thereof for the Diameter, 1.914, and for the Circumference 6.014 *ferè*) they wil be found ſeverally, 2.507, which cubed,

bed, yields 15.756617, &c. (*viz.* 15*li.* 12 *oun.* 1.69 *dr*) wanting now of the true weight, only 2.3 drams.

The artificiall Lines of meaſure, for the moſt ſpeedy diſcovering of the weight of a Spheare or Bullet of caſt-Iron, in Troy-weight, both libraall and unciall.

Now if any ſhall upon occaſion, make uſe of the Troy-weight in a Spheare of this metall (though indeed this kind of weight is not uſuall for any Metall beſides Gold and Silver, as I have noted before) then the artificial *Lines* of meaſure for the ſpeedy diſcovering of the weight thereof by the diameter, wil be 1.794 inch for libraall-weight, and by the Circumference, 5.636 inches: And the *Lines* for unciall weight, wil be for the *Diameter*, 0.784 inch *feré*, and for the Circumference, 2.462 inches.

And now albeit no art or induſtry of man, can make a Spheare of metall or other matter, ſo very exact and preciſe indeed, as it ought to be (according as I noted before from *Ghetaldus*) Yet conſidering that the weight of this and the other coarſer kind of Metals in any thing, is not ſo preciſely ſtood upon as the weight of *Gold* and *Silver*, (being pretious Metals) So the exactneſſe of a Sphericall body made of any of them for ordinary uſe (as a *Cannon-bullet*, which is commonly made of caſt-iron, as I noted before) is not ſo much ſtood upon, ſo as it come ſomthing neer the ſame: and indeed there is hardly any Cannon-bullet, or other bullet for ſhooting, but is ſo round (being caſt in a Mould) as that the Eye can hardly adjudge or diſcerne it to be not exactly orbicular or ſphericall; and ſo the weight thereof can be very litle miſtaken, being obtained by the diameter or Circumference of the ſame, taken in a certaine ſet Meaſure, either naturall, as Inches, or the like; or artificiall, as deduced therefrom, according as I have here ſhewed

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at large. And which ſeverall dimenſions with many other the like metrical Concluſions pertaining hereunto, and the Converſe of the ſame (and all cheifly in reference to Gunnery) I will next expreſſe proportionally by Number, (from our foregoing experiments) according to the aforeſaid Metall and Meaſure commonly uſed in this thing, and the Weight both *Avoirdupoiz* and *Troy*, and in each of theſe, both by the librall and unciall-weight together, which in the ſeverall Sections of proportions following, are (for brevity-ſake) noted by the letters of diſtinction, *l*, and *u*.

1. The Cube of the
- *Diameter*
- *Circumfer.*

in meaſure, is to the Spheare it ſelfe in weight, as 1. to

.1425566.	*l.*	*Avoirdupoiz.*
2.280906.	*u.*	*weight.*
.17314487.	*l.*	*Troy-weight.*
2.0777384.	*u.*	
.00459767.	*l.*	*Avoirdup.*
.07356273.	*u.*	
.0055841877.	*l.*	*Troy.*
.06701025.	*u.*	

Conversly.

2. The Sphear in respect of weight, is to the Cube of its

Diamet.	in respect of measure, as 1. to	7.0147552.	*l.*	*Avoirdup.*
		0.4384222.	*u.*	
		5.7755104.	*l.*	*Troy.*
		0.4812925.	*u.*	
Circumf.		217.50144.	*l.*	*Avoirdup.*
		13.59384.	*u.*	
		179.07707.	*l.*	*Troy.*
		14.923089.	*u.*	

cubique measure.

Hence,

3. The

Diamet.	is to the side of the Cube equall to the Sphear in weight, as 1. to	.522391.	*l.*	*Avoirdup.*
		1.316343.	*u.*	
		.5573609.	*l.*	*Troy.*
		1.276038.	*u.*	
Circumf.		.16628227.	*l.*	*Avoirdu.*
		.41900507.	*u.*	
		.1774135.	*l.*	*Troy.*
		.4061755.	*u.*	

Linear measure.

Then for the speedy discovering of the weight of a spherical or any other body whatsoever made of this Metall, by the solidity thereof, in the Measure aforesaid: and contrariwise

trariwiſe the ſolid content by the weight thereof, both *Avoirdupoiz* and *Troy*, and in each of theſe both librall and unciall weight, the Proportions will be as followeth.

1. *As* 1. to { .27226314. *l.* / 4.3562103. *u.* } *Avoirdupois.* } Weight.
1. *As* 1. to { .33068234. *l.* / 3.968188. *u.* } *Troy.* } Weight.

Contrariwiſe.

2. *As* 1. to { 3.672917. *l.* / 0.229557. *u.* } *Avoirdup.* } Solid meaſure.
2. *As* 1. to { 3.02405. *l.* / 0.25200418. *u.* } *Troy.* } Solid meaſure.

And ſo likewiſe for diſcovering the gravity of any Body of fine or forged *Iron* by the magnitude, in the foreſaid Meaſure, and *contrà*: the proportions will be from the forementioned experiments made upon this Metall, as followeth.

1. *As* 1. to { 0.3474. *l.* 4.169. *u.* } *Troy.* } Gravity.
{ 0.2861. *l.* 4.5777. *u.* } *Avoirdupois.* }

Conversly.

2. *As* 1. to { 2.878, *l.* 0.2399 *ferè u.* } *Troy.* } Magnitude.
{ 3.495. *l.* 0.218. *u.* } *Avoirdupois.* }

And thus having shewed our artificiall Mensuration in regular *Solids*, as well for gravity or ponderosity, as for solid (and superficiall) measure; I shall now close up this *Section* and *Part*, with our third theorematicall Proposition, (answering to the third principall problematical or practicall Proposition in the first Part) and the practicall demonstration thereof, in which our more particular, or speciall reason of the like dimension of all regular-like *Solids* (as particularly of a *Cylinder*) in the severall respects aforesaid (in reference to the producing of the artificiall *Lines* of measure, for performing the same) is contained.

THEO

THEOR. III.

Expreſſing particularly, the artificiall Lines for the ſolid dimenſion of a Cylinder, from our particular or ſpeciall ground and reaſon formerly declared: And conſequently, the Lines for the like dimenſion of a Cone, and all other regular-like Solids in generall.

IF the two proper Dimetients aforenamed, of a (right or erect) Cylinder, exactly adequate to the cubicall, (compoſing or reſolving) Parts of the Rationall *Line*, ſhal be obtained according to an exact congruencie or congruity; The ſame ſhall be the proper, reſpective artificiall Line of Cylindricall ſolidation, according to the Parts: And there wil be of them to the correſpondent or congruall Lines of dimenſion according to the whole Meaſure, the Reaſon that is of the Parts to the *Whole*; which is as the Reaſon of their reſpective Cubes.

THe like reaſon holdeth for the Cylindricall Circumference conjunctly with the *Dimetient* of altitude: and alſo for both theſe in the Cone, in reſpect of its baſiall Dimetient and Periphery with the *Dimetient* of altitude, for ſolid dimenſion, and with its Side for ſuperficiary dimenſion; and alſo for the baſiall Dimetient of a Cylinder with its *Side*, in reſpect of ſuperficiall dimenſion: And ſo the like with theſe, for all Pyramids and Priſmes conſtituted upon regular Baſes, in reſpect both of ſolid and ſuperficiary dimenſion, according to the ſeveral wayes formerly declared and demonſtrated in all theſe Figures: only reſpect being had in all theſe, to the quadrate parts of the

Ri-

Rationall Line for superficiary dimension, as is here to the cubique parts for solid dimension.

And the like reason wil here hold in all these solid Figures, in respect of weight or gravity in any Metall whatsoever, as for their solid measure, according to what I shewed before in the *Spheare*, and consequently in the other re-Bodies, being made of Metall (respect being here had to the parts of the weight proposed, as in the Theoreme it self, is to the parts of measure, as I shewed before in the like case upon the 2d Theorem) so as that the like artificial *Lines* of Measure being extracted for these Figures severally in reference to weight, according to any Metall, Weight, and Measure proposed; their gravities wil be thereby obtained in the same manner as their solid magnitudes or measures, according to the severall wayes formerly declared and demonstrated for the same.

And now becaus of the two several *Dimetients* (or dimensional Lines) here continually concurring, there occurreth some more variety, then in the two preceding Theoremes: Therefore I wil here give a full demonstration or illustration of this, in an Arithmetical manner; by which those two (with all the things necessarily depending on them) as well as this, may be plainly understood, seing they be all grounded upon one and the same reason.

THerefore, let the Rationall Line (in respect of its properly composing denominate parts) be R 12. So the Cube thereof, CR, 1728; to which be conceived or constituted a Cylinder, exactly agreable for magnitude or dimension, & so, as that its two foresaid *Dimetients*, be in exact Congruencie, or Congruity (such as the Greeks (*Euclid* & *Proclus*) cal ἁρμοσις, or ἁρμογὴ; the Latines, (as *Ramus*) *Con-*

Congruentia, or *Congruitas*: & ſo the greek words do generally ſignifie) which therefore we are here next to enquire. Therefore ſeing then that like *Solids* in generall, do mutually hold in a triplicate or cubique reaſon of their homologall Terms by *E* 8. *p.* 19 & 27; and more particularly, *E* 11, *p.* 33. and *E* 12. *p.* 8. and ſo in ſpeciall, Cylinders of their baſial Dimetients (or other like Terms) by *E* 12. *p.* 12. To this I aſſume another Cylinder, whoſe two *Dimetients* are already given in the ſame kind, with thoſe ſuppoſed in the former Cylinder, which (in the firſt Numbers from an unit, abſolute, making the whole Cylinder abſolute in all its dimenſions, according to the more ancient and vulgar Cyclometricall terms (in reſpect of the Baſe) but not the newer) let be (under the generall notion of a *Binomiall* in relation to the Rational Line taken as before) A = D R 12 + 2; whereupon wil ariſe the Cylinder, rationall and abſolute (according to the ſaid vulgar Cyclometrical Terms, in reſpect of the Baſe, and binomially in reference to the Cube of the Rational *Line* taken as before) C R, 1728 + 428; but according to the newer, and moſt approved Tetragoniſmall Terms, in reſpect of the baſe, and firſt in the more vulgar or *Metian* expreſſion noted formerly; for the performing of this operation after the more uſuall or vulgar way of Numeration, (and for that this expreſſion is finite and limitted in it ſelf, whereas the decimal expreſſion runs infinitly) it wil be irrational and inabſolute, C R, 1728 + $427\frac{15}{113}$ (which is by decimal converſion of the fractionall termes, C R, + 427.132743, &c, very nearly agreeing with that which is produced by the proper decimall Cyclometricall termes, in reſpect of the

Eucl. Axiom. com. 8. *Ram. Geom. lib.* 1. *el.* 9. *& Schol. mathemat. lib.* 8. *prop.* 4.

the base of the Cylinder; and that both from themselves simply or naturally, and also artificially, from their Logarithme, as shal be shewed by and by) then by *E.* 12. *p.* 12 aforesaid, and also by the reason of *E.* 7. *p.* 19. it holdeth in a triple or cubicall reason of the aforesaid Cylindrical Dimetients conjunctly, as the Terms homologall, thus;

CR 1728 + 427$\frac{15}{171}$: CR 1728 :: CR 1728 + 1016 (*viz.* CA = D. R 12 + 2 in CR 1728 + 427 $\frac{15}{171}$) : CR 1728 + 472$\frac{3841}{2431}$ or (by their greatest common Divisour, or number of exact commensuration, 686) 472$\frac{56}{347}$ in the least termes of the same reason (*viz.* CA = *D* in CR 1728) which is decimally, CR 1728 + 472.157746, &c. whose Root or Side irrationall or ineffable, (and binomiall in reference to the Rational Line, taken as before, and according to a decimall extraction of the Cubicall *Gnomon*) R 12 + 1.00062, &c. is A = D in CR 1728, being the thing first sought for; or the artificial Line according to the parts of the natural Line of measure proposed: which from the most admirable invention of the Logarithmes, is most speedily performed by a simple Composition and Resolution of Numbers, together with a Trichotomie of the Cubicall Logarithme; the first or greater Cylinder, being raised from the most exact Cyclometricall Logarithme, (in respect of the Base) answering to the most exact decimall Terms aforesaid.

CR

C R 1728 + 427.132560, &c.		3,3334739884
C R 1728	———	3,2375437381
C R 1728 + 1016. *viz.* C A = D, R 12 + 2	———	3,4383841070
Aggreg.	———	6,6759278451
C R 1728 + 472.157933, &c. *viz.* C A = = = D in C R 1728		3,3424538567. *Diſ.*
R 12 + 1.006225, &c. *viz.* A = = = D in C R 1728		1,1141512856. $\frac{1}{3}$ *Vc.*

Or again, ſeing that between two like Solids, there do neceſſarily intercede two mean Proportionals, by *E.* 8, *p.* 12, and 19; the two *Means* between theſe two *Cylinders*, will be found (binomially, in relation and compariſon to the foreſaid Rationall *Cube*, and according to a decimall enumeration of the cubick *Gnomon*) C R 1728 + 274.15289, &c. and C R 1728 + 132. 03231, &c. moſt compendiouſly and exactly, by Logarithmicall Computation, thus;

CR 1728 + 427.13256, &c. 3,3334739884 } S

CR 1728 ——— 3,2375437381 }

Dif. ——— 0,0959302503 } A

$\frac{1}{3}$ *dif.* ——— 0,0319767501 } A

CR 1728 + 132.0323, &c. 3,2695204882 lesser *Mean.*

CR 1728 + 274.152, &c. 3,3014972383 greater *Mean.*

Or the same may be obtained more plainly, (though not so readily) in a dis-junct manner, at two distinct operations, by the Analogie which holds from the Cube of one *Solid* to the Cube of the mean Proportional falling next to it, as doth from that Solid to the other propounded, between which the two mean Proportionals are required.

Then seing the greater *Mean* serveth here our present purpose; it followeth according to *E* 7. *p.* 19 thus; CR 1728 + 427.13256, &c. CR 1728 + 274.15289, &c. : : A═══D, *R* 12 + 2 : A═══D, *R* 12 + 1.0062, &c. as before: which appeareth plainly and briefly, by Logarithmicall or artificiall Numeration, thus;

CR

CR 1728 + 427.13256,&c. 3,3334739884 } S

CR 1728 + 274.15289,&c. 3,3014972383 } A

A==D, *R* 12 + 2 —— 1,1461280357 } A

Sum. —— 4.4476252740 } S

A==D, *R* 12 + 1.0262, &c. 1,1141512856. *Dif.*

Agreeing exactly with the firſt Logarithmicall operation.

Or again, the ſaid A==D in CR 1728 required; found more briefly yet, after this manner.

CR 1728 + 427.13256,&c. 3,3334739884 } S

CR 1728 —— 3,2375437381 } S

Dif. —— 0,0959302503

$\frac{1}{3}$ *dif.* —— 0,0319767501 } S

A==D, *R* 12 +2 given, 1,1461280357 } S

A==D, *R* 12 + 1.00622, &c. inquired. 1,1141512856 *Dif.*

Which being found out theſe three ſeverall waies; it followeth laſtly, according to the Conſectarie of the third Theoreme, in this ſimple Analogie.

R 12 : R 1 :: A ═ D, *R* 12 + 1.0062, &c. A ═ D, *R* 1 + .0838, &c.

Which last found Number, is for the said congruall Dimetients of a Cylinder, according to the unity of measure; and so for the artificial (or second Rational) Line according to the whole intire natural (or prime Rationall) *Line* in generall.

And what manner of working is used here, for the investigation of this artificiall Line in the Cylinder; the same is to be used for the like in the *Cone*: and which (though needlesse) I wil here further illustrate in the same.

Therefore, suppose here the foresaid Rationall *Line* (taken in its composing, denominate parts) R 12. and to its Cube, C R 1728, a (right, or Isoskelan) Cone exactly adequate (as the Cylinder before) having its two like proper *Dimetient*-lines exactly congruall (as those of the Cylinder) in the investigation or inquisition whereof conjunctly, consisteth the first operation. Therefore seing then, that as Cylinders (and other like Solids) so Cones, are respectively one to another, in a triple or cubicall Proportion of their Basiall Dimetients (or other correspondent *Termes* of dimension) by *E* 12. *p.* 12 aforesaid: To this be supposed another like Cone, of given or known Dimetients, and exactly congruall, as the former; which let be the same with those of the foregoing Cylinder, included in the *Binomie*, A ═ *D*, *R* 12 + 2; whereupon this *Cone* wil be subtriple that *Cylinder*, *viz.* according to the vulgar Numeration, from the *Metian* Cyclometricall Terms, as to the composition of the Base (under the common acception of an *Apotome*, in reference to the foregoing Rationall Cube) irrationall or unabsolute, C R 1728 ═ $1009\frac{211}{339}$ (which is by decimall conversion of the fraction-part,

1728

1728——1009.622418, &c. and which very nearly agrees with that which is produced by the moſt exact (decimall) Cyclometrical termes in regard of the Baſe, and that both naturally or ſimply from themſelves, and alſo artificially from their correſpondent Logarithme; being C R 1728——1009.622480. as will appear in the Logarithmicall operation following) Then by *E* 12. *p.* 12, and alſo by the reaſon of *E* 7. *p.* 19 before cited, it holdeth in a triplicate or cubique reaſon of the aforeſaid Conicall *Dimetients* conjunctly, as the Termes homologall, thus;

C R 1728——$1009\frac{211}{339}$: C R 1728 :: C R 1728 + 1016 (*viz.* C A = D, R 12 + 2 in C R 1728——$1009\frac{211}{339}$) : C *R* 1728 + $4872\frac{112148}{243339}$; or rather in the leaſt homologall fractional termes (by their greateſt common Meaſure, or Number of exact Symmetrie, 686) C *R* 1728 + $4872\frac{161}{339}$, *viz.* C A = *D* in C *R* 1728; which by decimall dinumeration, is C *R* 1728 + 4872.473239, &c. whoſe *Root* or ſide cubicall, and irrationall or inexplicable, is (binomially in reference to the foreſaid *Rational* Line, and according to a decimall eradication, or reſolution of the cubique *Gnomon*, *R* 12 + 6.7723, &c. for A = *D* in C *R* 1728, being the thing firſt inquired; or the artificial *Line* of meaſure, according to the parts of the natural Line propoſed: And which may moſt eaſily and readily be obtained by Logarithmicall or artificiall Numeration, by a ſimple Compoſition and *Reſolution*, or Proſthapheretical ſuppputation only; and that the ſeverall waies which I ſhewed before in the *Cylinder*; whereas the working out of theſe things by the natural Numbers (or the vulgar way of Numbring) is moſt laborious, and intricate, by reaſon of the many large and tedious Multiplications, *Diviſions*, and *Radicall* extractions, both quadrate and cubique, ariſing

from; which notwithstanding I have also done both in this of the *Cone*, and that of the *Cylinder*, and likewise in other Figures, where the same might be conveniently performed after this manner, as in the *Circle* and *Sphear*, for the finding out of their respective artificiall Lines of Mensuration before set forth.

But now to the operation of our present question in the *Cone*, by the artificiall or Logarithmicall Logistique; according to a cubicall proportion of the aforesaid homologall Terms, thus;

CR 1728——1009.622480	2,8563527	S
CR 1728 ———	3,2375437	A
CR 1728 + 1016, *viz.* CA=D R 12 + 2 in CR 1728—1009 .6224&c. ———	3,4383841	A
Aggreg. ———	6,6759278	S
CR 1728 + 4872.473798 &c. *viz.* CA=D in CR 1728	—3,8195751.	*Dif.*
R 12 + 6.75822 &c. *viz.* A=D in CR 1728	1,2731917.	⅓ *Vc.*

Which differeth from the number found by the naturall operation, in the decimall fraction, but very little, that giving it .7723 &c. this .7582 &c. which breedeth no sensible

ſenſible difference in the Concluſion of the worke; but however the number produced by the *Logarithmicall* operation is the trueſt; the ſecond aſſumed Cone (though the firſt in place here) being produced from the exacteſt Cyclometricall Logarithme, as to the compoſition of its Baſe, upon which it is raiſed and conſtituted.

Or againe ſecondly, by finding out the two Mean Proportionals between theſe two Cones, which is moſt ſpeedily and accuratly performed alſo by virtue of the *Logarithmes*, as before in the Cylinder; and ſo they will be found (under the two common *Apotomies*, in reference to the Rational Cube, and according to a deeimall *Analyſis* of the cubicall *Gnomon*) C R 1728——765.465285 &c. for the leſſer *Meane*; and 1728——438.3256268 &c. for the greater *Meane*, thus,

C R 1728	1009.62248 &c.	2,8563527	S
C R 1728	——	3,2375437	S
Dif.	——	0,3811910	A.
Subtriple	——	0,1270637	A.
C R 1728	765.46528 &c.	2,9834164, leſſer	A
C R 1728	438.3256 &c.	3,1104801. greater	M

Or the ſame may be found more diſjunctly, according to the Analogy before declared in the Cylinder, for the finding of the two *Means*; the reaſon being the ſame here.

Now the leſſer *Meane* ſerving here our purpoſe; it followeth

loweth thence Analogically thus : C *R* 1728——1009, &c. : C *R* 1728——765 . 465, &c. :: A——*D* in C *R* 1728——1009, &c. *viz.* *R* 12 + 2 : A——*D* in C *R* 1728, *viz.* *R* 12 + 6.7582, &c. as before ; as appeareth briefly and plainly, by this subsequent Logarithmeticall Calculation.

C *R* 1728——1009.6224, &c.	2,8563527		S
C *R* 1728——765.465, &c.	2,9834164	A	
A——*D*, *R* 12 + 2 ——	1,1461280		
Aggreg. ——	4,1295444		
A——*D*, *R* 12 + 6.7852, &c.	1,2731917	*Dif.*	

Exactly agreeing with the first Logarithmicall operation.

Or again thirdly and lastly, the same *Dimetients* in the *Cone*, C *R* 1728, found out conjunctly and congrually, most briefly of all, thus,

C R

CR 1728 —— 1009.6224 &c. 2,8563527 } S

CR 1728 —— 3,2375437 }

Dif. —— 0,3811910

⅓ —— 0,1270637 } A

A——D, *R* 12 + 2 given —— 1,1461280 }

A——D, *R* 12 + 6.7852, inquired 1,2731917. *aggreg.*

Which being had theſe ſeverall waies; it followeth laſtly thereupon, according to the reaſon of the Conſectary of the foregoing Theoreme, by way of Analogy, in theſe Termes; *viz.*,

R 12 : R 1 :: A——D, *R* 12 + 6.7852 &c : A——*D*, *R* 1 + .56318 &c. Or R 12 : R 12 + 6.7852 &c. :: R 1 : R 1 + .5632 *ferè*. Which laſt found Number is for A——D in the *Cone* anſwering to CR 1; and ſo for our artificiall Line of rectangle parallelepipedation (according to an exact quadrate Baſe) or parallelepipedall conſolidation of a *Cone* deſired, in reference to the whole intire natural Line of meaſure in generall.

And after the ſame manner may be found out the like artificiall Lines for the conſolidation of a Cone, and Cylinder, by their baſiall Peripheries and altitudinary Dimetients together, as hath been here done, by their baſiall and altitudinary dimetients together: (which baſiall *D*imetient and Periphery do continue the ſame throughout the *Cylinder*, it being of equall craſſitude, but continually alter in the *Cone*) And theſe Lines we ſhewed before in the practicall

demonſtration and uſe of the other two Lines of ſolidation pertaining to theſe two Bodies, to be for the *Cylinder*, of the Rationall Line, 2 + .3266 &c. and for the *Cone* 3 + .3531 &c. and theſe by the naturall way of working, from the *Metian* Cyclometricall termes, in reſpect of the Baſe of the Cylinder and Cone: and by the artificiall or Logarithmeticall way, I find the ſame to be for the Cylinder 2 + .32489 &c. (very little differing from the other) and for the *Cone*, exactly agreeing with the former: And therefore I ſhall not need (I conceive) to make any more adoe about the demonſtration hereof: And the ſame way holdeth for the extraction of the like artificiall Lines for the Superficiall dimenſion of the *Cylinder* and *Cone*, and alſo for the dimenſion both ſolid and ſuperficiall of all regular-baſed Pyramids and Priſms: And for all theſe Figures, not only in reſpect of ſolid and ſuperficiall meaſure, but alſo of gravity or weight, according to any Metall propoſed; as I have noted before upon the 3d. Theorme.

And all theſe things, together with the like in the other Figures, before-mentioned, may be yet more readily obtained, and that according to our generall ground and reaſon of this artificiall *D*imenſion (or of our artificiall metricall Lines) which was formerly ſaid to conſiſt in unity alone. For as in our particular, partiall, or ſpeciall Reaſon thereof, every particular *Meaſure* propounded (as the prime or naturall *R*ational Line) being conſidered according to its Parts, (compoſing or conſtituting, or otherwiſe dividing) every particular kind of Figure, or figurate Magnitude, meaſurable in this way, was conſequently ſaid to be conſidered in the firſt or ſecond Power of the ſame (as in the powers of *Number*) according to the nature and kind of the Dimenſion; for the producing of the artificiall

Lines,

*L*ines, (according to the three foregoing Theoremes) So here in our generall *R*eaſon, or *R*eaſon of the *Whole*, or of *Unity*; every *Meaſure* being conſidered in it ſelf ſimply and abſolutely, (as was formerly declared) every ſuch figurate Magnitude (as aforeſaid) is to be conſidered accordingly, in the like Powers of Unity (as of the whole intire Meaſure in generall) according to the nature of the Dimenſion. And ſo again, as in the performance of theſe *D*imenſions, by the naturall Meaſure propoſed (or the prime *R*ationall *L*ine in generall) in an Arithmeticall manner, by way of Proportion (according as hath been ſhewed in the ſeverall dimenſions beforegoing, among the dimenſionall Proportions) for the more immediate producing of the ſides of the Squares and Cubes, &c. equal to the Figures propoſed to be meaſured, and which are immediatly given by our artificiall Lines of meaſure; we begin with Unity ſimple, and proceed from that to Number ſimple or linear, being ſome *R*oot of a Figurate Number ſuperficial or ſolid; & ſo to the figurate Number it ſelf; as from ſome dimenſional line of a Figure (by which the ſame is propoſed to be meaſured in a quadratary or cubatory manner, according to the nature of the *D*imenſion; or by which the ſide of the equal Quadrat or Cube of the Figure is to be produced, either naturally in an arithmetical analogicall manner, as aforeſaid; or artificially, by a Line of meaſure convenient for the purpoſe) to the ſide of the equal Quadrat or Cube; and ſo come to the Content of the Figure it ſelf; Or we come to the ſame more immediatly, and alſo moſt naturally and properly; proceeding from *Unity* taken in the power thereof, prime or ſecond, (according to the nature of the dimenſion propounded) to Number figurate, and then from that to the correſpondent *R*oot thereof; as from ſome dimenſional *L*ine of a

Figure, in the aforeſaid powers thereof, to the Figure it ſelf; and then from that to the ſide of the equall Quadrat or Cube. *So* in this (for the immediate producing of the artificiall Lines) we proceed from Unity figurate in the firſt or ſecond power thereof, according to the nature of the *Dimenſion*, to a Number figurate or potentiall, accordingly, and from that to the correſpondent *Root* thereof; as from every kind of Figurate Magnitude in general, falling under our artificiall *Dimenſion*, to the like powers of ſome one of its dimenſional lines by which it is propoſed to be thus meaſured (or ſquared and cubed, or otherwiſe artificially meaſured, according to the nature and kind of the Magnitude propounded, and of the Dimenſion it ſelf) and ſo from thence to the ſaid dimenſionall line it ſelf of the Figure, for the artificiall Line of meaſure required: which as in all exactly ordinate or regular Figures, it is but one ſuch line ſimply; ſo in all regular like-Figures it is of two together according as we have lately ſhewed. And ſo we might hereupon raiſe a 4th. and generall Theoreme, if it were needfull; but that which I have already ſaid concerning the ſame, is very ſufficient.

And what hath bin here ſpoken for ſolid Figures, in reference to their ſolid meaſure, is to be underſtood in the ſame accordingly, for gravity or weight (according to any Metall, or other like ponderous matter propoſed) For as before, according to our particular or ſpeciall reaſon, every Integer of weight whatſoever propoſed, was ſaid to be conſidered in its parts taken ſimply in number; and ſo every kind of metalline, or other like ponderous (regular and regular-like) Body, was accordingly to be conſidered in the ſame, as in the like parts of Unity: So here the Integer of weight propounded, is to be conſidered ſimply abſo-

absolutely and intirely, as in the nature and reason of unity it self; and so every kind of metalline (or other like ponderous) regular and regular-like body, is to be considered accordingly, for the producing of the artificiall *Lines* of measure therefrom, for the speedy discovering of the gravity or ponderosity of any such Body proposed, in any magnitude whatsoever.

PART

PART III.

Containing that kind of Dimension, or metricall practice, which is commonly called, the Gauging of Vessells; after a most artificiall, exact, and expeditionall manner.

Sect. I.

Concerning the measuring or gauging of Vessells, in generall.

ANd what hath been here done for the solid Dimension of a *Cylinder* (or the dimension of a solid Cylinder) by artificiall Lines found out for the same, according to any Measure appointed: the like may be done for the liquid dimension of a concave Cylinder (or Cylindricall Vessell) for the more speedy finding out of the liquid Content, according to any Liquour and Measure given. As our Vessells for *Wine* and *Ale*, or *Beer* (which two, most com-

commonly come to be meaſured) and other liquid things, which though they be not abſolute Cylinders in themſelves, yet may be (and commonly are) conceived or ſuppoſed as Cylinders, (being reduceable by Art thereunto) for the better meaſuring of the ſame, by finding out and taking the *Meane* between the Diameter at the Head, and the Bung of the Veſſell (or otherwiſe the Meane between thoſe two Circles) and ſo obtaining the liquid Content thereof; which is commonly called *Gauging* of Veſſells; the Meaſure by which theſe Veſſells are thus valued or eſtimated, being uſually a *Gallon*, and which is the greateſt of our liquid Meaſures, and but the beginning, as it were, of Veſſell-Meaſure; but in *Wine*, and *Ale* or *Beere*, holdeth not one and the ſame; but differeth very ſenſibly, as is commonly known: And therefore we will next briefly ſhew the meaſuring or gauging of theſe Cylindricall Veſſells (or Spheroidall, as ſome will have it, this kind of Veſſel being more commonly taken for a *Sphæroides*, having the two ends equally cut off; though for mine own part, I conceive this kind of Veſſel may more properly be termed a *Cylindroides*, by the ſame reaſon that a *Sphæroides* and *Conoides* are ſo called; this having the ſame ſimilitude or reſemblance to an exact Cylinder, that thoſe have to an exact *Spheare* and *Cone*) by Lines of meaſure peculiarly appropriated and applied to them (as we did before in the *Cylinder* in general for ſolid meaſure) according to the different kind and quality of the Liquour, and ſo the different quantity or magnitude of the liquid Meaſure given, in relation to ſolid Meaſure in inches.

Sect.

SECT. II.

Setting forth the Quantities of the Wine and Ale-gallons, in reference to the gauging of Veſſels.

IT is generally holden by Artiſts about the City of *London*, that a *Wine-Gallon* containeth in its concave Capacity, 231 cubicall or ſolid inches, or is inſenſibly differing therefrom: But for the *Ale* or *Beer-Gallon*, I finde the ſame to be as generally controverted among them. Mr. *William Oughtred*, a reverend Divine, and moſt eminent Mathematician, before-named, after ſome experiments made by him to find out the ſolid content of this *Gallon* in inches, beſides the experiments of ſome others which came to his ſight, finding ſome difficulty therein, in regard both of the irregularity which he obſerved to be uſually in the ſeverall Standard-Gallons which he met with, and alſo their diſagreement one from another in their Contents, as himſelf confeſſeth and declareth in his foreſaid book of the Circles of Proportion, *Part* 1. *chap.* 9. looketh back there to the firſt ground and principle of our Engliſh Meaſuring from Barley-corns: and ſo at length he commeth to a rationall conjecture of the *Ale-gallon* (and that very neatly, and prety nearly alſo, as I ſhal ſtraightway ſhew) in cubique inches, according

ding to the number of the ſquare-parts or Feet in the common Statute-Perteh or Pole, *viz.* $272\frac{1}{4}$, as you may ſee in the place fore-cited.

But Mr. *John Reynolds* aforenamed alſo, (who ſeemeth to have been as induſtrious in this, as in many other mathematicall experiments) wil have this *Gallon* to contain $288\frac{1}{4}$ cubique inches; holding that the Wine-gallon (which he ſtrongly affirmeth to be 231 inches) is to the Ale-gallon in ſuch proportion preciſely, as 4 to 5; or rather for the reduction of Wine-meaſure to Ale-meaſure, as 5 to 4; which is according to Mr. *John Goodwyn* long agoe, in his little Tract entituled, *A Table of gauging*, publiſhed above 50 years ſince, and dedicated to the then Lord Major and Aldermen of the City of *London*: wherein he ſhewing how to reduce Wine-meaſure into Ale-meaſure, *& contrà*; ſaith, that 5 gallons of wine-meaſure make but 4 gallons of Ale-meaſure: with which very nearly agrees the opinion (not certain experiment perhaps) of ſome others, who wil make this Gallon to contain juſt 288 inches upon this ground, that a cube-Foot ſhould hold in its concave capacity juſt 6 Ale-gallons, and ſo conſequently one ale-gallon muſt contain juſt 288 inches, which that learned gentleman Mr. *Edm. Wingate*, a Barreſter of *Grayes-Inne* (a man eminent for his mathematicall abilities) firſt declared to me by word of mouth, and ſoon after I found the ſame noted in his book of the uſe of his *Rule* of *Proportion*, *chap.* 10. And theſe two jump ſo nearly together, as if one were borrowed from the other: but I declaring this to Mr. *Reynolds* at my firſt ſeing of him, he ſaid that he had not obſerved this thing.

This Mr. *Goodwyn* was Maſter in the Mathematicks to Mr. *Reynolds*, as himſelf hath told me.

Now as for Mr. *Oughtred's* Ale-gallon of 272¼ inches, the ſaid Mr. *Reynolds* indeed alloweth of ſuch a Gallon-meaſure, but not for any liquid thing, but for drie things, as Corne, Coals, Salt, and other dry things meaſurable by this kind of Meaſure; and ſo calleth it the drie Gallon-meaſure: And thereupon he wil have to be 3 ſeverall Gallons (or other like Meaſures) one for *Wines*, (which alſo ſerveth for oiles, ſtrong-waters and the like) Another for *Ale* and *Beer*, and a third for *Corne*, *Coales*, and the like; and this he maketh leſſer then the Ale-meaſure, whereas ſurely it ſhould rather be greater, if there be any difference at all between them: And theſe three ſeverall Gallon-meaſures, he compareth together, or differenceth by theſe three Numbers, *viz.* 28, 33, 35. as to ſhew their proportions one to another: *viz.* the *Wine-Gallon* (231) to the dry Gallon-meaſure (272.25) as 28 to 33, which is ſo in the leaſt terms, rationall or abſolute; but otherwiſe in the leaſt proportionall termes, irrationall or unabſolute, and finite or limited, I find them to be as 7 to 8.25. And the ſaid *Wine-Gallon* to his *Ale-Gallon* (288.75) as 28 to 35, which in the leaſt rationall termes, is indeed as 4 to 5; but otherwiſe in the leaſt termes irrationall (but finite or limitted) I find them to be as 1. to 1.25. and which again is integrally rationall, 100 to 125: And the dry-Meaſure to the Ale-meaſure, as 33 to 35, which cannot be abbreviated in terms rational.

And ſurely, evil Cuſtome ſeemeth to have brought up three ſuch diſtinct meaſures (and which the foreſaid Mr. *Wingate* hath alſo expreſſed to me) For at the *Guild-Hall* in *London*, where is generally holden to be the true Standard for theſe Meaſures, and ſo from which all others of the like kind throughout the Kingdome, are uſually derived

ved, there are but two ſuch diſtinct Meaſures only (as we have been there informed for a certain truth) *viz.* one for *Wines* (and ſo for ſtrong-waters, oiles, and the like) and the other for *Ale*, *Beer*, and drie things, as Corn, Coales, Salt, and the like; which latter is commonly called the *Wincheſter Meaſure*, and from this are taken the bigger drie Meaſures, as the *half-Peck* and *Peck*, and ſo on to the *Buſhell*, which is the greateſt of our drie Meaſures: Which ſaid Standard-Meaſures at the *Guild-hall*, the foreſaid Mr. *Reynolds* confeſſed to me (going to him on purpoſe to receive ſome ſatisfaction from him about the Wine and Ale or Beer-meaſures (which was in *June* 1646, and then he gave me in writing under his hand, the ſolid content of the Ale-gallon to be 288¾ inches, and ſo its Proportion to the Wine-gallon, to be exactly as 5 to 4 (or for the reduction of Ale-meaſure to wine-meaſure, as 4 to 5) that he had never made any triall of them, (neither could I find that any other had, or if they had, it was ſurely to ſmall purpoſe) but only of thoſe Meaſures in the Tower of *London* (which he pleads for to be the moſt ancient and true ſtandard Meaſures) and at *Cowpers-Hall*, and ſome other ſuch places, which ſeem to be but ſome particular Cuſtomary meaſures, differing from the generally received Standard-meaſures at the *Guild-hall*.

And therefore to be fully ſatisfied in this point, concerning the true *Wine*, and Ale or Beer Meaſures, according to the common Standards, (and more eſpecially about the Ale or Beer-meaſure, finding ſuch a diverſity of opinions concerning the ſame, and in ſo vaſt a difference, as that between *Mr. Oughtred's* and *Mr. Reynold's* Ale-gallon, being 16½ cube-inches) My ſelf and one *Mr. Baptiſt Sutton*, (a man well known in the City among artiſts)

did agree to go together to the *Guild-hall*, where he was wel acquainted with the keeper of the Standard-meaſures and Weights, who otherwiſe I found to be very nice and ſcrupulous in ſhewing of them; and for our further ſatisfaction herein, we made known our intention to the foreſaid *Mr. Wingate*, who much approved of the ſame, expreſſing his deſire alſo of it: And ſo *Auguſt* 9th. 1645; we repaired together to the *Guild-hall*, carrying along with us two large ſquare glaſſe-vials, which we firſt weighed in a Gold-ſmiths Ballance by Troy-weights, (as being the beſt) which were ſupooſed to be exact enough; and afterwards filling the two braſſe-ſtandard-Gallons for *Wine*, and *Ale* or *Beer*, with fair water from the Ciſterne, and that with all poſſible preciſeneſſe, we powred forth the ſame with the like accurateneſſe, into the ſaid two glaſſe vials, and then weighed the Glaſſes with the water in them by the ſame weights: and ſo comparing the weight of each Glaſſe alone, with the weight of the glaſſe and water together, we found the Wine-Gallon of water to weigh $117\frac{1}{4}$ ounces-Troy, and the Ale-gallon of water $140\frac{9}{16}$ ounces, (which laſt, according to the common diviſion of the Ounce-troy by penny-weights, is 140 ounces, 11 penny-weights, and a quarter) which do hold in proportion (from the leſſer to the greater) as 10000 11937, which comes very near, as 5 to 6. Then ſeing that Weight and ſolid meaſure do hold in proportion one to another, ſo as that one may be deduced from the other, as I have ſhewed before; if we compare theſe two Gallon-weights of water, with ſeverall experiments made by my ſelf, and *Mr. Reynolds*, ſeverally (and conferred together) for the finding out of the true weight of water in relation to its ſolid meaſure in inches, (or for the comparing of its gravity

vity and magnitude together, which thing is moſt admirable and excellent uſe, as I ſhall ſhew more afterwards) we ſhal thereby diſcover the ſolid capacity of the ſaid two Gallon-veſſels in inch-meaſure; which is the very groundwork of Gauging.

Now as to the foreſaid experiments; the ſaid *Mr. Reynolds* did firſt (amongſt other things to this purpoſe) cauſe a Veſſel to be made of Wood, by an exact * Workman, in the forme of an oblong rectangle *Parallelepipedum*, (or long Cube as ſome term it, though improperly, as they call an oblong rectangle *Parallelogram* a long *Square*) whoſe Baſe was $4\frac{1}{2}$ inches ſquare, and the heighth, depth, or length, (which you will) 14 inches, and ſo the ſolid capacity thereof, $283\frac{1}{4}$ inches; and which was cloſed up at both the ends or baſes, ſaving that in the middle of one end, was made an hole for the powring in of water, and which was no bigger, then that he might guelle in the filling thereof to a drop or two of water, more or leſſe: which Veſſell therefore being preciſely filled with fair ſetled Rain-water (as being the fitteſt, as I ſhal ſhew afterwards) and then as preciſely weighed by Troy-weights, he found the water thereof alone to weigh 12*li.* and $5\frac{1}{4}$ oun. or $149\frac{1}{4}$ ounces troy. And he not being contented with this own experiment, he cauſed ſuch another Veſſel to be made, every way like and equall in its dimenſions with the former, and that by the ſame Workman; which he filling with the like water, found it to agree in weight exactly with the former. But yet he not reſting fully ſatisfied with theſe two experiments, he procured ſuch another Veſſel to be made, by another Workman, of the very ſame Dimenſions with the former; which he fil-

* *Mr. John Thompſon* in Hoſiar Lane.

ling with the like water as aforesaid, found the weight of the said water alone to be 12*li.* and very near 6 oun. Troy, (or $12\frac{1}{2}$ pounds-Troy *ferè*) exceeding the former weight about $\frac{3}{4}$ of an ounce, and which he conceived to be the truer, (notwithstanding the exact agreement between the two former experiments) by comparing these experiments with some other of the like kind, which had been made before by himself, or some other body; And this difference of weight seemeth to proceed chiefly from some difference of measure in the Inch, by which the two first Vessels, and the last were made, being done by two several Workmen. And therefore (considering the difficulty in a work of this nature, in respect of the nicety and curiosity of the experiment) he comparing these with some observations which I had then made by the bye, to this purpose; we concluded together at length, that the neareſt and indifferenteſt weight of the water exactly filling up the foresaid Vessel of $283\frac{1}{2}$ cubick or solid inches, would be 12*li.* and $5\frac{1}{2}$ oun. Troy (or $149\frac{1}{2}$ ounces) and this to stand good.

And then after this, I got an exact cubical Vessel to be made of throughly seasoned wood, with all the accuratneſſe & preciſeneſſe that could be, being 6 inches the in-ſide, (or the baſe thereof exactly 6 inches square) and so the whole Cube in its concave capacity, exactly 216 inches; and which then, to keep it from sucking in water in any part, or any water to soak into it, was well primed all within, with a thin oile-colour (yet of a sufficient body) having afterwards a Cover put on it, with a little hole in the middle thereof, about $\frac{1}{4}$ of an inch wide, as the foresaid Veſſels of Mr. *Reynolds* had: And which cubicall Vessell I then filling with all the exactneſſe and preciſeneſſe that might

might be, with fair ſetled Rain-water, at *Gold-ſmiths-Hall*; and ſo having the ſame as exactly weighed by the Standard Troy-weights; I found the weight of the water alone (deducting the weight of the empty *Veſſel* it ſelfe firſt of all had, from the weight of the veſſel and water together) to be upon the very point of 114 ounces, or 9 *li.* & $\frac{1}{2}$. without any conſiderable difference therefrom: and thus I found it to be, at two ſeveral trials. Now according to the two firſt obſervations of *Mr. Reynolds*, aforeſaid; 216 inches of the forenamed water, ſhould weigh 113.7 oun. Troy, *viz.* 113 oun. and 14 p.w. which comes ſhort of our obſervation, by about a quarter of an ounce; and according to his laſt obſervation the ſame ſhould weigh 114.3 oun. *ferè*, *viz.* 114 ounces, and neer upon 6 p.w. which exceeds the weight found by us, juſt ſo much as the other wants of the ſame; So that the weight of this cubical body of water produced by our experiment, falleth directly in the middle between the ſeverall weights of the ſame deduced from his foreſaid experiments, upon one and the ſame kind of veſſel. And according to this our moſt exact obſervation; the weight of 283.5 cubick or ſolid inches of the foreſaid water, (being the content of each of *Mr. Reynolds* his three foreſaid veſſels) will be 149.625 ounces troy exactly, which is 149 oun. 12 p.w. and an half. And this is the very arithmetical *Mean* beween his two firſt obſervations, agreeing one with another, being 149.25 oun. and his laſt, being 150 oun.

And ſo now according to this experimental Concluſion of mine own; I ſhall proceed exactly in the ſubſequent operations upon the Wine and Ale-Gallons: For ſo the weights of the two ſeverall Gallons of water aforeſaid, being compared ſeverally with this experiment, the ſolid ca-

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pacity of the Wine-gallon, will be found 223.105 inches, and of the Ale or Beer-gallon, 266.329 inches.

But I not resting fully satisfied with this one experiment in the said standard-Gallons (though we conceived the same to be performed with as much care and diligence as might be) and so desirous to trie the same thing over again, to see how nearly two severall trials would agree to confirm the matter; knowing that two testimonies upon any thing are much better then one; I again moved the said *M. Wingate* and *Mr. Sutton* (whom I still desired as witnesses to what was done) for another triall of this thing, and that divers times; but could not accomplish my desires herein, til about two yeares after: And so in *July* 1647, I and *Mr. Sutton* went together again to the *Guild-hall* (*Mr. Wingate* having promised to go along with us, but was hindred by other occasions) carrying along with us two other great glasse-Vials like the former, into which first powring the foresaid Standard-gallons of water exactly filled, and then weighing the said Glasses with the water in them severally, by the great standard-Ballance there, with Avoirdupois-weights, and afterwards the empty Glasses severally (being wel dried first) by the same weighes, and so comparing them together as before; we found the weight of the Wine-gallon of water alone, to be 8*li.* 1 oun. 3 dr. avoirdupois (or $129\frac{3}{16}$ ounces avoirdupois) and of the Ale-gallon of water, to be 9*li.* 9 oun. 12 dr. (or $153\frac{3}{4}$ ounces avoirdupois) which are in Troy-weight (according to the most exact Proportions of the Avoirdupois weight to the Troy-weight before noted, *viz.* 14 to 17. and 56 to 51) 9*li.* 9 oun. and 5.223 dr. or 13 p.w (which is 117.65 oun. troy)

2d. experiment upon the wine & ale-gallons.

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the Wine-gallon ; wanting of the first observation or experiment (*viz.* 9*li.* and 9.75 oun. or 117.75 oun.) only 0.1 oun. troy, which is 2 p.w. or $\frac{4}{5}$ of a dram-troy, which difference is of no moment. And for the Ale-gallon, 11*li.* 8 oun. and $\frac{4}{25}$ of a dram-troy, or $\frac{25}{76}$ of a p.w, *viz.* 10 $\frac{5}{7}$ gr. (which is 140$\frac{5}{234}$ oun. or 140.0223 oun. troy) wanting also of the first experiment (*viz.* 11*li.* 8 oun. and 11$\frac{1}{4}$ p.w. or 4$\frac{1}{25}$ dr. or 140,$\frac{9}{16}$ oun. or 140.5625 oun. troy, (about half an ounce-troy, and which difference is of small moment in the matter of gauging; but yet this latter experiment is the truer, as more nearly agreeing with the other experiments and observations following.

Now these two Gallons of water in this second experiment, are in proportion (from the lesser to the greater) as 100000 to 119013, which comes neer the former proportion: And being compared with the foresaid experimentall Conclusion made by me, (for the weight of water in reference to solid inch-measure) will give the solid content of the Wine-gallon, 222. 9 inches, and of the Ale-gallon, 265.3 inches; which wanteth of the former solid measure in the Wine-gallon, not fully $\frac{1}{7}$ of an inch, and in the Ale-gallon, about 1 inch. Which difference between these two experiments, especially in the Ale-gallon, though in the matter of Gauging the same can breed no sensible errour or difference, as will afterwards plainly appear, when we come to shew our gauging-Lines: yet for my further and fuller satisfaction in this nice and curious piece of art, so much handled and controverted by Artists, as I said before; and that I might come as neat the matter as possibly might be; I urged again for another triall: And thereupon in *November* next following, I again repaired to the *Guildhall*, carrying with me two other large glasse-vials, (differing

3d. experiment upon the Wine & Ale-gallons.

much in form from the other before uſed, though indeed this be nothing materiall to the purpoſe,) which I firſt cauſed to be weighed ſeverally by the ſtandard-*avoirdupois* weights there; and then (with the help of the keeper of the Standards) filled both the ſtandard-Gallons with fair water from the Ciſtern, with all the accuratneſſe that might be, as before; and which with the like accurateneſſe being poured out into the ſaid two Glaſſes: I cauſed the Glaſſes with the water in them, to be weighed ſeverally by the ſame weights and *Ballance*, and that as exactly as might be, and thereupon found (by the Compariſon aforeſaid) the Wine-Gallon of water to weigh alone, 8*li.* 1 oun. 11 dr. avoirdupois (or 129 $\frac{11}{16}$ oun. avoirdup.) and the Ale-gallon of water, to weigh 9*li.* 9 oun. 15 dr. avoirdupois, (or 153 $\frac{15}{16}$ ounces) which do exceed the ſecond obſervation, in the weight of the wine-Gallon, 8 dr. or half an ounce avoirdupois, and in the weight of the Ale-gallon, only 3 dr. or $\frac{3}{16}$ of an ounce avoirdup. And theſe two laſt Gallon-weights of water, do hold in proportion (from the leſſe to the greater) as 100000 to 118699 *ferè*, which is very little leſſe then the former proportions. And theſe alſo being collated with our foreſaid experiment of 216 cube-inches of water, to weigh 114 oun. troy (or 9*li.* and an half) which is in Avoirdupois-weight according to the neareſt proportions of the Troy-weight to the Avoirdupois, before declared and demonſtrated, 125 $\frac{9}{17}$ oun. (or 7 $\frac{14}{17}$*li.*) do give the ſolid capacity of the Wine-Gallon, 223.784 inches, and of the Ale-gallon 265.629 inches; which do exceed the firſt experimentall obſervation, in the Wine-gallon, by 0.679 inch only; & the ſecond, by 0.86 inch; and doth want of the firſt obſervation in the Ale-gallon, only 0.7 inch; and exceeds the ſecond, by 0.3 inch only; which laſt is very little.

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But I being desirous to be further satisfied in the weight of these two last Gallons of water: So soon as I had performed the same at the *Guild-hall* by the Avoirdupoiz weights there; I caused the said Glasse-*Vials* with the Gallons of water in them, to be streightway carried unto *Gold-smiths-Hall*, to be tried by the great *Standard*-Ballance of Troy-weight there (as being the most exact kind of weight) where first weighing each Glasse together with its water, and afterwards each Glasse alone (being throughly drie) I found (by comparing the one with the other as before) the weight of the Wine-gallon of water alone, to be 118$\frac{1}{16}$ oun. troy, *viz.* 118 oun. and $\frac{1}{2}$ dr. or 1$\frac{1}{4}$ p.w. (which make *9li.* 10 oun. 0.5 dr. or 1.25 p.w.) and the weight of the Ale-gallon of water alone, to be 140 oun. and 4$\frac{1}{4}$ p.w. or 1.7 dr. (which is *11li.* 8 oun. and 4.25 p.w. or 1.7 dr. Troy) And these two Gallon-weights do hold in proportion (from the lesse to the greater) as 100000 to 118761, which exceeds that which was produced by the avoirdupois-weight, (*viz.* 118699 *ferè*) by 62 parts of 100000. And being conferred with our foresaid experiment for finding the proportion between the ponderall and dimensionall quantity of water, or its gravity and magnitude; will give the solid content of the Wine-gallon, 223.697 inches, and of the Ale or Beer-gallon, 265.666 inches: which exceeds the first experiment, in the wine-gallon, by 0.59 inch; and the second by 0.776 inch; and wants of the third experiment in the same, from the avoirdupois-weight by the ballance, only 0.087 inch. And it wants of the first observation in the Ale-gallon, 0.66. inch; and exceeds the second by 0.36 inch only; and this third by avoirdupois-weight from the ballance, only by 0.04 inch *ferè*, which is as much as nothing.

The foresaid severall Proportions between the *Troy* and *Avoirdupois* Weights, cõpared again together, & examined by the Ballance upon another Experiment.

And here having a fit occasion and opportunity, I shal (by way of *Digression*) speak somwhat more concerning the Proportions between the Troy and Avoirdupois weights, for a further confirmation & verification of what was said formerly, and also demonstrated upon a Cannon-bullet concerning the same: And therefore the weight of these two last Gallons of water, taken first by Avoirdupois-weight at the *Guild-hall*, (*viz.* 8*li.* 1 oun. 11 dr. or $129\frac{11}{16}$ oun. the Wine-gallon, and 9*li.* 9 oun. 15 dr. or $153\frac{15}{16}$ oun. the Ale-gallon) being converted into Troy-weight, and that first, by the more common termes of proportion, *viz.* 60 to 73 for pound-weight, or 80 to 73 for ounce-weight, wil give the Wine-gallon, 9*li.* 10 oun. 6 p.w. and 19.125 grains exactly, or 118 oun. 6 p.w. and 19.125 gr. or 118 oun. 2 dr. and 43.125 gr. troy: & the Ale-gallon, 11*li.* 8 oun. 9 p.w. 8.62 gr. or 140 ounces, 9 p.w. 8.62 gr. or 140 oun. 3 dr. & 44.62 gr. troy, which do exceed the true weight taken from the Ballance at *Goldsmiths-hall*, by 5 p.w. and 13.125 gr. or 2 dr. and 13.125 gr. in the Wine-gallon, and by 5 p.w. and 2.62 gr. or 2 dr. and 2.62 gr. in the Ale-gallon. And then by the other terms of proportion, *viz.* 14 to 17 for libra ll weight, or 56 to 51, for uncial-weight, the Wine-gallon wil be 9*li.* 10 oun. 2 p.w. and 3.96 gr. or 118 oun. 2 p.w. and 3.96 gr. or 118 oun. 0.5 dr. and 21.96 gr. troy: and the Ale-gallon, 11*li.* 8 oun. 3 p.w. and 20.7 gr. or 140 oun. 3 p.w. and 20.7 gr. or 140 oun. 1 dr. and 32.7 gr. troy; which differ from the true weight of the Ballance, (by way of excesse) in the Wine-gallon, but 21.96 gr. or 22 gr. *ferè*; and (by way of defect) in the Ale-gallon, only about 9 gr.

which

which differences are very inconſiderable, the greateſt of them not amounting to a penny-weight, whereas the leaſt of the differences produced by the vulgar proportions, is above a quarter of an ounce-troy. And hereby it is again moſt manifeſt, that theſe latter Proportions between theſe two kinds of weight, are much the truer, and ſurely the neareſt and trueſt that may be found, and are therefore generally to be received.

And theſe our experiments beforegoing for the diſcovering of the ſolid Contents of the foreſaid Wine and Ale-gallons at the *Guild-hall*, may be confirmed by ſome other experiments which I afterwards made upon the ſame. For before my ſecond obſervation of this thing, by the weight of the Gallons of water, I cauſed a concave *Cube* to be made of Braſſe, of 4 inches the Side exactly, and ſo the whole *Cube*, 64 inches; into which (being ſet level) my ſelfe, and Mr. *Sutton* aforenamed, together pouring out the two Gallons of water in the Glaſſes which we had from the Standard-Veſſels at the *Guild-hall*, as aforeſaid, with all the accurateneſſe and preciſeneſſe that might be (at ſeverall times) we found firſt the Wine-gallon to fill the *Cube* three times, and then halfe the *Cube* (as neerly as we could poſſibly meaſure it) which being computed, doe make 224 cube-inches exactly, for the ſolid content of the Wine-gallon; And then we found the Ale-gallon to fill the *Cube* 4 times, and moreover to ariſe to ſuch an height of the ſaid *Cube*, (*viz.* 0.7 inch) as being computed, did make 11.2 cube-inches; all which together do give 267.2 cube-inches for the ſolid capacity of the Ale-gallon. Both which do ſo neerly agree with the former experiments (eſpecially in the Wine-gallon) as that

4th. experiment upon the Wine and Ale-gallon.

this experiment may ſufficiently confirme the former; this being as plaine and demonſtrative an experiment as may be. And if it had been performed by a cubicall *Veſſell* ſo large, as might have received into its concave capacity each of the Gallons of water wholly at once (which indeed I afterwards wiſhed had been done, but my foremen-tioned Cubicall Veſſell of wood was made long after the triall of this Experiment) then the ſame might probably have yet come neerer the truth; for that the often filling of this ſmall *Cube*, might cauſe ſome ſmall errour, which by a more large or capacious Veſſel, might have been avoided, wherein the ſolid content of each *Gallon* might have been had at once, by the heighth (or depth) of its water; though indeed the difference between this which we tried, and ſome of our former experiments (in the Wine-gallon) is in a manner inſenſible, and between all of them, in reſpect of both Gallons, is altogether inconſiderable. And this experiment by the braſs *Cube*, I afterwards tried again by my ſelf, and found it to differ as much as nothing from the former. But indeed to have the ſolid Contents of the Wine and Ale-gallons ſo very exactly and preciſely by their liquid Contents, as can be imagined according to the ſtrictneſſe of Art, is (I may ſay) impoſſible, unleſſe the Standard-Veſſels were ſo narrow-mouthed, as that in the filling thereof, one might be able to gueſſe at a few drops of water, which in the Standard *Veſſels* at the *Guild-hall* (and I thinke in all others) cannot be done, they being ſo wide at the mouthes or tops, as that a ſpoonfull of water more or leſſe in the filling thereof is hardly diſcernable, or ſo much more, as might breed the difference of halfe an ounce more or leſſe, in the weight of the water, and ſo conſequently of one inch more or leſſe, in the ſolid content; ſeeing that one inch cubick or ſolid of water weigheth

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(as wee ſhall ſhew anon) halfe an ounce-troy, at leaſt: And yet in all theſe ſeverall experiments and obſervations compared together, the greateſt difference of ſolid meaſure is but about one inch and an halfe, and that in the Ale-gallon: but yet ſetting aſide the experiment made by the braſs *Cube*, (which is too large;) the greateſt difference in the ſame wil be but one inch ſolid, which wil breed no ſenſible errour in the matter of gauging, as I ſaid before. And which may ſhew how very near the matter we have come, for the diſcovering of the true Contents of the common ſtandard-Gallons for *Wine* and *Ale* or *Beer*, and ſurely as near the truth as can wel be gone.

But yet that there might no likely way be left unattempted, for the diſcovery of this thing, I wil add to the former, one experiment more, being very demonſtrative, which I made laſt of all in the foreſaid braſſe ſtandard-*Ale-gallon* at the *Guild-hall*, by taking the proper linear *Dimenſions* thereof, it being indeed an exact ſegment of a *Cone* (or Calathoidall) the internall or concave ſuperficies from the top to the bottome, being very ſtraight and ſmooth, aſwell as the externall or convexe Superficies, and alſo exactly circular throughout; only a little ſhelving or arching at its meeting or connexion with the bottome, and this not preciſely plain, but rather a little hollowiſh, yet not ſo much as to make any ſenſible errour in giving the ſolid capacity of the Veſſel, as wil ſtraightway appear, by comparing the ſame with the obſervations before-going, and alſo one other obſervation following after. Which ſtandard-gallon Veſſel, as it is ſomewhat like in form to one of thoſe which Mr. *Oughtred* ſpeaketh of in the fore-

5th. experiment in the Ale-gallon alone, by taking it's Dimenſions.

cited

Circles of Proportion. *Part* 1 *chap.* 9. *Sect.* 4.

cited place of his *Circles of Proportion*, concerning Gauging (as being ſhewed unto him, and alſo the meaſures thereof firſt given him, by that great Antiquary Mr. *William Twine* of *Oxford*, whom he ſaith to have undergone great pains and charge in finding out the true Contents of our Engliſh Meaſures (and whom I wel knew at *Oxford*, being of the ſame Houſe with him) So alſo it comes very near it in all its dimenſions: For by the Diameters of the top and bottome, and the height of that Veſſell which they together meaſured, (and which you may ſee in the fore-mentioned book) they found, that the ſame would contain in its concave capacity, 268.85 cubick inches, which exceedeth the meaſure of our Gallon-Veſſel, produced by the experiment of the braſſe Cube aforeſaid, but little more then one inch and a half; and the meaſure deduced from the firſt experiment by weight, about two inches and an half; and that which was deduced from the ſecond experiment, by weight, (being the leaſt of all) by about three inches and an half: But indeed, beſide that the ſides of that ſtandard Ale-gallon were a little arching (as Mr. *Oughtred* ſaith) he obſerved divers other irregularities in the ſaid Veſſell, which might wel hinder the diſcovering of the true Content thereof, by ſome few inches.

Now the linear dimenſions of the Standard *Ale-Gallon* at the *Guild-hall* (which I took as exactly as I could, and I believe inſenſibly differing from the truth) I found to exceed thoſe of Mr. *Twine's* Veſſell (taken both by him and Mr. *Oughtred* together, as I noted before) in the Diameter of the top, but 0.33 inch; and in the Diameter of the bottome, but 0,1 inch; and to want of that in the perpendi-

pendicular height (or the depth) 0.8 inch; by which I find the ſolid dimenſion of this Conicall Veſſel (or of this decurtate or detruncate Cone) to be 265.5 inches, according to a multiple or conjunct compoſition of the aggregate of the two Baſes, and their meane Proportionall (produced moſt exactly by Logarithmicall ſupputation) with a trient of the Altitude, thus,

greater Baſe,	36.637960	A	Square inches.
leſſer Baſe,	22.902210		
meane Proport.	28.967055		
Aggreg.	88.507225	M.	
⅓ *altitude,*	3		
Solidity.	265.521675.		cube-inches.

Which comes very neer the ſolid Content of this Veſſel produced by all our former experiments; eſpecially the ſecond and third, from which it differs as much as nothing; the one of them, giving 265.3 inches, and the other 265.6 inches.

And all theſe our experiments in the *Ale-gallon*, may yet be further confirmed by another obſervation or experiment, being taken from the *half-Peck*, wch they hold at the *Guild-hall*, to be equall in Content to the Ale-gallon, as being taken therefrom (and ſo I find it to be) and which I ſhall

6th. Experiment in the Ale-gallon, taken from the half-Peck: and that from the Buſhell.

deduce from the *Buſhell*, according to the Dimenſions thereof, eſtabliſhed by an Act of the *common-Councell* of the City of *London*, and yearly publiſhed by authority of the *Lord Major*, which ordains the breadth or wideneſſe of a Buſhell to be 19 inches, and the depth $7\frac{1}{2}$ inches; which being caſt up according to a Cylindrical dimenſion, wil be fonnd to contain in in its concave capacity, 2126.5 cubick or ſolid inches, whoſe $\frac{1}{8}$ for the half-Peck, is 265.8 inches, from which the ſolid Meaſure of the Ale-gallon, found by moſt of our former experiments, doth inſenſibly differ in a manner, eſpecially that of the third experiment (by weight both avoirdupoiz and Troy, from the ballance) which gave the ſame, 265.6 inches; or ſomewhat nearlier, that from Troy-weight, 265.7 *ferè*. And if we ſhal mediate between that of the firſt obſervation, being the greateſt (except that of the braſſe *Cube*, which is too large) *viz.* 266.3 inches, and that of the ſecond, being the leaſt, *viz.* 265.3 inches, (as is uſually done in ſuch like caſes, where ſeverall obſervations or experiments made upon one and the ſame thing, do a little differ, and as they for the moſt part will, let all the art and induſtry be uſed that may be) the Mean between them, wil be juſt 265.8 inches, for the ſolid capacity of the Ale-gallon, exactly agreeing with that of the half-Peck.

And nearly agreing with the foreſaid Buſhell, I found the Content of a ſtandard-Buſhell of *Queen Elizabeth*, which I was informed to be in the hands of the City-Founder, which was made of Braſſe, in the laſt year but one of her reign *viz.* *Anno* 1601 having her Inſcription or Title about it; and ſomwhat reſembling a ſegment of a Cone, it being wider at the top then at the bottome, whoſe dimenſions there fore I took with all the accuratneſſe that might

might be, and found the *Diameter* of the top (or upper baſe) to be one foot, and 7.5 inches, or 19.5 inches: the Diameter of the bottom (or lower baſe) one Foot and 5 inches, or 17 inches: and the depth (or height) 8 inches (or more accuratly, 8.1 inches) which being caſt up, according as the *Ale-gallon* beforegoing; the ſolid capacity thereof wil be found, 2122.165 inches; thus,

greater Baſe,	298.6476	Square inches.
leſſer Baſe.	226.9801	
mean Proport.	260.3595	
Sum.	785.9872	
$\frac{1}{3}$ *altit.*	2.7	
	55019104	
	15719744	
Solidity,	2122.16544	Cube-inches.

Which wanteth of the London-Buſhell, (being 2126.465 inches) 4.3 inches: But indeed this Buſhel was made a litle turning or winding outwards near the edg or top therof, (wch was made very ſharp or thin) which might give ſo much more in the ſolid content thereof, as to make it equall with the City-buſhell; and ſo I ſuppoſe this London-buſhell was firſt intended to be made equall with that of Queen *Elizabeth*, as nearly as might be; And indeed the true Diameter of the top (from edge to edge) I found to be 19.6 inches, according to which if the ſolid capacity of the

Buſhell ſhould be computed, the ſame would be found, 2134 inches, but that is too large: and the other comes nearer the truth: And the 8th. part of this laſt, for the half-peck, and ſo for the Ale-gallon, wil be but $266\frac{3}{4}$ inches.

There came into the hands of the City-Founder, together with the foreſaid ſtandard-Buſhell of Queen *Elizabeth*, a ſtandard Ale-gallon of the ſame Queen, made of Braſſe in the very ſame Year of her Raigne, having alſo her Epigraph about it: & which, I went with an intent chiefly to have meaſured; but indeed before my comming thither, it was ſold away to a Town in Yorkſhire called *Whitby*: But the ſame having been compared with the ſtandard Ale-gallon at the *Guild-hall*, by the meaſure of water; was found to agree therewith, without any conſiderable difference, both as one of the Founders men, and alſo the under-keeper of the Standards for the City, told me, who were preſent at the triall thereof: which may be a means to confirm that at the *Guild-hall*: And theſe two ſtandard Gallons, as they were both of a like capacity, ſo alſo of a like form.

Another meaſure of an Ale-gallon, Mr. *Oughtred* makes mention of in the place fore-cited, as being preſented to him alſo by the foreſaid Mr. *Twine*, it being (as he calleth it) a Standard-Gallon of Queen *Elizabeth*, which the ſaid Mr. *Twine* had tried by another *Veſſell* made of braſſe, in manner of a *Parallelepipedum*, whoſe baſe was exactly ſix inches ſquare, and the *Sides* divided into inches, and twentieth parts: into which he pouring out the ſaid ſtandard-Gallon filled with water, found it to ariſe unto ſuch an height therein (*viz.* 7.6 inches) as being computed, would give 273.6 cubique inches for the ſolid content of the ſaid Ale-gallon. And hereabout Mr. *Oughtred* conceives might be the true Content of the other Ale-gallon meaſured by him and Mr. *Twine*.

All

All which experiments and obſervations aforegoing, (with ſome others made by Mr. *Oughtred*) may ſufficiently demonſtrate, the true *Ale-gallon* not to be of ſo large a capacity, as to contain 288 cubick inches, and upwards, as Mr. *Reynolds*, (and ſome others) will have it.

As for the diſcovering of the ſolid meaſure of the Standard *Wine-Gallon* at the *Guild hall*, in a geometricall manner, by the Linear dimenſions thereof, as before of the *Ale-gallon*; the truth is, I attempted not the ſame, in regard of the irregularity which I found in that Veſſel, by the much arching or curvity of its Sides (whereby it is much like to that *Ale-gallon* which Mr. *Oughtred* and Mr. *Twine* thus meaſured; only this is wider at the bottome then at the top, whereas that was wider at the top then at the bottome, as the *Ale-gallon* at the *Guild hall*) which makes it to differ from a ſegment of an exact *Cone*, and ſo may rather be taken for a ſegment of a *Conoid*.

But having by our two laſt experiments (eſpecially the 5th.) moſt plainly and manifeſtly diſcovered in a geometricall manner, the dimenſionall quantity of the *Ale-gallon* in inch-meaſure, as neerly as may be (which two moſt plain and demonſtrative experiments do not conſiderably differ, the difference between them being but .3 of an inch-cubique) and then by the 2d. and 3d. experiments, the ponderall quantity thereof, in reſpect of the weight of the water exactly filling the ſame, (which two experiments alſo do not conſiderably differ; the difference being but 3 dr. avoirdupois, as we there ſhewed, or about $1\frac{1}{3}$ dr. troy, being neer $\frac{1}{6}$ of an ounce-troy) eſpecially the 3d. experiment (which I take to be the moſt exact for this *Gallon*, in this reſpect) we may thereby (collating them together) be able to diſcover that which was experimented by

me, from my large cubicall Veſſell of 216 inches: and alſo confirme the ſame; and ſo conſequently likewiſe confirme the ſolid Contents of the *Wine* and *Ale-gallon*, firſt of all diſcovered or produced thereby: For the ſolid meaſure of this *Ale-gallon*, being found moſt plainly and demonſtratively by our two laſt obſervations or experiments, to be very nigh 266 inches; and the weight of the water exactly contained in it, found moſt neerly (by the 3d. experiment) to be (in Troy-weight) 11 *li.* 8 oun. and neer about 4 p.w. or 140.2 ounces: the weight of the water exactly contained by our foreſaid Cubicall Veſſel, will be found thereby (according to geometricall proportionality) 113.86 oun. which wants of that found by us, only 0.14 oun. which is but about 3. p.w. And ſo likewiſe may the ſame be very neerly confirmed by the *Wine-Gallon*, according to our 4th. and moſt plain demonſtrative experiment for the ſame (by the braſſe concave Cube) by which the ſolid meaſure thereof being found 224 inches; and by the three former experiments (conferred together) the weight of the water contained exactly in the ſaid Gallon-veſſel, very neer about 9 *li.* and 10 oun. Troy, or 118 oun. the weight of the water exactly contained in the foreſaid large cubicall Veſſel of 216 inches, will be found thereby (according to the foreſaid proportionality) very nearly as before, *viz.* 113.78 oun. which wants of the true weight found by us, but 0.22 oun. and which is only about 4 p.w. or $\frac{1}{7}$ of an ounce-troy.

Moreover for a further confirmation of this thing, and that very nearly, I ſhall produce another experiment, which I made upon the foreſaid braſſe-Cubicall Veſſel of 64 inches: For ſo ſoon as I and Mr. *Sutton* aforenamed, had meaſured out the Standard *Wine* and *Ale-Gallons* thereby,

as

as aforeſaid; we firſt weighed the ſaid concave *Cube* by a ſmall Ballance of his, with Troy-weights, and then again filled the ſame with fair water, as preciſely as poſſibly we could; which we alſo weighing together, found the additional or differentiall weight (take it which way you will) to be (as nearly as we could poſſibly gueſſe) 33.5 ounces, for the water alone; And afterwards I being deſirous to make a further and exacter triall of this experiment, I carried this *Cube* unto *Goldſmiths-hall*, where I procured the ſame to be weighed firſt alone, by the exact Standard Troy-weights, with all the accuratneſſe that might be; and then filling the *Cube* with fair ſetled water, as preciſely as poſſibly I could, had the ſame weighed together, with the like accuratneſſe; and ſo by collating theſe two ſeverall weights together, found the weight of the 64 inches of water alone, to be exactly in a manner as before, *viz.* 33.5 oun. (or but ſome few grains over) and this the *Aſſay-maſter* judged to be the neareſt weight thereof that could be given, conſidering the wideneſſe of the Veſſel, to be ſuch, (and that fully open on one ſide) as that ſome few drops of water more or leſſe in the filling thereof, were not very diſcernable. Now according to our former experiments, the weight of this cubicall body of water, wil be found, 33.7 oun. troy; (or rather 33.8 *fere*) which comes prety near the other: but indeed this laſt being deduced from our experiments made by Veſſels of a much greater capacity (or by a greater quantity of water) muſt needs be the trueſt; ſo that the other is ſomewhat wanting of the true weight. And theſe our experimentall Concluſions upon the weight of water in reference to its ſolid meaſure; or the comparing of its quantity ponderall with its quantity dimenſionall, I ſhall alſo afterwards

wards as neatly confirm by a manifeſt experiment made laſt of all by me, upon a ſolid body of a known magnitude in inch-meaſure, in reſpect of its gravity taken both in the aire and in the water, and the ſame compared together.

SECT. III.

Containing the practice of Gauging, according to our artificiall way of meaſuring: together with the naturall Dimenſion (by way of compariſon) for a confirmation of the ſame.

HAving by the ſeverall waies and means before-going, diſcovered the neareſt quantities of the common ſtandard-Gallons for *Wine* and *Ale* or *Beer*, in ſolid inches; and that of the *Wine-Gallon*, to be at the moſt but 224 inches, and of the *Ale* or *Beer-gallon*, but 266 (which two are in proportion very nearly as 5 to 6; being as 5 to $5\frac{15}{16}$) We ſhall now proceed to the practice of Gauging it ſelf; or the diſcovering of the liquid Contents of *Veſſels* for *Wine* and *Ale* or *Beer* in Gallon-meaſure; and that in the moſt eaſy and ſpeedy manner that may be, according to our artificiall way of meaſu-

meaſuring, being here, as the ſolid dimenſion of a *Cylinder*, before declared and demonſtrated, (as I have already noted) by the like artificiall Lines of meaſure: though indeed the reaſon thereof cannot be exactly referred to the 3d. Theoreme before-going, which is for Cylindricall Dimenſion in generall; nor yet to our other, or generall Reaſon of meaſure, fully expreſſed ſoon after that; according as the prime Integer of *Veſſel-meaſure (a Gallon)* is here conſidered; to wit, in the nature of a ſolid (Cylindricall) body, expreſſed in ſome certain ſolid meaſure, according to the capacity thereof: whereas elſe the ſame being conſidered either ſimply, abſolutely, and intirely in it ſelf, as ſuch a liquid *Meaſure*, or inats Parts compounding, &c. (as being by themſelves apart take for other leſſer or inferior liquid Meaſures) like as the Integer of meaſure (or of Weight) in generall, was formerly; then wil the Reaſon of this Dimenſion (as in reference to the quantities of meaſure in the artificiall ganging-Lines) be the ſame with that of Meaſure (or Weight) in generall; eſpecially according to our firſt or generall acception or conſideration thereof; but wil exactly agree with that of weight, in both the reſpects or Reaſons aforeſaid. And thus the quantity of the artificiall ganging-Line for Wine-meaſure (according to the Gallon of 224 inches) wil be 6.58 inches; and for Ale or Beer-meaſure, (according to the Gallon of 266 inches) 6.97 inches. But yet conſidering the loſſe that happens in Wine by the dregs or lees thereof, and much more in Ale and Beer, by the frothing and otherwiſe; therefore we have thought it meet and convenient, to allow one inch more at leaſt in the meaſure of the Wine-gallon, and 4 inches in the Meaſure of the Ale or Beer-gallon; and ſo make the Wine-gallon to be 225 inches, and the Ale-gallon

270 inches; which we conceive to be the moſt neer and indifferent meaſures (in a generall reſpect,) for theſe two Gallons, that may be: And theſe two do hold in proportion, exactly as 5 to 6: And in this proportion we might ſuppoſe they were firſt intended, conſidering how very neer the ſame, the true Contents thereof do come: And therefore according to theſe two meaſures, we ſhall proceed in the worke of Gauging. And the length of our artificiall gauging-Line ſerving hereunto, will be for Wine-meaſure, 6.59 inches, and for Ale or Beere-meaſure, 7.00 inches: which being divided decimally (as all the former Lines) into 10, or rather 100 parts, or more, (and ſo turned over ſeverall times upon a Rod or Rular of a convenient length) and then the *Diameters* and lengths of Veſſels taken thereby; their Contents ſhall be obtained in Gallon-meaſure immediatly, as the ſolid Contents of Cylinders were formerly had, according to any Meaſure appointed.

But ſeeing theſe *Veſſels* are not exactly Cylindricall (as I ſaid at firſt) but Cylindroidall (or as others will have them, Spherordall) and ſo the Diameter altereth between the middle and the end of them, or the bung-hole, and the head; Therefore muſt a way be firſt had for the finding out of ſuch a mean *Diameter*, as may reduce this unequall Body into an abſolute Cylinder, or ſo neer the ſame as poſſibly may be; that ſo the true Contents thereof may be had, or very neer the ſame: By which mean Diameter, muſt not here be underſtood a *Meane* either arithmetical or geometricall, but a third ſort of *Meane* differing from them both: the diſcovering of which, is the firſt and main thing in the practiſe of Gauging, next after the diſcovering of the Contents of the *Wine* and *Ale-gallon* in ſolid inch-meaſure. And which we ſhall ſhew, moſt eaſily and

ſpeedily

ſpeedily how to performe, by this firſt Rule next enſuing.

RULE I.

How to find the meane or equated Diameter of any Veſſell.

Augment the difference of the two Diameters, by .7, and add the Product to the Diameter at the *Head*: the aggregrate ſhall be the meane or equated Diameter.

Example.

Diameter at the	*Bung*,	32.95	*dif.* 6.59	inches.
	Head,	26.36	.7	
		4.613.	*Product.*	
Equated Diameter.		30.973.		

But Mr. *Oughtred* neglecting, and indeed wholly rejecting a meane Diameter, hath regard to the meane Circle (as it were) between that at the *Bung*, and that at the *Head*, being compoſed of $\frac{2}{3}$ of the Circle at the Bung, and $\frac{1}{3}$ of the Circle at the Head, and thereby reduceth the *Veſſell* to a Cylinder; and which two portions of thoſe two Circles, he ſheweth how to find moſt readily, by this twofold Analogie, *viz.*

See *Circ. of proport. part.* 1. *chap.* 9.

1. *As* 1. *is to* 0.5236: *So is the Square of the Diameter at the Bung, to* $\frac{2}{3}$ *of the Circle at the Bung.*

2. *As* 1. *to* 0.2618: *So the Quadrat of the Diameter at the Head, to* $\frac{1}{3}$ *of the Circle at the Head.*

Or, for the more exactneſſe in working, theſe Proportions might better be ſomewhat extended in the Terms; And ſo the firſt of them, will be, as 1. to .523599 *ferè*: And the ſecond, as 1. to .261799.

And ſo accordingly in this our Example, the Square of the Diameter at the Bung, being 1085.7025; the $\frac{2}{3}$ of the Circle at the Bung, wil be 568.47, &c: and the Square of the Diameter at the Head, being 694.8496; the $\frac{1}{3}$ of the Circle at the Head, wil be 181.91, &c. and which wil alſo be had, by finding the whole Circles at the Bung and Head, according to their Diameters: For the whole Circle at the Bung, wil be found, 852.7087, whoſe $\frac{2}{3}$ is 568.47251 and the whole Circle of the Head, wil be found 545.7336, whoſe $\frac{1}{3}$ is 181.9112 as before: which two portions or ſections of theſe two Circles being added together, do make, as it were, the mean Circle, between them, 750.3837 which muſt ſtand for the equated Baſe, as it were, of the Cylindrical Veſſel, ſuppoſed to be reduced by this means to an abſolute Cylinder: the Diameter of which Circle therefore, may not unfitly be called the mean or equated Diameter; and which wil be found to differ but very little from ours, it being 30.910 *ferè*, and ours is 30.973, and ſo the difference only about 0.06 inch, which in my ſlender judgment is ſcarcely conſiderable: And therefore I ſee no

juſt

just cause why Mr. *Oughtred* should so much inveigh against Mr. *Gunter*, for his mean Diameter, as he doth, in the forecited place of his *Circles of Proportion*: though I must needs confesse this way proposed by Mr. *Oughtred*, for finding a mean Circle, is the most rationall and demonstrative. And which way of Dolial Dimension, is (amongst others) laid down as the most facile, though a little in another manner, by that great Geometrician, of famous memory, Mr. *Henry Briggs*, in his *Arithmetica Logarithmica*, (first written and published by him in Latine) *cap.* 24. *viz.* by finding the Circle at the Bung (or of the middle crassitude of the Vessell, as he cals it) and at the Head, by the Diameters thereof; & so raising frō them and the heighth, (or length) of the Vessel, two exact Cylinders, and then taking the difference between them, whose $\frac{2}{3}$ being added to the lesser Cylinder; or the $\frac{1}{3}$ subtracted from the greater; shall give the solid capacity of the Vessell; as I shall shew by and by, among the other operations of this kind.

Now the arithmeticall *Mean* between the Diameter at the Bung, and the Head, in this our example, is 29.655. and the geometricall *Mean* (or the mean Proportionall) is 29.47, which is lesse then the other, and that is lesse then the aforesaid mean Diameter.

Having thus shewed most briefly how to find out the mean or equated Diameter; I shall next shew as briefly how to find out the Content of any Vessell in Gallons, either of Wine or Ale and Beer, according to the common or naturall measure by Inches, and that by a twofold Analogisme, in reference to the two aforesaid Gallon-measures.

Rule II.

How to find the Contens of any Veſſel in Gallons.

As 286.5 *ferè* (if Wine-meaſure) or 343.8 *ferè* (if Ale or Beer-meaſure) to the length of the Veſſell in inches: So is the Square of the equated Diameter, to the Content in Gallons.

The like Analogiſme for Wine-meaſure, according to the Gallon of 231 inches, will be by the Number 294: and for Ale-meaſure, according to the Gallon of 272¼ inches, by the number 346.6. And here from our Gallon-meaſures, the gauge point (according to M. *Gunter*) would be for Wine-meaſure, 16.93 inches; and for Ale-meaſure, 18.54 inches: But with theſe points we meddle not.

The reaſon of theſe two Numbers, or of this Analogiſme, is deduced from that of a Circle to its circumſcribing or *Diametral* Quadrat, which is vulgarly, as 11 to 14 (but more accurately, 1000000 to 1273239, or in the neareſt rationall termes, according to *Metius*, as 223 to 284; but more truly, according to his Cycloperimetricall termes, it will be in termes irrationall, as 88.75 to 113) Wherefore if the cubick inches contained in a Gallon, be augmented by the conſequent terme, and the product be reſolved by the antecedent, the Quotient ſhall yield the firſt analogiſmall terme.

Now for a triall of this Rule: Suppoſe a Veſſell to be in length, 39.54 inches; the *Diameters* at the Bung and Head as before: the Square of the mean or equated *Diameter* (found by our way) is 959.326729, which multiplied in the length, gives 37931.77886, &c. cubique or ſolid

ſolid inches, which divided by 286.5 (if Wine-meaſure) giveth in the Quotient, 132.40688, &c. gallons of Wine: or being divided by 343.8, (if Ale or *Beer*-meaſure) giveth 110.339 gallons of Ale or Beer. For the mean or equated *Diameter*, being by us, 30.973 inches, the Circle anſwering thereto (for the equated baſe of the Veſſell) is 753.45345 ſquare or ſuperficiall inches, which multiplied into the length, gives the ſolid content of the *Veſſell* (as if it were a juſt Cylinder) 29791.54945, &c. cubique or ſolid inches; which being divided by 225 (if Wine-meaſure) gives in the Quotient, 132.4068, &c. gallons of wine, as before; or by 270 (if Ale-meaſure) gives 110.339 gallons of Ale or Beer, as before alſo. But now according to Mr. *Oughtred*, the mean Circle between that at the Bung & that at the Head (for the equated baſe of the Veſſel to be reduced to a Cylinder) being but 750.3837 inches, the ſame multiplied by the length of the *Veſſell*, will produce the ſolid Content thereof, but 29670.17149 inches. And which will alſo be produced by Mr. *Briggs* his way before declared: For ſo the Circle at the Bung, or middle of the *Veſſell* (as the greateſt Circle of the *Veſſel*) *viz.* 852.7087, &c. inches, being multiplied by the length of the *Veſſell*, 39.54 inches, will produce the ſolid content of the greater of the two Cylinders before-mentioned, 33716.10396, &c. inches. And the Circle of the Head, or end of the *Veſſel* (as the leaſt Circle of the *Veſſel*) *viz.* 545.7335, &c. inches, being multiplied by the ſame length of the *Veſſell*, will produce the ſolidity of the leſſer Cylinder aforeſaid, 21578.30654 &c. inches. All which are moſt accurately and eaſily produced by the Logarithmical Logiſtique, and not otherwiſe; or elſe not without a great deale of pains and trouble of Calculation; and either of which

here

here to ſhew, is altogether needleſſe and ſuperfluous. Now the difference of theſe two Cylinders, is 12137.79742, &c. inches, whoſe $\frac{1}{3}$ *viz.* 4045.93247, &c. being deducted out of the greater Cylinder, or $\frac{2}{3}$ thereof, *viz.* 8091.86494, &c. being added to the leſſer Cylinder; there will either of theſe wayes, reſult 29670.17149 inches, for the ſolid capacity of the *Veſſel*, as before: and which you may ſee performed all together, in this ſubſequent operation;

greater Cylinder,	33716.10396	S.
leſſer Cylinder.	21578.30654	S.
dif.	12137.79742	S.
$\frac{1}{3}$.	4045.93247	A.
Veſſell it ſelf.	29670.17149	A.
$\frac{2}{3}$ *dif.*	8091.86495 *ferè.*	A.
Veſſel it ſelfe,	29670.17149, as before.	

Which divided by 225, (for Wine-meaſure) affoordeth 131.8674, &c. gallons of Wine: or by 270 (for Ale-meaſure) yieldeth 109.8895 gallons of Ale or Beer, which our accompt exceedeth in the Wine-meaſure, about 0.5395 gallon, which is ſomewhat more then half a gallon: and in the Ale-meaſure, 0.4495 gallon, which by converſion or reduction into the proper parts of liquid Meaſure, is about 3 pintes, and an half, and which differences in ſo big a *Veſſel*, are ſcarcely conſiderable.

But now to apply theſe naturall operations in veſſel-meaſure, to our artificiall way of Gauging, that thereby the uſe

uſe of our gauging-Lines may be the better underſtood, and the verity thereof demonſtrated; I wil lay down the ſeverall dimenſions of the fore-ſuppoſed *Veſſell*, according to the ſame, delivering firſt a brief Rule for the uſe thereof, which is thus;

Firſt, take the diameter at the Bung and Head of the *Veſſel*, by theſe Lines, and thereby get the mean Diameter as before is ſhewed: Then, multiply the Square thereof by the length of the *Veſſell*, taken by the ſame Lines; and the Product ſhall be the Content in Gallons.

1 Example, in *Wine-meaſure*.

Diameter at the	*Bung*, 5.00	*dif.* 1.00	
	Head, 4.00	.7	
		0.70	*Product.*
Mean *Diameter.*		4.7 | 0	
		4.7 | 0	
		329	
		188	
Quadrat.		22.09 | 00	
Length of the *Veſſel.*		6 | 00	
Content in Gallons,		132.54 | 00	*Product.*

Which fraction above 132 gallons, gives ſomewhat more then halfe a gallon, *viz.* 4.32 pintes exceeding our former meaſure of 131.40688 gallons, by about one pinte: and the meaſure of 131.867 gallons, (deduced from the

 ſolid

ſolid content of the Veſſell in inches, found by Mr. *Oughtred's*, and Mr. *Brigg's* way;) by ſomewhat more then 5 pintes. But indeed the Diameters and the length of this *Veſſel*, here ſet down from the artificial gauging-Line, are a little larger then what they really ſhould be; according to the quantities of the ſaid dimenſional lines of the *Veſſel*, laid down before from the natural or vulgar Meaſure, being arithmetically compared with the ſaid artificial *Line* taken in its true quantity from the naturall Meaſure, according to a more ample or numerous partition thereof: and ſo they give the liquid content of the *Veſſel* in gallon-meaſure, ſomewhat more then otherwiſe they would. But theſe falling ſo very neerly upon whole parts, or Integers of the artificiall gauging-Line; therefore for more plainneſs and readineſs of working, I thought good to expreſſe them artificially in whole Numbers only.

And the liquid content of this Veſſel, would be according to the ſolid content of the Wine Gallon, commonly taken to be * 231 inches; but 128.4423, gallons; which is 128 gallons, and ſomewhat more then $3\frac{1}{2}$ pintes: and which wanteth of the true meaſure, (being 131.8674 gal. according to the foreſaid Standard-Gallon of 225 inches) 3.425 gal. which by reduction of the parts, is 3 gal. and $3\frac{2}{7}$ pintes.

* The artificiall gauging-Line will be according to this Gallon, 6.65 inches.

a Exam-

2 Example, in *Ale* or *Beere-meaſure.*

Diameter at the { *Bung,* 4.7 } { *dif.* 0.9
{ *Head,* 3.8 *ferè* } .7
0.63 *Product.*

Equated Diameter. 4.43 *ferè.*

Square. 19.6249 *ferè.*
Length. 5.6

1177494
981245

Content in Gallons. 109.89944 *ferè.*

Which parts above 109 gallons, do yield 7 pintes, and neer upon $\frac{1}{7}$ of a pinte: and which doth want of our former meaſure of 110.339 gallons, about 3 pintes and an halfe; but exceedeth the meaſure of 109.8895 gallons (deduced from the foreſaid ſolid content of the *Veſſel* found by Mr. *Brigg's* and Mr. *Oughtred's* way) only about 0.08 pinte, which is altogether inconſiderable. But yet the more parts theſe Lines are divided into, the more exactly ſtill will be produced thereby, the content of any *Veſſel* in gallon-meaſure, according to what was ſaid and demonſtrated in all the other *Dimenſions* beforegoing, upon the ſeverall artificiall Lines of meaſure: So that this kind of Gauging-Line, is as exact as any whatſoever.

And as I formerly ſhewed in the ſolid dimenſion of a Cylinder, how the ſame might be performed artificially, not

by the *Diameter* & the height (or length) thereof together, but alſo by the Circumference with the height: So here in like manner, may the Content of any Veſſell be had in Gallon-meaſure, not only by the mean *Diameter* with the length thereof; but alſo by finding a mean Circumference between that at the Bung and that at the Head; whoſe Square being multiplied in the length of the Veſſel, ſhall give the liquid capacity thereof immediatly in gallons: And the quantity of the artificiall Line of meaſure, ſerving hereunto, will be for Wine-meaſure (according to to our foreſaid Gallon) 14.14 inches, and for Ale or Beer-meaſure, 15.03 inches. But becauſe the uſuall way, by the *Diameters*, is the eaſieſt and readieſt, and alſo that the Circumferences of the in-ſides or Cõcaves of *Veſſels* (eſpecially at the middle or bung) can hardly be taken; therefore I will uſe no more words about this thing.

But here it may perhaps be expected, that I ſhould ſhew the ready manner of taking the *Diameters* of *Veſſels* at the Bung and Head, and their lengths, by our gauging-Lines or Rods; but this being a thing eaſily underſtood by the pregnant Practitioner, and the ſame alſo fully ſhewed in books particularly for Gauging, long ſince publiſhed; I ſhall paſſe them by, having ſaid ſufficient for the practice of Gauging; it being not mine intent and purpoſe, here to ſet down every particular Circumſtance pertaining to Gauging; but briefly to ſhew the making and uſe of theſe new artificiall gauing-*Lines* or Scales, and that according to our new experiments and obſervations for the meaſure of *Wine* and *Ale*, or *Beer*.

A-TACTO-

A-TACTOMETRIA.

OR

An Appendix, *for the dimenſion,*

(or the diſcovering of the ſolid quantity or capacity,) of any irregular kind of Body whatſoever; after the moſt exquiſite manner that may be.

HAving now fully finiſhed the Dimenſion of regular and, regular-like Magnitudes in generall, according to our artificiall way of meaſuring, (or more artificiall kinde of practicall Geometry) at firſt propoſed; together with the natural or vulgar, by way of metricall compariſon: and of the Solids both regular and regular-like, aſwell in reſpect of gravity, or quantity ponderal (according to any Metall

aſſigned) as of magnitude, or quantity menſurall: And then in the laſt Part, the like dimenſion of that irregular kind of concave Body, (or Veſſell) which is chiefly uſed for the containing or keeping of our moſt common potable Liquours, *viz.* Wine, Ale, and Beer; I ſhall now at laſt (by way of an *Appendix*) add ſomewhat, for the moſt eaſie and exact diſcovering or obtaining of the ſolid capacity, or dimenſionall quantity, of any other kind of Body whatſoever, how irregular ſoever it be; whoſe *D*imenſions can in no wiſe be taken by a Line of meaſure (eſpecially ſo as it be but of a reaſonable bigneſſe) and ſo whoſe ſolid content cannot be obtained in any certain Meaſure propoſed, after the manner of geometricall Menſuration. And this I ſhall ſhew from our forementioned experiments made upon the moſt common liquid Body, for the comparing of its gravity and magnitude, or ponderall and dimenſionall quantity together; drawing firſt from thence ſome apt and brief Notes or Concluſions, for the more eaſie and ready performing of this kind of *D*imenſion. And ſo from thence we gather, the neareſt and trueſt weight of one Inch cubique or ſolid of clear or fair ſetled water in generall, to be 0.527 oun. troy, or rather, 0.528 *ferè*, being 0.52777, &c. which is but little more then halfe an ounce, it being (by converſion or reduction into the proper parts of this weight) 4 dr. or 10 penny-weight and 13 gr. preciſely in a manner. And yet afterwards, for further curioſity in this thing, I cauſed to be made of braſſe, a concave *Cube* of one inch, open on one ſide, but having a lid or Cover made very exact, to clap cloſe on it, with an hole in the middle thereof, no bigger, then to receive through it a ſmall drop of water, that ſo I might be able to fill the ſame preciſely to the leaſt drop: (but indeed this

this *Cube* was made too ſcanty, and ſo wanted of it's true magnitude, as I found by comparing it with other exact cube-inches; and it being filled with fair water, and weighed by a very curious Ballance at *Goldſmiths-hall*, as preciſely as might be; the weight of the water alone, was but half an ounce troy, & about a gr. over, and this I tried twice over: though yet for all this, I wel knew that the true weight of one inch of water (or of any other humid or liquid body) could not be ſo exactly deduced from a Veſſel or Body of one inch only, as from a Veſſel or body containing many inches; in regard of the exceeding nicety and curioſity in the making of ſo ſmall a Veſſel, and then in the filling & weighing of the ſame; over there is in a larger Veſſel: & for that in the filling of ſo ſmal a *V*eſſel, the leaſt drop of liquour more or leſs, is of ſome moment, wheras in a large *V*eſſel, a few drops more or leſſe are not conſiderable, ſo as to make any ſenſible or conſiderable difference in the weight of one inch being deduced therefrom. (Nor is our foreſaid cubical *V*eſſel of 64 inches, ſo very ſufficient for this purpoſe, as to ground ſo nice & curious an experimentall Concluſion thereupon.) And therefore we muſt proceed herein, *à majori ad minus*, from a greater quantity to a leſſe, both for the diſcovering of the weight by the meaſure (as before) and alſo of the Meaſure by the weight: and not *à minori ad majus*, from a leſſe to a greater; for that wil not hold good in this thing. But indeed in a ſolid Body, of what matter ſoever it be, the thing wil hold juſt contrary to the former, in both the foreſaid reſpects: For herein, we muſt rather go *à minori ad majus*, then otherwiſe: And ſo the weight of one inch cubick or ſolid, may more exactly be had from a Body of one inch cubick only (being exactly made, and which I find is hard to do) then

from

from a Body of many inches; and conſequently, the gravity or ponderoſity of any bigger body (of the ſame matter) may be had moſt exactly and ſpeedily therefrom: For thus *Gheraldus* in his experiments before-mentioned, for the comparing of ſundry Metals together, in gravity and magnitude, and ſo to determine exact proportions between them; began upon ſmal quantities, and from thence deduceth greater quantities: And ſo in particular, for finding the gravities of ſpherical Metalline bodies by their Diametrall magnitudes, hee firſt began with a Sphear of a ſmall magnitude; Or rather, becauſe hee could not have ſuch a body exactly made (as I noted formerly) he procured a metalline Cylinder to be made, (which might be more exactly done) of equall *D*iameter and altitude; and the ſame to be but two inches from the *R*oman Foot; and thereby he obtained the gravity of a Sphear, of the ſame metal, whoſe Diameter was equal with the *D*iameter, and altitude of the Cylinder, &c. and from this he deduceth the gravity of any other *S*phear of the ſame Metal, of what magnitude ſoever, as I formerly ſhewed. And ſo likewiſe have I ſeen our Countrey-man Mr. *Reynolds* beforenamed, in making the like experiments, for finding the proportions of gravity, &c. in Metals, perform the ſame by Bodies of one Inch cubique only, holding it a ſurer way, ſo to obtain the weight of one inch cubick or ſolid of any Metall (or other ſolid ſubſtance) then by a body of a greater magnitude: Though yet (for mine own part) I conceive there may not be altogether ſo much curioſity or difficulty in this, as in the former; but that the weight of a ſolid body of one inch in magnitude, may as (if not more) nearly be deduced from the weight of a body of a greater magnitude or dimenſion, (of the ſame matter)

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as the weight of a liquid or humid body of one inch in magnitude or bulk, can be deduced from the weight of a liquid body (of the ſame kind) of a greater bulk or craſſitude; by how much, the weight of any ſolid body in generall, may be had more exactly then the weight of any liquid body; ſith that a ſolid body is weighed immediatly by it ſelf in the Ballance or Scale, without the help of any other thing to contain it; whereas a liquid body (eſpecially one ſo thin and fluid as water is) cannot wel be weighed by it ſelf (unleſſe it be in a ſmal quantity) but by means of ſome Veſſel to hold it, whoſe weight muſt alſo be had firſt or laſt by it ſelf (though indeed it is not abſolutely materiall or requiſite to take notice of the ſame, as to any certain, known, regular, denominate weights, unleſſe the Veſſel be laſt of all weighed by it ſelf, for otherwiſe any irregular indenominate weights will ſerve to poize it, if it be firſt weighed alone, before it receive the liquid Body whoſe weight is required. *But* this by the way.

Now one inch of water weighing, 0.5277, &c. oun. *Troy*, *viz.* 0.528 oun. *ferè*; the ſame wil be found in *Avoirdupois*-weight, to be 0.5795 oun. (which are in libral or pound-weight, 0.04398 *li.* troy, and 0.03622 *li.* avoirdupoiz) And from the ſame experiment I find, that a body of water of 40 inches, will weigh 21 oun. *troy* almoſt exactly, and 23 oun. *avoirdupois* almoſt as nearly; and that 36 inches of water wil weigh 19 oun. *troy* exactly, but which wil be in avoirdupois-weight, $20\frac{44}{51}$ oun. or 20.86, &c. And ſo contrariwiſe, arguing from the gravity or ponderall quantity of water, to its magnitude or dimenſionall quantity; the ſolid meaſure of one ounce-*troy*, will be (in

The dimenſionall and ponderall quantity of water compared together ſeverall ways.

unciall or inch-meaſure) 1.8947 inch; and of one ounce-*avoirdupois*, 1.72556 inch; (and conſequently in Pedall or Foot-meaſure, 0.001096, for Troy-weight; and 0.00099859 for Avoirdupois-weight.) And ſo the ſolid meaſure of one pound-troy of water, wil be, *Unciall*, 22.7368; and of one pound-avoirdupois, wil be *Unciall*, 27.609: and the ſame wil thereupon, be *Pedall*, 0.013158 *fere*, for troy-weight, and 0.015977, for avoirdupois-weight. And ſo from our foreſaid experiment made by the large cubicall Veſſel, which was an *Octant* of a *Foot* cubique; the weight of a Body of water of a juſt *Foot* in magnitude, (or the weight of a cube-Foot of water) wil be eaſily, 912 oun. Troy, and which is juſt 76*li*. Troy: And this wil be in Avoirdupois-weight $1001\frac{21}{31}$ oun. or 1001.41176 oun. and ſo $62\frac{10}{17}$*li*. or 62.588*li*.

But indeed, of all theſe termes of proportion or compariſon, between the gravity and magnitude, or the ponderall and dimenſionall quantity of water, thoſe of 21 ounces, to 40 inches (in reſpect of Troy-weight, which is here the beſt) are the fitteſt for a generall uſe: for that I find them to anſwer indifferently to all the ſeverall experiments that have been made by me, and Mr. *Reynolds* ſeverally, for the foreſaid purpoſe; which ſaid ſeverall experiments, though in the weight of one Inch of water, and ſo in the ſolid meaſure of one Ounce, they may make ſome ſmall difference, ſo as ſomwhat to alter the decimal numbers, by wch the ſame are expreſſed (the ſaid numbers being produced or extended beyond two or three figures in the fraction; and in which all our beſt experiments do concurre, *viz*. as to make the weight of one cubick inch of water, 0.527 oun. troy, or very near thereabout; and ſo the ſolid meaſure of one ounce-troy of water, about 1.895 cube-inch) and

ſo

ſo to make ſome litle differẽce in the operations performed thereby, eſpecially the greater that the quantity of water uſed, is; yet they all agree in this, as to give to 40 inches of water the weight of 21 oun. troy, (and ſo contrarily, to 21 oun. troy of water, the ſolid meaſure of 40 inches,) without any conſiderable difference: And then beſides that theſe two numbers thus anſwering each other reciprocally, they may alſo more eaſily be borne in memory, then the decimall numbers, which denote the gravity of one inch of water, and the magnitude of one ounce of water: And the like wil be for 23 ounces to 40 inches, where the operation is performed by Avoirdupoiz-weights, for want of Troy-weights. But yet again after all this, conſidering the foreſaid experiment of the large cubick Veſſel, made by me, was as exact, (I may be bold to ſay) as wel may be, or as need be deſired; then the foregoing quantities of 19 oun. troy of water to 36 inches of the ſame, (which are thoſe of 114 oun. and 216 inches, in the leaſt Terms) may be generally uſed, (being as eaſily born in memory as thoſe of 40 inches and 21 oun.) to expreſſe the Proportion between the gravity and magnitude, or the ponderall and dimenſionall quantity of water; and ſo for the producing or diſcovering of the ſolid meaſure of any irregular body, as I ſhal next of all ſhew.

Having therefore thus diſcovered the weight of water in generall, in relation to its ſolid meaſure, &c. (and that as nearly we conceive, as may be, or need to be) or the proportion between the gravity or ponderall quantity, and the magnitude or dimenſionall quantity thereof, and the ſame laid down here in all the fitteſt terms that may be, for a generall and ſpeedy uſe; we may thereby be able eaſily and exactly to diſcover the ſolid content of any

How to find out the ſolid capacity of any irregular Body, by the weight of water.

ſolid Body whatſoever, how inordinate or irregular ſoever it be; by conferring it with the foreſaid common liquid Body, according to magnitude; and that is, by finding out or diſcovering ſuch a body or quantity of water, as is equal in bulk, magnitude or craſſitude, wᵗʰ the ſolid body, whoſe meaſure is required: Which water being exactly weighed, and then the dimenſionall quantity or ſolid meaſure thereof, deduced from its weight, by means of the foregoing terms of proportion for that purpoſe: the ſame ſhall be alſo the ſolid meaſure of the ſolid body required. And this may be performed by any irregular kind of Veſſel whatſoever, by filling the ſame exactly with fair water, and then ſinking in the ſolid body, ſo low at leaſt, as wholly to cover it: for ſo the water forced out of the Veſſel by the ſolid body, ſhal be equall in bulk or magnitude thereunto; the ſolid body now taking up the ſame room or ſpace in the Veſſel, which that did. And the quantity of water ſo forced out of the *V*eſſel, may be found, either by firſt weighing the *V*eſſel full of water, and then the *V*eſſell with the remainer of the water left therein, after the taking out of the ſolid body, (for here the weight of the *V*eſſell alone need not be had) whoſe difference of weight ſhal be the weight of the water effluxed: or elſe (which is much eaſier and exacter, eſpecially if the *V*eſſel be ſo great, as that it cannot conveniently be weighed) by making a ſmall hole near the top of the *V*eſſel, or a nick or notch in the very edge, brim, or top thereof; and ſo filling the *V*eſſel exactly

How to finde the exact quantity of water (or other liquid body) which is equall in magnitude or dimenſion to any ſolid Body given.

exactly up to the ſaid hole or notch, through which the water forced out by the ſolid body may run, and ſo be received into ſome other ſmaller *Veſſel*; which may be done exactly to a drop, if there be a quill or ſome ſuch like thing as a pipe or ſpout, cloſely faſtened in the ſaid hole or notch, whereby to convey the water clearly away into the other *Veſſel*, without ſpilling any; for ſo the weight of this water (which is equall in magnitude to the ſolid body) being taken; and then the ſolid dimenſion thereof produced thereby, (according to ſome of the foreſaid proportional Concluſions for this purpoſe) the ſame ſhal be the ſolidity of the body, given to be meaſured. And herein, it maketh no matter, whether the ſolid body be ſunk into the *Veſſel* of water, any lower then juſt to cover it, or not; for it comes all to one paſſe, ſeing that it ſtill takes up but the ſame room or place in the water; and ſo conſequently cauſeth but the ſame quantity of water to flow out. And ſo may the ſolid meaſure of any part aſſigned, of any ſolid body be had, by ſinking that part only into the *Veſſel* of water, and then obſerving the quantity of the water effluxed thereby; for this ſhal be of equall bulk or magnitude with that part of the ſolid body; and therefore the ſolid content thereof being found out by its weight (as before) the ſame ſhal be the ſolid content of the ſaid aſſigned part of the ſolid body, as was required.

And thus alſo may the capacity of any irregular concave Body or Veſſel whatſoever, be diſcovered in ſolid meaſure, by the weight of the water exactly filling the ſame (or the ſolid meaſure of any part thereof aſſigned, by the weight of the water exactly filling that part) if the bigneſſe thereof hinder not the convenient weighing of it; acccording as I formerly ſhewed for diſcovering of the concave capacities

cities of the City-Standard-Gallons for Wine and Ale or Beer, in ſolid inch meaſure; and both which Veſſels were of an irregular forme, eſpecially the Wine-Gallon, whoſe dimenſions could not be rightly taken by a Line of meaſure, as I then noted.

How, to find the gravity of the Water (or other liquid body) which is equall in magnitude to a ſolid body given, by the gravity of the ſolid body only. And ſo the manner of weighing a ſolid body in Water.

Or again, the weight of the water equall in magnitude to any irregular ſolid body, whoſe meaſure is deſired, may be had by the weight of the ſolid body only; the ſame being firſt exactly weighed in the Ballance or Scale, after the uſuall manner; and afterwards out of the Scale, in water; which is (according as * *Ghetaldus* teacheth) by hanging the ſolid body by one of the Scales of the Ballance, into the water, ſo as it may hang freely therein, being covered therewith, and that by an horſe-haire, (which he ſaith to be, in a manner, of equall weight with water, and ſo neither addeth to, nor taketh from the gravity in the Body to be weighed) or many ſuch haires joyned together, if the gravity of the Body require them; but with due conſideration had of ſuch part thereof (in reſpect of weight) as hangs out of the water, that is (as he ſaith) as hangs out of the Scale to which the Body is faſtened, even to the Body it ſelfe; by putting the like quantity of haire in the other Scale where the weights muſt be put; leſt that the many haires together, ſhould add ſome weight to the Body, (and ſo neglecting that part of the haire which is wound about the body, or with which the

* *Archimed. promot.* poſt exempl. 1 prob. ſeu 8 prop.

the ſame is bound up, becauſe that is ſuppoſed to be equally weighty with the water, as being with the Body in the water, and ſo to have no gravity therein) for ſo the Scales ſhall become equiponderant; and then the Body hanging freely in the water (and ſo as neither of the Scales touch the water) the ſame ſhall be ponderated or poiſed, as if it hung in the aire; and will be in the water ſo much lighter then it is in the aire, as is the gravity of that portion of water, which is equall in magnitude or bulke to the ſolid body: and which *Archimedes* demonſtrateth, and from him *Ghetaldus* in the forecited *prop.* of his *Archimed. promot.* And therefore the difference betweene the gravity of the ſolid body taken in the aire, (which is the true gravity thereof) and the gravity in the water, ſhall be the gravity of ſo much water, as is equall in magnitude to the ſolid body. But this muſt be underſtood of ſuch a ſolid body or Magnitude, as is heavier then water; that is, being put therein, will ſinke, or be carried down into the ſame, of its own accord: Whereas elſe, if it be a ſolid body which is lighter then the water, that is, being put into the ſame, will not ſink, or deſcend down of it ſelf; then the gravity of ſuch a portion of the water, as is equall in magnitude to this ſolid body, muſt (in this way) be had by the adjection or appoſition of ſome other ſolid body, which is weightier then water, (whoſe gravity both in the aire and water are firſt had, as alſo the gravity of the lighter body in the aire) ſo as theſe two ſeveral Solids being joyn'd together, may make up (as it were) one Body; wch being let down into the water, may voluntarily ſink or deſcend

Archimed. lib. 1. περὶ τῶν ὀχουμένων (ἐφ' ὑγρῷ) ſeu de iis quæ inſident aut vehuntur in humido, ſive liquido, (Vel ἐφ' ὕδατος, in aqua, ut vulgò dicitur) prop. 7.

ſcend: And both which Solids thus put together in one, wil be lighter in the water, then the heavier *Solid* alone, (as *Ghetaldus* ſheweth in the fore-named place) And wil be alſo lighter therein, then they are in the air, (both their true weights being taken together) by ſo much, as is the gravity of the water equall in magnitude to them both, (according to that of a ſolid body alone which is heavier then water) And this ſhall exceed the gravity of the water equall in magnitude to the lighter body alone, by ſo much, as is the gravity of the water equall in magnitude to the heavier body; and therefore the gravity of the water equall in magnitude to the lighter body (which was required) is thereby immediatly obtained. And what hath been here ſpoken concerning *Water*, the like underſtand of any other liquid or humid body, in which a ſolid body may be thus weighed. And indeed this latter way for the finding of the gravity of the water (or other liquid body) wch is equal in magnitude to a ſolid body given, by the gravity of the ſolid body only (& therby to diſcover the quãtity or capacity of the ſolid body, according to a certain meaſure appointed, as before; which is in my judgment, the moſt excellent and beneficiall uſe that can be made of it) is much more neat, then the former, but withall much more curious and difficult in the performance; and therefore I ſhal rather refer the Practicioner to the firſt way, as being very plain, eaſie, and exact.

But now, that I might be found not more wanting in the practice and experience of theſe things, then of the other beforegoing (for with the bare ſpeculation of things I could never yet well content my ſelfe, though they were never ſo likely in themſelves, (as many men do, and ſo take up all things

things upon truſt) I wil here produce a manifeſt experiment made by my ſelf, of both theſe waies, under one, in reference to the diſcovering of the dimenſionall quantity of a ſolid body, by ſuch a body of water as is equall in magnitude thereto, whereby the verity of all theſe mathematicall Concluſions may plainly appear. And therefore to this end and purpoſe, I ſought to get ſome ſolid body of Metall or Stone, of a reaſonable bigneſſe, which might not ſuck or receive water into it, and be alſo wrought into ſome ſuch regular form, as that the ſolid content thereof might be exactly obtained by a Line of meaſure, according to the plain and uſuall way of meaſuring, and alſo ſuch an one, as might eaſily be hanged in a ſtring, without any fear of ſlipping out when it was in the water; as ſome Priſmall or Cylindricall body, or the like; but which (though with much ſeeking up and down) I could not meet with ready to my hand, nor otherwiſe eaſily obtain: And ſo after all this adoe, I could procure none that might any way ſerve the turn, but one which was of a ſphericall form, and that was a Marble-bullet, made as round and ſmooth as might wel be imagined, (and which therefore was the moſt troubleſome for the performance of this experiment, that could be, being the moſt difficultly hung in a ſtring, of any other kind of body) whoſe diameter I therefore firſt took moſt exactly by a Callaper Compaſſe, and applying the ſame to a Diagonal Scale of an *Inch*, I found it to be 4.95 inches, and from thence the ſolid Content, 63.5 inches: which being done; I immediatly carried the Bullet to *Goldſmiths-hall* (where I was to make my experiment) and there got the ſame exactly weighed in the Scale; and ſo found the

A double experiment upon the foregoing Concluſions. *Apr.* 12. 1649.

weight thereof, 89 oun. 2 p. w. (which are *7li.* 5. 1 oun. troy) Then for the diſcovering of the weight thereof in water, as alſo for the performing of the other Concluſion, for the finding out of the quantity of water equall in magnitude thereunto, both under one: I got preſently an earthen Veſſell, very broad and deepe, which held about a good Paile-full and an half, or more; cauſing firſt a ſmall hole to be drilled through it neer the top, and then a pipe of Tinne to be fitted thereto, and faſtened therein ſo cloſe, as that no water might paſſe through the hole beſide the pipe; and then I next prepared ſeverall ſmall lines or ſtrings of horſe-taile haires twiſted together, with which I girted the Bullet ſeveral times about, till it would hang firmly, and not ſlip out (and which I found very tedious and troubleſome to doe, by reaſon of the ſmoothneſſe and weightineſſe of the body, and the ſlipperineſſe of the horſe-haire together) continuing the ſaid ſtrings or lines out together from the bullet to a convenient length, and tying them together, whereby the bullet might hang down in the water from the Ballance: which done; I faſtened the ſaid ſtrings (not to one of the Scales, as *Ghetaldus* ſeemes to intimate, but) to one end of the Beame, where the Scale is hung, which I conceived to be the beſt way; eſpecially conſidering the weightineſſe of the Body: And then becauſe in that reſpect, it required a ſtrong Beame whereon to hang, and that the Scales belonging to the great Beame would be troubleſome, and an impediment in the way, in reſpect of their largeneſſe: therefore I, and the Aſſaymaſter of *Gold-ſmiths-hall* together, conceived it beſt to put in their place a ſmall paire of Scales, which might be ſufficient to hold the Weights; and ſo the Body might hang freely down by the Scale without interruption; which being

being done; I ſet the foreſaid earthen *Veſſel* under that end of the Beame on which the Body hung, and then filled the ſame with faire water exactly up to the hole therein, till the water entred the pipe, (letting it run a few drops till it ſtaied of it ſelfe, for ſo I was ſure of the exact filling thereof to a due height) under which I then ſet a glaſſe-viall with a little Funnell in it, which might receive the water forced out of the Veſſell by the bullet, to a drop; and then ſinking the ſaid Body gently into the water, (affording it ſo much ſtring therein, as might keep it fully covered all the time) I had the *Weights* put into the other Scale, together with neer about ſuch a quantity of horſe-haire, as was contained in the ſeveral ſmall ſtrings joyned together, by which the bullet hung (neglecting that part thereof which was wound or wrapped about the bullet in the water, for the reaſon before alledged) and ſo the bullet hanging freely in the water, we found it to weigh 55 ounces and 3 quarters, Troy (which is 4 *li.* and 7. 15 oun.) and thus I let it hang in the water duly poiſed, till all the water was run out into the Glaſſe, which it could make to flow out; for this water thus effluxed, was of equal magnitude with the ſaid Bullet. And this being finiſhed, I repeated one part of my experiment the next day, by filling up the *Veſſel* againe to the hole thereof exactly as before; & then ſetting another Glaſſe with the Funnell in it, under the pipe, I ſunk the ſaid bullet into the water, letting it abide therein, till it had forced out all the water that it could, to the laſt drop (not one drop running beſide the pipe) & then afterwards weighing the ſaid ſeveral Glaſſes of water effluxed out of the *Veſſel* by meanes of the bullet, I found them to differ inſenſibly; the weight of the water alone

being 33 ounces, and 8 p.w. or 33.4 oun. troy, (which nearly agrees with our experiment made upon the braſſe Cubicall Veſſel of 64 inches aforeſaid, whoſe water we found, to weigh 33.5 oun. troy) Now the difference between the weight of the Bullet in the air, or the Scale, 89.1 oun. (or 89 oun. 2 p.w) and its weight in the water, 55.75 oun. (or 55 oun. and 15 p.w.) is 33.35 oun. (or 33 oun. 7 p.w.) for the weight of the water equall in magnitude with the bullet; which you ſee differs (defectively) from the weight of the water forced out of the Veſſel by the bullet, only 1 p.w. which is not conſiderable: And yet had I been ſo curious in weighing the bullet in water, as to have put in the Scale where the weights were, the ſame quantity of horſe-hair, as precisely to an hair, as were contained in the lines or ſtrings by which the bullet hung, except thoſe which were about it in the water (according as *Ghetaldus* directeth) then haply might the weight of the water equall in magnitude with the bullet, produced thereby, have exactly agreed with the weight of that which was forced out of the Veſſel by the bullet; but indeed I think there wanted about ſo much hair in the weight-Scale as might have made the bullet to weigh in the water, 1 p.w. leſſe: But I conceived there was no need of ſo great curioſity in ſuch an experiment as that was, eſpecially upon ſo great a body; neither do I conceive it abſolutely neceſſary & requiſite to perform the ſame by horſe-hair, but that ſilk or thread might ſerve the turn, putting ſo much in the *Scale* with the weights, as is uſed about the body to be weighed, and this can breed no ſenſible errour, unleſſe it be for the performing of ſome very nice and curious experiment indeed; as to find the exact difference and proportion between the weight of a Metall (as Gold or *Silver* eſpecially)

or

other thing, in the aire, and in the water, and the like; then indeed to uſe horſe-haire, and in that ſtrictneſſe and preciſeneſs, as *Ghetaldus* ſpeakes of, will be altogether requiſite, for the reaſon before delivered: And now you ſee how theſe two experimentall Concluſions do manifeſtly confirm one another, & ſo both of them do confirm our former experiments for the weight of water in relation to its ſolid meaſure, (and ſo for the ſolid meaſure thereof in reference to its weight) for thereby the weight of 63.5 inches of water, (the ſame as the ſolidity of the Marble-Sphear aforeſaid) wil be upon the point of 33.5 ounces troy: And ſo (to come to the point in hand, which is the diſcovering of the ſolid capacity of this ſpherical body in inch-meaſure, by the weight of the water which is of equal magnitude with the ſame) the ſolid meaſure of 33.4 oun. troy of water (the quantity of water agreeing exactly in magnitude with the Bullet) wil be thereby 63.3 inches, for the ſolid content of the bullet, which wanteth of the ſolidity found at firſt by the *Diameter* 63.5 inches, only $\frac{1}{5}$ of an inch. And indeed the ſolid Content thus found by water, I may adjudge to be the truer meaſure, in regard there was a ſmal flatting in one place of the bullet, which might well make the ſolid content thereof leſſer by ſo much then it was, being taken according to a full rotundity or ſphericalneſſe every way, without any flatting in the ſame, and according to which the *Diameter* was

Ghetaldus in *Archimed. promot. poſt prop.* 19. ſpeaking of his new artifice, whereby he would find out the quality of Gold from the gravity only which it hath in the air, and in the water; ſaith, that the weight of pure Gold, which is in the air 19, will be in the water 18. And the weight of Silver, which in the air is 31, will be in the water 28: And ſo the weight of air, which is in the air 9, wil be in the water 8.

taken. And therefore in any ſolid body, of how regular a form ſo ever it be, where there is any ſuch flatting, dent, or hollowiſhneſſe, or other like defection in any part of the ſuperficies thereof, which may diminiſh ſomewhat of the true ſolid dimenſion which it naturally obtaines and ought to have, according to ſuch a forme (and which yet will be fully deduced from its linear Dimenſions, according to a plain way of meaſuring, as if there were no ſuch defecti-on) there the ſolid Content (in the ſtate the body then is) wil be moſt truly diſcovered by water (as I have here ſhewed two ſeveral waies) for that the water wil ſearch out the true quantity of thoſe defections, which a *Line* of meaſure cannot, and ſo give the ſolid content of the Body accordingly.

And this way for diſcovering the meaſure of any irregular Solid, muſt needs be generally better then that which Mr. *Diggs* hath delivered in his * *Stereometry*; to be performed by an exact Cubical, or other rectangular priſmal or parallelepipedal *Veſſel*, with water; and that in the uſual way of meaſuring; by taking the *Dimen*ſions of the *Veſſel*, as to the finding of the ſuperficiall content of it's rectangle Baſe, taken according to the concavity thereof; and then noting the aſcent of the water, in the Veſſel, when the ſolid body lies covered therein, and alſo when it is out, whoſe difference being infolded with the ſuperficicies of the water (or of the foreſaid rectangle Baſe of the Veſſel) gives the ſolid meaſure of the water equal in bulk or magnitude with the ſaid Body, and ſo withall of the Body it ſelf; and which way *Clavius* alſo ſpeaketh of, as being the way vulgarly noted and uſed by

* *Pantomet. lib. 3. cap. 14.*

Clav. Geom. pract. l. 5. c. 11.

by Artiſts: whereas the way delivered here by us, may be performed by any kind of *Veſſel* whatſoever, how irregular ſoever it be (as was the *Veſſel* by which I performed my foregoing experiment upon the Marble-Sphear or Bullet, for the diſcovering of its ſolid meaſure by water) and not only for the diſcovering of the Content of any irregular ſolid or groſſe body, but alſo of any irregular concave body, according to a ſolid dimenſion, as I have both ſaid and alſo experimentally ſhewed before. But however that common way delivered by Mr. *Diggs* and alſo by *Clavius* and ſome other *Latin* Authours, need not to be tied or confined wholly to ſuch a kind of *Veſſel* as they ſpeak of, but the ſame may be as wel perform'd by any other kind of priſmal, or any Cylindrical *Veſſel*, ſuch as a Pail or other circular *Veſſel*, being exactly made: but that indeed the Baſes of theſe Veſſels, wil not be altogether ſo readily obtained as thoſe of an exact cubicall, or other rectangular Veſſel, unleſſe it be by our artificiall way of meaſuring, where the baſe of the *Veſſel* is equilateral, or exactly circular. And of this way I would have produced an experimentall example, aſwell as of the other, and that by the ſame regular body, to have compared them together, if I could have met with ever a *Veſſel* fit for the purpoſe: but however the thing being ſo very plain and perſpicuous of it ſelf, there needs no example either from experiment or otherwiſe, to illuſtrate it.

Objection. concerning the difference of gravity in Water, in reference to the foregoing Dimenſion of Bodies.

But now whereas after all this, it may be objected, that all water weigheth not alike; but that different kinds of water (as Rain-water, Fountain-water and River-water) are of a different gravity; and therefore our foregoing experimental Concluſi-

ons

ons for the weight of common water in general, in relation to its ſolid meaſure (or for the comparing of its gravity and magnitude, or ponderal and dimenſional quantity together) cannot hold generally true? To this I anſwer; that albeit theſe ſeveral waters *Anſwer.* do uſually differ ſomewhat in gravity, (which I cannot deny, but muſt needs acknowledge, and that not only from natural reaſon it ſelfe, but alſo from my owne experience, which I ſhall now come to ſhew,) yet not ſo much, as to make any notable, or conſiderable difference in the ſolid content of a Body produced ſeverally thereby (eſpecially Rain and River-water) for which end thoſe Concluſions were aymed at, and intended by us; but that any one of them may be indifferently uſed in this thing, without the committing any conſiderable errour, as I ſhal ſhew by and by, in diſcovering the gravities of ſeveral waters and comparing them together. And *Marinus Ghetaldus* in all his ſeveral experiments made upon Water, for the comparing of it and divers other bodies both ſolid and liquid, together, in reſpect of gravity and magnitude, ſpeaketh only of water in generall, without any difference or diſtinction.

Now the water by which Mr. *Reynolds* made his experiments upon his 3 ſeverall parallelepipedall Veſſels of 283$\frac{1}{2}$ inches, formerly mentioned, to find the true weight thereof, was (as he told me) fair ſetled Rain-water; And ſuch was alſo the water, by which I made my experiment upon the cubicall Veſſel of 216 inches, as I noted formerly, it being the moſt ſimple kind of water, and ſo generally the beſt for that purpoſe: for ſo that learned Mathematician *W. Snellius* going about to diſcover the true weight of the *Rhynland-Foot*, in its cubicall capacity (which he wil

have

have to be exactly equall to the old * Roman-Foot) in water, that thereby he might (as it ſeemed to him) tranſmit the meaſure thereof ſo much the more certainly and eaſily to poſterity, (like as many eminent men together had done before at *Rome*, for the determining and eſtabliſhing of the exact meaſure or quantity of the ſaid Roman Foot (as *Ciacconius* out of *Latinus Latinius* his

* *Suel.* in *Eratoſth. Batav. l.* 2. *c.* 2.

And where he pitcheth upon the ſame Roman Foot (by name,) for the trueſt, which Mr. *Greaves* and moſt others do; namely, *Pes Colotianus* aforementioned: but differeth in the magnitude thereof from Mr. *Greaves*; for that he makes it greater then the Engliſh Foot; whereas Mr. *Greaves* makes it leſſer. For he ſaith that the *London-Foot*, which is generally uſed throughout *England*, and whoſe meaſure was taken from the Iron Standard of 3 Feet, kept in the *Guild-hall*, and ſo tranſmitted to him, is

968 parts, ſuch as the *Leyden-Foot*, vulgarly called there the *Rhyn-land*-Foot, (which he makes exactly equall with the foreſaid Roman-Foot) is 1000, *Eratoſth. Bat. l.* 2. *c.* 1. whereas Mr. *Greaves* makes the very ſame meaſure of the Engliſh-Foot to be 1034.13, ſuch as *Pes Colot.* is 1000; and the *Rhyuland*-Foot (or that of *Snellius*) to be 1068.25 of the ſame parts. And ſo this Foot to be 1033 ſuch parts as the ſaid engliſh Foot contains 1000 (and the foreſaid Roman-Foot to be but 967 ſuch parts as was formerly ſhewed) and conſequently the Engliſh Foot will be from hence, 968 ſuch parts as the *Rhinland* Foot is 1000, which agrees exactly with the obſervation of *Snellius*. But indeed *Snellius* takes the dimenſion of *Pes Colot.* chiefly from the bare deſcription and deſineation thereof made by *Pelletier* in his Commentaries upon *Vitruvius*, *Architect. l.* 3. *c.* 3. (who there ſaith, that he found this Foot to agree with that of *Stadius*) preſuming upon that typographicall Trente, which he had received about the contraction of letters and Lines upon the paper, after their impreſſion, by a 60th part. For ſo ſaith he in *Eratoſth. Bat. l.* 2. *c.* 1. before-cited, *Charta uda dum prelo ſubiicitur & typum patitur, ipſa preſ-*

surâ & humore quem anteà imbiberat, non nihil extenditur & seipsâ fit amplior, quæ postmodùm siccata, iterùm contrahitur, & simùl linearum mensuras quas receperat, justò exhibet minores. Pars enim sexagesima typorum & formarum longitudini excusis decedit, quemadmodùm à diligentibus & peritis Typographis sciscitando edoctus sum. Which last must needs be erronious and uncertaine, and cannot hold generally true, as reason it selfe (besides the experience of my selfe and others) may plainly demonstrate, according to what I formerly said concerning this point.

* observations of the Roman Foot reporteth, who was one of the eight that made the experiment together at *Rome*) and so two others, by name, *Lucas Pætus*, and *Villalpandus*, by two other Vessels; conceiving it to be a surer way to discover (or recover) the same, then by haires, grains of Corne, digits, palms, and the like) did above all other waters, chuse Rain-water, because that (saith he) being fallen from the heaven, brings down with it no taincture of any earthy dregs; and moreover for that it seemed to be alike in a manner, in all places; and this hee used after many daies setling, being thereby made very pure and cleare.

See M. *Greaves* his Discourse of the Roman Foot. *pag.* 12. 13.

And then, the water with which I measured the Standard-*Wine* and *Ale-gallon* at the *Guild-hall*, to find the weight thereof, and thereby the solid Content in inches, was from the new *River* of *Ware* (commonly called *Middleton's River*, and which water is in a manner of equall weight with Rain-water, as I shall shew by and by) as also that at *Goldsmiths-hall*, with which I measured my two brasse concave Cubes, the one of 64 inches, and the other of 1 inch, to find the weight thereof: (and the lesse Cube also another time with rain-water) and also by which

which I performed my laſt experiment there, upon the weighing of a ſolid body in water, &c.

As for *Rain-water* compared with *Fountain*, *Spring*, or *Well-water*, in reſpect of gravity; *Snellius* obſerved the proportion to be, as 1000000 to 1007522; and of Rain-water naturall, to the ſame artificiall, or diſtilled, to be as 1000000 to 997065: For that he might have his Rain-water exactly defecated, or cleanſed & purified from all earthy dregs or grownds, it ſeemed good to him (as he ſaith) to uſe for the experiment of the exact weight of a cubicall Foot of water, chiefly that which might again be collected into it ſelf from vapours and exhalations, and ſo betooke himſelf to Chymicall diſtillation, according to that form, which they uſually call by a technicall or artificiall term or expreſſion, *Balneum Mariæ*, *ſeu Maris*, becauſe that is in no wiſe forcible or violent, but very gentle; And then moreover hee uſed pure Fountain-water, to trie what his cubicall Foot might alter in gravity, in theſe three waters: And this experiment he performed by a Cylindricall Veſſel made of braſſe with all the accurateneſſe that might be, having its altitude and Diameter equally

Eſt in æquali mole ratio aquæ pluviæ ad diſtillatam, quemadmodum 1000000 ad 997065 pluviæ autem ad puteanam, ut 1000000 ad 1007522. Eratoſth. Bat. l. 2. c 5.

The laſt of which I find indeed to be ſo, according to the Weights of thoſe two waters ſet down by him, or more compleatly, as 10005000 to 10075216. But the firſt of theſe I find according to his weights of the waters in the Veſſell by which he made the experiment, (and ſo of his cube-Foot of the ſame waters, which I exactly deduced from thence) to be as 1000000 to 997117. being more compleatly, as 10000000 to 9971167.

ſemi-pedall, from whence he deduced the weight of a cube-Foot in Water.

But if we will be ſo very curious concerning the difference of gravity in ſeverall waters; then we may as well queſtion, whether all water of the ſame kind, be of the ſame gravity or not (except Rain and Snow-water, which we conceive to hold alike in all places, without an ſenſible difference) as whether all Fountain, and all River-water do weigh alike, eſpecially the firſt of theſe two, but that it may alter and differ in gravity as well as in other things, according to the different nature and quality of the Earth where it is engendred, and of the veines and paſſages thereof, through which (as it were through Channels) it runs, and ſo the various matter wherewith it is mixed; and more eſpecially, if one water be a meer ſimple water, and another be a minerall (for ſo Naturaliſts do uſually diſtinguiſh waters) for then theſe wil more ſenſibly differ in gravity; and ſo wil ſeverall minerall waters among themſelves, being of a different vature: And indeed moſt of our River-water ſeems to come from Fountaines or Springs; and alſo for that *Ariſtotle* gives to them both, the ſame originall of generation, to wit, from vapours and fumes (or air) in the caves and paſſages of the earth, condenſed and concreted into water by the coldneſſe of the Earth. And therefore ſeing that the generation of fumes and vapours, (and thence of water) under the earth, is continuall; it followeth, that the flowing, and the courſe of Rivers is perpetuall: ſo that a *River* is, as it were, no other thing then the water of a perpetuall Foun-

Ariſtot. l. 1. Meteor. c. 13.

Magir. Phyſiol. peripatet. l. 3. c. 4. Com. & l. 4. c. 7.

Fountain or Spring, continually running on in a great body or bulk.

Keckerm. Syſtem. phyſic. l. 2. c. 15.

But now as for the difference of gravity in waters homogeneall, (or of the like kind and denomination) I ſhall prove the ſame by experiment, from four ſeverall Fountains, or Springs (and partly from two Rivers) in comparing them with Rain-water ſeverall times: and which is as much as (if not much more then) any other hath done in this kind, that I could ever yet hear of: And the waters which I firſt tried by way of ponderall compariſon; were firſt, *Rain-water*, as being the Baſe of the Experiment, in regard of what hath been ſaid thereof before; and ſo to which all the other waters are here compared; and which I received into a clean *Veſſel* as it fell from the Clouds, that ſo I might have it as pure as might be, without the mixture of any earthly matter: And the Fountain-waters which I now uſed, were from the Conduit in *Grayes-Inne* Fields, commonly called by the name of *Lambs-Conduit*, which is a pure Spring of it ſelf; and from the *Standard* in *Cheap-ſide*, which is conveighed under ground by pipes, from a *Spring* at *Padington*, being a *Village* about 3 miles diſtant from *London* to the N-W. (as alſo is the water of the other two Conduits at the two ends of the ſaid Street, from the ſame Spring-head) and yet is received as pure and clear at the ſaid Conduit, as if were taken at the Spring-head it ſelf: And then the River-waters were from the *Thames*, and from the new River of *Ware*, or *Middleton's* River aforeſaid. And with theſe I tried *Snow-water*, (which was pure Snow as it fell, having never touched the ground nor other thing, before it came in the *Veſſel* where it was put, and there diſſolved.) Which other ſeverall Waters afore-

ſaid, I took the pains to ſee always fetched from the right places, thereby to avoid all errours and miſtakes which might happen by truſting to any meſſenger alone; and ſo had them carried to *Goldſmiths hall* (where I was to make this experiment alſo) leting them ſtand there a ſetling for 2 dayes: (though indeed the Fountain-waters being ſo very pure and clear, needed not ſo much ſetling) And the *Veſſel*, by which (as a *Standard* or *Gage*) I firſt tried theſe ſix ſeverall waters, was the ſame by which Mr. *Reynolds* had formerly made the like experiments (though not upon ſo many waters, unleſſe artificiall waters, as diſtilled ſtrong-waters, and wines, & ſome other liquid ſubſtances, as he told me) and that was a glaſſe-Phiall which held almoſt 3 quarters of a Wine-pinte, having the neck thereof done about vvith lead or pevvter, and a top or Coyer of the ſame metall made to ſcrew on, vvhich upon the propoſall of my intention to him concerning this experiment, and his good liking and deſire of the ſame, he courteouſly offered to lend me, and I as courteouſly accepted; for I could not then meet vvith one ſo fit for that purpoſe as that vvas, ſave thoſe vvhich vvere too ſmall; for this vvas at the ſmalleſt. And ſo the 21th. of *March* 164$\frac{8}{9}$, I made my experiment at *Goldſmiths-hall* by the ſaid Glaſſe vvith all the exactneſs that poſſible I could, both in filling the ſaid Glaſſe vvith the ſeverall vvaters one after another as they vvere vveighed (beginning vvith *Rain-water*) ſtill ſcrevving on the ſaid top, to keep in the vvater from falling out, as nearly as I could, & then in the vveighing of them; & thereby I found firſt the Snovv-vvater to vveigh

1 experiment for the gravities of ſeverall waters, and the ſame compared with Rain-water. Mar. 21. 164$\frac{8}{9}$.

2 graines leſſe then the *Rain-water*, and the two *River-waters* to weigh each of them 1 gr. more then Rain-water; and ſo alſo the Standard-water in *Cheap-ſide*, (and thereupon theſe 3 to be of equal weight) & the *Lambs-Conduit*-water to weigh 4 gr. more then Rain-water. But indeed finding this Glaſſe to be very uncertaine for the performing of this ſo nice and curious an experiment, in regard both that as I ſtill ſcrewed on the top or Cover thereof after the filling it, to keep in the water that none might fly out, there ſtill iſſued forth ſome water, and beſides alſo that the mouth of the Glaſſe was ſomewhat too wide, (conſidering the ſmalneſſe of the Glaſſe) for the filling of it with every ſeveral water exactly alike to the leaſt drop as was requiſite to doe in ſo ſmall a Veſſel; ſith that I obſerved, that one ordinary drop of water more or leſſe would alter the weigh full 2 graines more or leſſe; which made me continually to iterate the experiment, by the ſaid Glaſſe upon every ſeveral water, by filling up the Glaſſe againe, and then drying the out-ſide thereof before I put it againe into the Scale. But however being doubtful of the ſame, and that I might make this experiment with all the exactneſſe that might be; I did thereupon for my further ſatisfaction, repair to the Glaſſe-houſe in *Broad-ſtreet*, and there cauſed a Glaſſe-Phiall to be preſently blown before me, which might hold a *Wine-pinte*, as neer as could be gueſſed (for this I conceived would be a convenient bigneſſe, ſo as it being filled with water, might be conveniently weighed in a ſmall Ballance that would turne upon ſome ſmal part of weight, as that was which we had uſed for the other Glaſſe, being a very nimble and ſubtile Ballance, that would turne upon the 4th. or 5th.

part

part of a graine, which was as much as could well be expected from a ballance to weigh a Veſſel of this capacity being filled with water or other liquour,) and was alſo blown round, ſomewhat after the manner of an *Urinall*, but made flat at the bottome, ſo as to ſtand upright, and very ſmooth and even at the top, and with ſo ſmal a mouth or orifice, as that it might be alvvaies filled alike to the leaſt drop; for that, when I came to fill the ſame, not one drop could be ſhaken out, till a little was firſt ſuckt out with a quill, (inſomuch as that one might have carried it full of water in their pocket, with the mouth downwards, and not have ſpilt one drop therein:) and which being thus exactly made for the purpoſe; I weighed the very ſame waters therein, which I had done before by the other Glaſſe, having let them ſtand all the vvhile in their ſeveral Glaſſes, vvith an addition of tvvo other Fountain-vvaters, vvhich in the mean ſpace I had been told of, as being generally accounted the tvvo fineſt vvaters about *London*; the one neer the *Poſtrne-gate* on *Tower-hill*, called the *Poſtern-Spring*; the other on the backſide of St. *Giles-Church* at *Cripple-gate*, called there by the name of *Crowders Well*; both vvhich are commonly accounted exceeding good for all manner of ſoare eyes; and have a very pleſant taſte, like that of nevv-milke, (eſpecially that of * *Crowders-Well*, vvhich much exceeds the other for ſapour and gravity) and vvhich therefore for their vertue and gravity above the reſt, may be taken for mineral. And ſo by

* Upon the weighing of this water, the Aſſay-maſter of *Gold-ſmiths-*

this

2d. Experiment, for the gravities of ſeverall waters &c. *Mar.* 31. 1649.

this nevv *Glaſſe*, and the foreſaid Ballance, I found firſt the *Snow-water* to vveigh one grain lighter then *Rain-water*; as alſo the nevv *River-water* (and ſo theſe two to vveigh alike) and the *Thames-water* not to differ ſenſibly in gravity from Rain-water, and the Conduit-water in *Cheap-ſide* to be 2 gr. heavier then Rain; and and the *Lambs conduit-water* to be 3 gr. heavier then Rain; and then, the *Poſtern-Spring-water*, to weigh 7 gr. more then Rain; and *St. Giles*-water, (or that of *Crowders-Well*) to weigh 12 gr. or halfe a pennny-weight more then Rain-water; and ſo to be weightier thē the *Poſtern-Spring-water* by 5 gr. And ſo as I weighed each water, I continually iterated my experiment upon the ſame, by putting out a few drops, and then filling up the Glaſſe againe; and ſo afterwards ſtill drying the Glaſſe throughly on the out-ſide, I againe committed the ſame to the Ballance, and ſo found the ſeveral waters to weigh as before; ſave only the *Thames water* now weighed one gr. more then Rain-water, which whether it was in the filling of the Glaſſe, or in the weighing, I cannot

Hall, by name, Mr. *Alexander Jackſon* (who was pleaſed with much courteſie, humanity, and patience, to aſſiſt me in theſe and all other the experiments which I there made; as to the work of the Ballance, or the matter of weighing, and which he performed with his own hands, with all the accuratneſſe that might be) told me that he once knew an ancient man in this City, who whenſoever he was ſick, would drink plentifully of this water, and was thereby immediatly made well: and ſo, being overcome with Drink (as he often would be) would preſently drink of this water to make him ſober, as finding it to be the moſt ſpeedy remedy.

cannot juſtly ſay: But ſoon after this it happening to rain, I took ſome pure freſh rain-water, and withall, ſuch other of the aforeſaid Waters as were neer the place of obſervation, freſh againe, *viz.* the Conduit-water in *Cheapſide* and the two foreſaid *River*-waters, and (after due ſetling) tried them over againe by the ſame Glaſſe and Ballance; as alſo the Snow-water which I had uſed before (for freſher I could not then get) and found them all to weigh exactly as at firſt by this Glaſſe; and ſo the *Thames water* to be of equal gravity with Rain-water.

3d. Experiment for the gravities of ſeverall waters. *Apr.* 7th.

And then I having a ſmal Glaſſe ſtanding by me, whoſe mouth was rather leſſe then that of the pinte-glaſſe and held little more then 2 ounces-troy and an halfe; I thought good to make an experiment by the ſame, upon ſome of the foreſaid waters, *viz. Rain* & *Snow-water*, & the two *River-waters:* (becauſe theſe did neerlieſt agree one with another) and thereby found the Snow-water to be halfa grain lighter then Rain-water; and the two *River*-waters, to be each of them of equal weight with the Rain-water as before. And this was performed by a very ſmal and ſubtil Ballance, vvhich vvould eaſily be turned vvith one mite, or the 20th. part of a Grain.

4th. Experiment for the gravities of ſeverall waters. *Apr.* 21.

And thus having diſcovered the difference of gravity in ſeverall waters by ſmaller Veſſels, (or quantities of water) I conceived it very convenient after all this, to make one experiment or obſervation more upon all the foreſaid ſeverall waters, by a much larger quantity; conſidering that theſe ſmaller quantities were not ſo ſufficient to diſcover the

the difference of gravity, as to ground or determine Proportions of gravity thereupon; and ſo that the greater the quantity of the waters was, the greater & more apparent ſtill would be their difference of gravity; and ſo to ſee how this would agree in Proportion with the other: And to this end I got the largeſt vitreall *Veſſel* or *Viall* that I could meet with fit for the purpoſe; which was one that held near about five wine-pintes and an half, having a very ſmall neck, done about with lead or pewter, and a top or Cover to ſcrew on very cloſe (like the Glaſſe by which I made my firſt obſervation in this kind) and which in the ſcrewing on, would not force the leaſt drop of water out of the Glaſſe, being exactly filled; and the mouth of the Glaſſe not being half an inch wide: And ſo having provided all the foreſaid eight waters freſh again (except the Snow-water, which could not be had freſher then that which I uſed before) and the ſame duly ſetled; I found by this Glaſſe (from the great Standard-ballance, which would ſenſibly turne upon one grain) firſt the *Snow-water* to weigh 8 gr. leſſe then *Rain-water*; and the two *River-waters* to be equi-ponderant with the Rain-water: and the Conduit-water in *Cheap-ſide*, to weigh 14 gr. more then Rain; and the *Lambs-conduit-water* to weigh 24 gr. (or 1 p.w.) more then Rain: and the *Poſtern-Spring* water to be heavier then Rain-water, by 2 p.w. and 9 gr. (or 57 gr.) and laſtly, the water of *Crowders-Well*, to be weightier then Rain-water, by $3\frac{1}{2}$ p.w. or 84 gr. and ſo to exceed that of the *Poſtern Spring* in grauity, by 1 p.w. and 3 gr. *viz.* 27 gr.

5th. experiment upon ſeverall waters, in reſpect of gravity. *May* 3d.

So that now from theſe ſeverall exact experiments and

obſervations, it is manifeſt, that all waters homogeneall (or of the ſame kind) are not of the ſame gravity, but do ſenſibly differ therein, as we have here proved from four ſeverall Fountain or Spring-waters, all of them differing in gravity one from another; and that in the very ſame continued order and proportion in a manner, from all the ſeverall experiments by which they were tried: as that of the Conduits in *Cheapſide* (from *Padington-Spring*) to be the lighteſt; and the next above that, the water of *Lambs Conduit* (in *Graies-Inn* fields) and then the next to exceed that, the *Poſtern-Spring* water (on *Tower-hill*) and then the heavieſt of all, the water of *Crowders-Well* at *St. Giles-Cripplegate*; and which thing I was very deſirous to demonſtrate: And therefore conſequently, that no certain, poſitive Proportion can be determined between *Rain* and *Fountain*, or *Pluviall* and *Puteall* water, as *Snellius* hath done; and which I cannot but wonder, that he (ſo intelligent an artiſt) ſhould do, from one ſingle experiment or obſervation only, having uſed but one Fountain-water (by what I can perceive from him) as if he took all Fountain or Puteall waters to be of the ſame gravity. And that which he uſed in that experiment, ſeemeth to be much heavier then the heavieſt of thoſe fountain-waters which we have here made uſe of; as appeareth by his comparing it with Rain-water, the proportion being (as I ſhewed before from him) as 1000000 to 1007522; whereas the proportion of *Rain-water* to the weighteſt of the Fountain-waters which we have here experimented, wil be (by our laſt and largeſt experiment, which is the beſt for that purpoſe) but as 1000000 to 1002104 *ferè*, and from which the Proportion between theſe two waters, deduced from the obſervation made by my new-pint-Glaſſe, wil but

little

little differ, (and that by way of defect) it holding there from Rain-water, to S^t. *Giles-Well-water*, as 1000000 to 1001925 *ferè*, both which by a millenary contraction, wil be in a manner the ſame, *viz.* 1000 to 1002; and by a decumillenary contraction or abbreviation, the one wil be as 10000 to 10019, and the other (and better) as 10000 to 10021. and both which by arithmeticall mediation, wil be as 10000 to 10020; and ſo between 1001925 and 1002104, the intermedian proportion arithmeticall, will be as 1000000 to 1002014.

But indeed, if vve ſhould have yet further tried the ſame by a larger quantity of the vvaters: then probably might the proportion have ſtill riſen higher; and ſo have come ſomevvhat neerer that of *Snellius*, according as vve have ſtill obſerved from our ſeveral experiments; that as the veſſel or quantity of each particular vvater hath been greater, ſo the higher hath riſen the proportion of gravity betvveen them, (though very little) according as the difference of gravity hath become ſomevvhat greater then perhaps it ſhould: For that ſurely, vvhether the ſame be experimented by a leſſer or larger quantity; yet the ſame proportion of gravity ſhould ariſe, according as the the difference of gravity ſhould be proportionably the ſame; and vvhich hath neerly happened in theſe ſeveral obſervations by ſeveral quantities of vvaters, except thoſe of the firſt experiment by the imperfect Glaſſe. And vvhich vvould continually happen, in caſe the gravity of each particular water could ſtill be taken in every ſeveral Veſſel, or Quantity, ſo very preciſely to the leaſt part of vveight, as might be imagined: but vvhich, conſidering that every Ballance (as an Inſtrument or Organ, conſiſting of divers parts) is continually ſubject to mutation up-

on every ſmal occaſion, (eſpecially thoſe ſmaller, nice, nimble, and ſubtil Ballances, ſuch as we uſed in moſt of our experiments of this kind, which by the ordinary breath of one's mouth or noſtril (any thing neer at hand) or the leaſt motion of the aire, or noiſe, or ſound, vvil be ſenſibly diverted from their due courſe and poſiture, to vvhich they tend) and ſo not infallible; therefore the ſame cannot well be expected, though notvvithſtanding vve here continually uſed all the ſedulity and ſollicitude that could be; taking continually the gravity of each particular water, upon an exact equilibrity or equiponderancy of the Scales, according to the moſt preciſe perpendicularity or rectitude of the Index, or Tongue of the Ballance; as neerly, as by the ſight might poſſibly be adjudged; inſomuch as that we could not perceive any ſenſible errour or miſtake to be committed therein. And moreover for a further avoiding of errours herein; I performed each particular experiment upon the ſeverall waters (according as they are ſet down orderly before) ſtil at one time, according to one and the ſame ſetting, or rectifying of the Ballance; conceiving it to be more convenient and ſure ſo to do, then at ſundry times apart, for that there are hardly any *Scales* to be met with, but at ſeverall times, wil require a ſeveral ſetting or rectification.

But for the determining of a certain, poſitive Proportion between *Rain* and any other [illegible], (or between any other two particular waters) it is beſt ſurely to uſe as large a quantity of thoſe waters, as can conveniently be weighed, (conſideration being had of the bigneſſe and ponderouſneſſe of the Veſſel to contain them) But the vitreall Veſſel or Viall, by which I made my laſt obſervation, would

would hold welnigh as much as the æreall Cylindricall Veſſel by which *Snellius* made his obſervation, as I have computed it, by comparing his Weight and Meaſure with ours.

And by moſt of our ſeverall obſervations, we find the two River-waters aforeſaid to be of like gravity with Rain-water, without any ſenſible difference, and ſo to be equiponderant in themſelves. And for Snow-water, we find the ſame by every particular obſervation (even from the leaſt veſſel or quantity of water, where the difference of gravity was leaſt diſcernable) to be ſenſibly lighter then Rain-water, & ſo the lighteſt of all; & in which, our experimentall obſervations wil agree with naturall reaſon it ſelf, which ſheweth *Snow* to be a much lighter ſubſtance then *Rain*; and in which all * Philoſophers do agree; that the Cloud or matter of which Snow is engendred, is more hot and drie then that of which Rain is, and alſo more aery, and ſo the humour or moiſture reſolved out of Snow, is aery, and very light, and as it were a froth or ſome, whereupon it ſo nouriſheth and cheriſheth the Earth: And becauſe it contains much of aire, it behooveth that it ſhould contain alſo much of warmth and moiſture, although yet it is drier then water. And ſo *Ariſtotle* likeneth Snow to Froth or Foame (& alſo calleth it ſo) in reſpect of its whiteneſſe, which he ſaith to ariſe chiefly by means of thoſe parts thereof which

* Zanard. *Diſput. de Univerſo elementari, part. 3. quæſt. 22. Aliam diſpoſitionem habet nubes nivis ab ea quæ eſt aquæ: nam nubes pluviæ habet, quod ſit magis denſa, magis obſcura, & magis unita in unum locum; Nivis autem nubes eſt tenuis & ſubtilis, & cùm multum de aereo continet, eſt clara & quaſi alba, & per aerem ſparſa, &c.*

are

Tenet multum de calido aereo nubes nivoſa &c. Zanard. *ibidèm.*
Ariſtot. *lib.* 2. *de generat. animal. cap.* 2.
Plin. *nat. hiſt. lib.* 17. *cap.* 2. & Keckerm. *Syſt. phyſ. lib.* 6. *cap.* 9. *Theor.* 1. & 2. *de nive.*
Colleg. Conimbr. *in* Ariſtot. *meteor. Tract.* 7. *cap.* 5.
Magir. *phyſiolog. peripat. lib.* 4. *cap.* 6.

are more aery; for that for this reaſon alſo, Froth (ſaith he) is white; and ſo water having oile mingled with it. And ſo alſo *Plinie* calleth Snow *aquarum cœleſtium ſpumam*, which his engliſh Tranſlatour Dr. *Holland* interpreth, the ſome or froth of Rain-water from Heaven, concerning which ſee *Keckerman.* And the *Conimbricenſians* (according to *Ariſtotle, lib.* 1 *Meteor. cap.* 11. and *lib. de mundo, cap.* 4) ſay, that *Snow* is a Cloud congluciated, or frozen together in a friable denſity; and which Cloud obtaines ſo much a greater ſiccity then that which is changed into water, by how much it congeales or grows together by the power or efficacy of the more prevalent Cold; ſith that the Cold while it bindeth, doth expreſſe, or force out the moiſture, &c.

Seeing therefore that the matter of Snow, is more hor, and dry, and more aery thin and ſubtile then that of Rain; and conſequently more light; and that Snow-water is no other then Snow diſſolved, and ſo ſtill retaines the nature of Snow: therefore alſo will Snow-water be neceſſarily lighter then Rain-water.

The Proportion of *Rain*-water to *Snow*-water.

As for the Proportion between theſe two Waters, I may from all our ſeveral obſervations and experiments beforegoing, conclude the ſame to hold generally from the heavier to the lighter, as 10000 to 9998. For

9998. For by 2 or 3 ſeveral obſervations made by the new Glaſſe of almoſt a wine-pinte, where the gravities of theſe two waters (and ſo the difference of gravity) were found ſtill the ſame; the proportion will be as 100000 to 99984 *ferè*: and by the laſt obſervation, being made by the great Glaſſe of about $5\frac{1}{2}$ wine-pintes, holding about 7 pounds-troy of water, it will be as 100000 to 99980 *ferè* (and which I take to be the truer,) and if we will take the intermediate proportion arithmetical, the ſame will be 100000 to 99982 *ferè*. Or the former proportion will neerly happen, if we ſhall mediate between that which will be produced from the leaſt Glaſſe of all, (which was that of about $2\frac{1}{2}$ oun. troy) being as 10000 to 9995 compleatly, and that which we have here produced from the greateſt Glaſſe of all, *viz.* 10000 to 9998 in a manner compleatly; which will be 10000 to 9997.

And now from theſe our ſeverall experiments foregoing, for diſcovering, for diſcovering the gravities of the ſeveral kinds of common, ſimple, or naturall Waters, as in reference to the finding out of the ſolid quantities or capacities of Bodies altogether inordinate or irregular, which wil not in themſelves admit of an ordinary or regular kind of *Dimenſion*, but their contents muſt be obtained by ſome extraordinary or unuſuall kind of way, as we have lately ſhewed; it is manifeſt, that the way here propounded by us for the ſame, may be performed by any ſuch kind of Water in generall, as we have here tried, according to the experiments formerly laid down by us for that purpoſe, from Rain-water, (and ſo upon which this our Areometricall practice is grounded, as being the moſt indifferent water for a generall uſe in this kind:) for that in our laſt obſervation, which was made by the greateſt quantity of

the ſeverall Waters afore-named, weighing about 84 oun. or 7℔. Troy, the difference of gravity between Rain-water and the heavieſt of the Fountain or Spring-waters, was but $\frac{1}{35}$ p.w. or 84 gr. And which ſaid quantity of water is much more then double the quantity of that which was found to be of equall magnitude with the foreſaid ſpherical ſtone-body, and ſo from whoſe gravity we obtained the content of the ſaid ſolid body, in the meaſure propounded; & therefore had the ſame been thus inquired from both thoſe Waters, that is, the lighteſt (except Snow-water) and the heavieſt, (though the Water by which it was done, we have ſhewed to be of like gravity with Rain-water) there could have been no conſiderable difference therein.

The Conclusion.

For the more ſpeedy abſolving or expediting of all the foregoing Dimenſions in generall.

And now for the more eaſie and ſpeedy performance ſtill of all the Dimenſions, and metrical Concluſions contained and mentioned in this Book, I ſhall ſubnect this one thing as a *Coronis* to the whole Work, by way of advertiſement and advice to the practical Reader that is not yet acquainted with the moſt compendious waies of arithmetical Calculation, which is, that he would uſe continually all along with his geometrical Lines, or Lines of meaſure, (whether natural or artificial) that moſt excellent artificial arithmetical Line (as I may ſo terme it) or Line of Numbers (ſo called

called by the authour and contriver thereof, Mr. *Edm. Gunter*, being deduced by him from the Logarithmical or artificial numbers, and to being, as it were, a *Line* or Scale of Logarithmes) by which all Mathematical Queſtions and Concluſions to be wrought by Proportion, (as indeed what are there, which may not be reduced to a way of proportion) are caſt up, or computed in a moſt compendious manner (as it were geometrically or mechanically) meerly by *Scale* and *Compaſſe*, without the labour of the pen: For all thoſe our artificial Dimenſions, which conſiſt meerly upon the ſquaring of ſome one number, or of one ſimple quadrature only, (ſuch as are of all ordinate or regular *Planes* or Superficies, and whether taken ſimply in themſelves alone as Figures, or many of one kind conjunctly, as conſtituting the total Superficieties of regular *Solids*,) will be abſolved by one extent of the Compaſſes only, being doubled, or once repeated upon the ſaid *Line*, according to the Analogiſm of *Multiplication*. And that will be, as from an *Unit* upon the ſaid Line of numbers, to the linear number or term taken by the artificial Line of meaſure, or *Line* of quadrature, (for ſome dimenſional line of a Figure whoſe ſuperficial Content is required therefrom, as repreſenting artificially the ſide of the equal Square) ſo from that, to the quadrate number, for the ſuperficies of the Figure.

See alſo Mr. *Edm. Wingates* Rule or Scale of proportion, of the like kind. And Mr. *Oughtred's* Circles of Proportion; or rather the ſame converted into a Spirall Line.

And all thoſe artificiall Dimenſions which depend meerly upon the cubing of ſome one number or term (ſuch as are of all ordinate or regular *Solids*, for their ſolid meaſure) wil be abſolved by one and the ſame extent of the Com-

paſſes being trebled, or twice repeated upon the ſaid Line of artificiall Numbers, according to the foreſaid Analogiſme (in a compound or double Multiplication) which wil be, as from an *Unit*, to the linear number or term taken by the artificiall Line of meaſure, or Line of Cubature, (for ſome dimenſionall line of a regular *Solid* given to be meaſured thereby, it repreſenting artificially the ſide of the equall Cube) ſo from thence to the *S*quare thereof; and from that to the Cube, for the ſolid content.

And all thoſe artificiall *D*imenſions which conſiſt not of cubing, but yet of a twofold Multiplication; the one whereof is a Quadrature, (ſuch are of all regular-like Solids, as Cylinders, Cones, and the like) wil be abſolved by two ſeverall extents of the Compaſſes, each of them being doubled or once iterated upon the ſaid Line of numbers; which wil be firſt, as from an *Unit* to the quadratary term taken by the artificiall Line of meaſure, repreſenting ſome dimenſionall line of the ordinate Baſe; So from that, to the Quadrat thereof, for the artificiall Baſe: Then, as from an *Unit* to that Quadrat; So from the other number or term taken by the ſame artificiall Line of meaſure, for the *Axis* or Altitude of the ſolid Figure to be meaſured; to the ſolid area of the Figure. Or it wil be, As from an *Unit*, to the number for the *Axis* or altitude; So from the quadrate number for the Baſe, to the rectangular ſolid number, for the whole ſolid Figure it ſelf. And here, if the two foreſaid lines of a regular-like Solid, to wit, the *Axis*, & the baſiall line (whatſoever it be) do happen to be equall (as they very rarely do) then wil the *D*imenſion be cubicall, as that of an exactly ordinate *S*olid by ſome one of its dimenſional lines; and ſo be abſolved upon the Line of Numbers accordingly.

Then

Then laſtly, for the ſuperficiall Dimenſion of theſe regular-like Solids, conſiſting of one Multiplication only, and that commonly of two unequall Terms, and therefore not an exact Quadrature; the ſame wil be abſolved by one extent of the Compaſſes, once repeated upon the ſaid Logarithmeticall Line, or Line of Proportion, according to the Analogiſm of *Multiplication*; which wil be, as from an *Unit* to one of the two linear Numbers or terms taken by the artificiall Line of meaſure (which do repreſent ſome one dimenſionall line of the Baſe, and the ſide, or other line upon the ſuperficies of the ſolid Figure, according to the nature thereof;) So from the other, to the rectangle ſuperficiary Number, for the ſuperficiall Area of the ſolid Figure (the Baſe thereof being ſecluded) And here alſo, if the ſaid two dimenſionall lines of the regular-like *Solid*, do happen at any time to be equall (as they very ſeldome wil) then this *Dimenſion* wil be exactly quadratary, as the ſuperficiall dimenſion of an exact regular *Solid*, by ſome one of its dimenſionall lines.

And the like with theſe, underſtand for the more ſpeedy computing of the gravities, or ponderall quantities of regular and regular-like metalline bodies, being inquired out artificially in the ſame manner as their ſolid meaſures; and therefore, as there the laſt proportionall number or term upon the ſaid Line or Scale of Numbers, denoteth ſolid meaſure, here it wil denote gravity or ponderoſity.

And ſo likewiſe in the work of Gauging, for the ſpeedier computing of the liquid Contents of Veſſels in Wine or Beer (as was of the ſolid content of a Cylinder, or any other regular-like Solid, from our artificiall way) where, after the mean Diameter of the Veſſel being had, according to the artificiall Gauging-Lines; it wil hold upon the

foreſaid *Logarithmical* Line (or Line of Numbers) by a twofold extent or opening of the Cõpaſſes, thus; *viz.* firſt, as from an *Unit* to the mean Diameter, So from that to the Square thereof; Then, as from an *Unit* to that Square; So from the length (or heighth) of the *Veſſel* to the liquid content thereof in Gallon-meaſure. Or as from an *Unit* to the length of the Veſſel, *So* from the Quadrat of the mean or equated *Diameter*, to the liquid meaſure aforeſaid.

And ſo with the like expedition by this Line, underſtand all Atactometrical operations to be abſolved, in computing the ſolid quantities of irregular bodies from the gravity of the equal body of water, according to the Terms of Proportion or compariſon between the gravity and magnitude, or the ponderall and dimenſional quantity of this liquid Body, delivered by us in our Atactometrical Diſcourſe.

And ſo all metrical operations ariſing here frõ the natural *Meaſure*, will be thus expedited, according to the dimenſional Proportions delivered in this Book, both for meaſure and weight (and all others of the like kind, not here particularly expreſſed) according to what I ſaid before. And thus I put a Period to theſe my mathematicall Contemplations and Exercitations.

Soli Deo univerſipotenti, qui omnia (ut loquitur Sapiens) menſurâ, numero, & pondere diſpoſuit, ſit gloria, honos & laus, in ſæcula, & omnem ſempiternitatem. AMEN.

vxo.l.iij.§.j.rem autē quam vedi meretrici r
Sed pignus non pignoraticia:sed rei ven.po
ob tur.cau.l.ij.non in tex.sed in glo.b.quia co
Quilibet alius possessor.ff.ad exhi.de eo
Sau.¶ Casus.Nec tria ca.intelligunt
causa delectatiōis.Di-
citur ergo hic q esau nō
venaret nisi pctōr fuis-
set. Nam piscatorē sanc-
tum inuenimus:sed ve
natorem non.

a ¶ Venator. Dū vena-
tor permanet.ne obici-
atur de eustachio.l.di.
ferrum.

b ¶ Piscatores. Sz q̄re
phibetur venari ⁊ non
piscari:quia forte pisca
tio fit sine clamore:ve-
natio non. vel q̄ maior
est delectatio i venatio-
ne.dum enim quis est in
venatione nihil pōt de
diuinis cogitare.

Venatio
prohibet:
sed non
piscatio.

Quid pdest.¶ Ca-
sus.Dicitur hic q
non prodest abstinentia
ad vitā eternā si q̄s lu-
xuriari in venationib⁹
velit. Ideo dicitur i se
quenti cap.immo quod
plus est non solū de pec
cato:sed de peccatis ser
uorum suorum quos se
cum ducit.

c ¶ Jeiunare. Efficaci-
ter ad salutem.

d Ar putatz.Seruu-
los.nō tñ serui de
bent ei obedire.xj.q.iij.
nō semp:excusantur ta-
men quo ad q̄d. vt.xxij.

Sau ve
nā pctō
nō inuenimus
ctis:sanctum
piscatores in
tē Ambro
quadrageſim
Quid
scribi
natibus:abst
rare peccatis
De
Ar pu
fratre
culo nō ad ec
btō ⁊ martyr
sed surgens c
disponit rhe
saltus siluas
los inq̄ secū
magis ad ec
et voluptatib
cumulat alie
se futuruz ta
de perdition
tē Ambro
Que sint
Quand
On se

amus. postea pctōribus: qb⁹ debem⁹ subuenire in eo q̄ sunt
ines: ⁊ nō in eo q̄ sunt peccatores. qd̄ per sequentia probat.
Prohibemur Aliter debemus.
Donare. ¶ Cus. In istis quattuor ca. s. donare. qui venato
ribus. q̄ venatorib⁹. vidēt. rep̄hendit Augustin⁹ illos q̄ do
nant histrionibus vena
toribus ⁊ meretricib⁹:
ea tm̄ causa vt malā ex-
erceant artem: non attē
dētes in eis naturā dei:
sed nequissimam artem
sed si donarēt eis: scilz
quia homines sunt non
esset peccatum.

prophete. Dispersit dedit
pib⁹. Personaꝝ: vt primū
us: deinde pctōribus quib⁹
nē dare ꝓhibemur°: nō quia
nies sunt: sed quia pctōres.
nde Augustinus super Jo-
iem parte secunda.
Immane pctm̄ est res suas
onare histrionibus.
Donare res suas histrio-
nibus ᵖ vitiuz est imma-
ne nō virtus: et ꝑtis de talib⁹ ᵠ
sit frequens fama cum laude
ia laudatur peccator ʳ in desi
erijs anime sue: et qui iniqua
erit benedicitur.
tem in psalm. ca. ij.
Grauiter delinquunt qui ve
tori aliquid donant.
Qui venatorib⁹ ˢ donant
quare donāt? dicāt mihi
are dāt venatori? hoc in illo
ant in quo nequissimus est.
c in illo pascit: hoc in illo ve
unt: ipsam nequitiā publicāt
ectaculis hominū: q̄ dant hi-
onib⁹: qui donant meretrici
ᵗ quare donāt? nūquid non
hoibus donātur? nō tm̄

p ¶ Histrionibus. Isti
histriones prohibētur
a promotiōe. xxxiij. di.
maritum ⁊ a corpe chri
sti. de consc. dist. ij. pro
dilectione. et infames
sunt. iiij. q. j. c. j. et. ff. de
his qui not. īfa. l. j §. ait
pretor.

q ¶ Talibus. Dare cau-
sa vane glorie pro exer-
citio sui vitij īmane pec
catū est: sed in necessita
te nō. Idem de venato
ribus ⁊ meretricib⁹. in
talibus enī nō vitiū est
fouendū: sed natura su-
stētanda est. Sed quid
de cytharistis ⁊ alijs q̄
varijs vtunt instrumen
tis nunq̄d est licitū au-
dire ea vel tangere vel
talibus ꝑsonis aliquid
dare? ⁊ quidē si ad volu
ptatem vel luxuriā: nō
quia deū manib⁹ stimu
lat qui populū vocibus
delectat. vt. xcij dist. in
sancta. [illegible]

ps. cxj.

Histrio-
nes a pro-
motione
ꝓhibētur

ps. ix.

Dare tur-
pib⁹ an sit
ꝑmissum.

Cythari-
ste et alij
varijs vtē
tes instru-
mētis an
peccent.

Pri

epetere nõ po
ssum. vt. C. de
uenitur tunc
§. heres.
de his qui ve

nator^a erat:
r erat. Et p
in scripturis
aliquẽ venat
uenim⁹ sanc
ius in omeli
1. De eod
dest ieiunare
s: ⁊ luxuriari
nere a cibis ⁊ e
Item.
odem.
atis illũ ieiuna
: qui primo di
esiam vigilat:
sctã loca ꝑqui
gregat seruu
a: canes ꝓdu
perlustrat. se
pertrahit: for
lesiam festinã
⁹ suis peccata
ia: nesciens re
n de suo delicto
e seruorum.
. in lib. de offic
n liberalitate
a.
tis est be